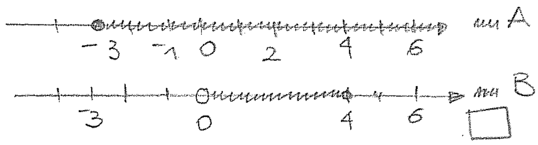
Università di Trento — Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
PRIMA PROVA INTERMEDIA DI ANALISI MATEMATICA
a.a. 2013 – 2014 — Rovereto, 31 ottobre 2013

FILA A

a1) $A = [-3, +\infty[\qquad B =]0, 4]$

$A \setminus B = [-3, 0] \cup]4, +\infty[$.



a2) $A = \{x \in \mathbb{R} : x^2 < 1\} =]-1, 1[$.

(i) F (ii) V (iii) F (iv) V.

a3) "C'è almeno uno studente iscritto nell'a.a. corrente al primo anno del CdL in STPC che non si è iscritto alla PPI di AM o non si è presentato alla prova."

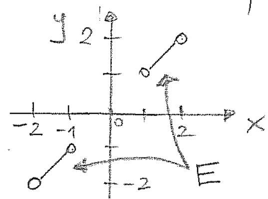
a4) $ax^2 + 2x - 1 = 0$ ha due soluzioni se $a \neq 0$ e

$a \in]-1, +\infty[\setminus \{0\}$. $\quad x_{1/2} = \dfrac{-2 \pm \sqrt{4+4a}}{2a} \iff 4 + 4a > 0$

$a > -1$

a5) $E = \{(x,y) \in \mathbb{R}^2 : 1 < x^2 < 4,\ y = x\} = \{(x,y) \in \mathbb{R}^2 : -2 < x < -1,\ y = x\}$

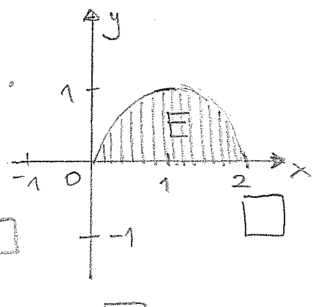
$\cup \{(x,y) \in \mathbb{R}^2 : 1 < x < 2,\ y = x\}$.



a6) r passante per $P = (-1, 2)$, $Q = (-1, 3)$

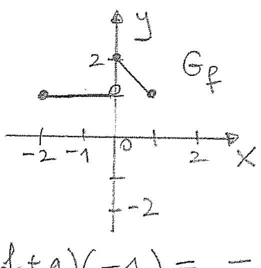
ha eq. $x = -1$.

a7) L'insieme delle coppie $(x,y) \in \mathbb{R}^2 : 0 \leq y \leq -x^2 + 2x$.



a8) $\dfrac{x^2+1}{x+1} \leq 0 \iff x + 1 < 0 \qquad S =]-\infty, -1[$.

a9)



$f([-2, 1]) = [1, 2]$.

a10) $(f+g)(-1) = -1 - 1 = -2$; $\quad \left(\dfrac{f}{g}\right)(1) = \dfrac{3}{1} = 3$.

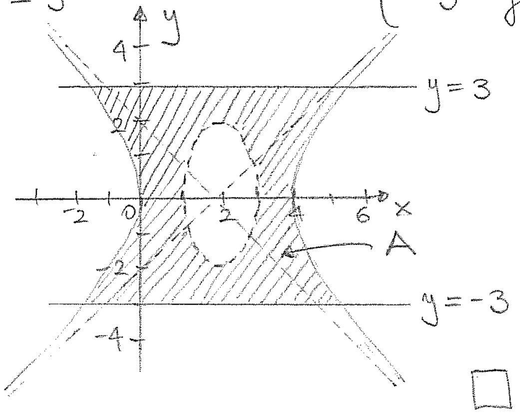
b1) i) non $\left\{ \forall x_1, x_2 \in \mathbb{R},\ \left[(x_1 \neq x_2) \Rightarrow (x_1^2 \neq x_2^2)\right]\right\} \Leftrightarrow$

$\exists x_1, x_2 \in \mathbb{R}:$ non $\left[(x_1 \neq x_2) \Rightarrow (x_1^2 \neq x_2^2)\right] \Leftrightarrow$

$\exists x_1, x_2 \in \mathbb{R}:\ (x_1 \neq x_2)$ e $(x_1^2 = x_2^2)$.

ii) Osserviamo che non A è vera: basta prendere $x_1 = -1 \neq x_2 = 1$ e $x_1^2 = x_2^2$.

b2) i) $\begin{cases} x^2 - y^2 - 4x \leq 0 \\ 4(x-2)^2 + y^2 > 4 \\ y^2 \leq 9 \end{cases} \Leftrightarrow \begin{cases} (x-2)^2 - y^2 \leq 4 \\ (x-2)^2 + \frac{y^2}{4} > 1 \\ -3 \leq y \leq 3 \end{cases} \Leftrightarrow \begin{cases} \frac{(x-2)^2}{4} - \frac{y^2}{4} \leq 1 \\ (x-2)^2 + \frac{y^2}{4} > 1 \\ -3 \leq y \leq 3 \end{cases}$



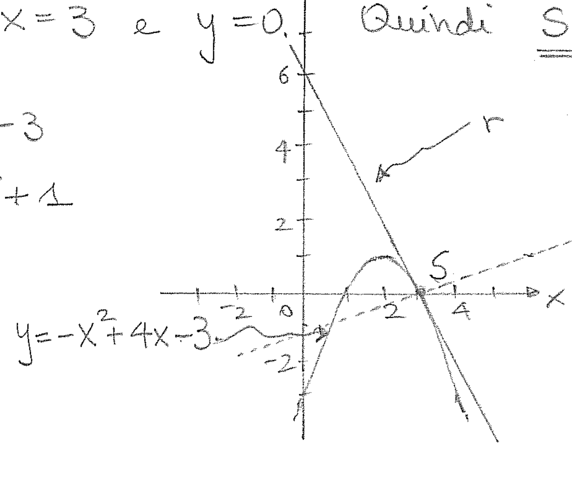
ii) $(-1,0) \notin A$; infatti $(-1,0)$ non soddisfa $x^2 - y^2 - 4x \leq 0$ (poiché $5 \nleq 0$).

iii) $y = k$ con $k < -3$ o $k > 3$.

b3) i) $y = -2x + q$ tocca la parabola P di eq. $y = -x^2 + 4x - 3$ in un solo punto se e solo se $-2x + q = -x^2 + 4x - 3$ ha un'unica soluzione. Si ha $0 = -x^2 + 6x - 3 - q$, da cui $x_{1/2} = \dfrac{-6 \pm \sqrt{36 + 4(-3-q)}}{-2}$; ne segue che $0 = -x^2 + 6x - 3 - q$ ha un'unica soluzione se $36 + 4(-3-q) = 0$, da cui $\underline{q = 6}$.

ii) Si ha $x = 3$ e $y = 0$. Quindi $\underline{S = (3,0)}$.

$y = -x^2 + 4x - 3$
$= -(x-2)^2 + 1$



iii) $r' : y = 0 + \frac{1}{2}(x-3) \Rightarrow \underline{y = \frac{1}{2}x - \frac{3}{2}}$.

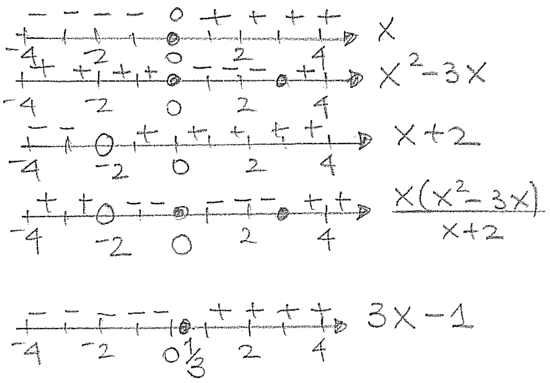
iv) $V=(2,1) \quad S=(3,0) \quad d(V,S)=\sqrt{2}$.

Eq. della circonferenza di centro il vertice e raggio $\sqrt{2}$:

$$\underline{\underline{(x-2)^2+(y-1)^2=2.}}$$

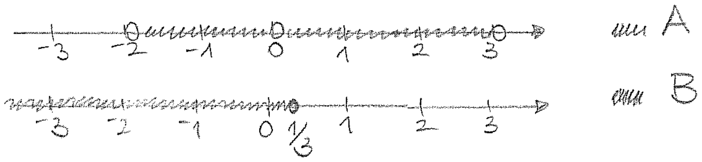
b4) i) $A=\left\{x\in\mathbb{R} : \frac{x(x^2-3x)}{x+2}<0\right\}$

$= \underline{\underline{]-2,3[\ \setminus \{0\}}}$;



$B=\left\{x\in\mathbb{R} : \frac{\cancel{x^2}+3x-\cancel{x^2}-1}{x^2+1}\le 0\right\}$

$= \underline{\underline{]-\infty, 1/3]}}$.

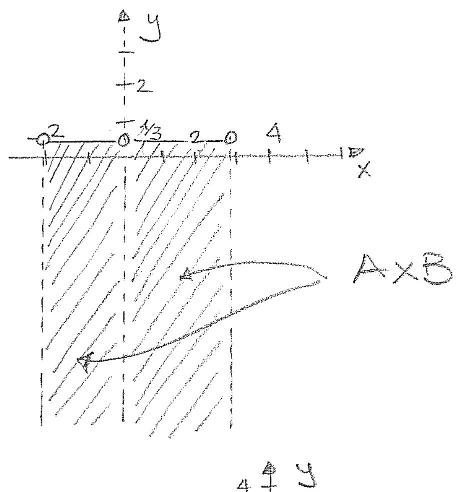


A e B non sono insiemi disgiunti, poiché $A\cap B\neq\emptyset$.

ii) $A\cup B = \underline{\underline{]-\infty,3[}}$; $\mathbb{R}\setminus B = \underline{\underline{]\frac{1}{3},+\infty[}}$.

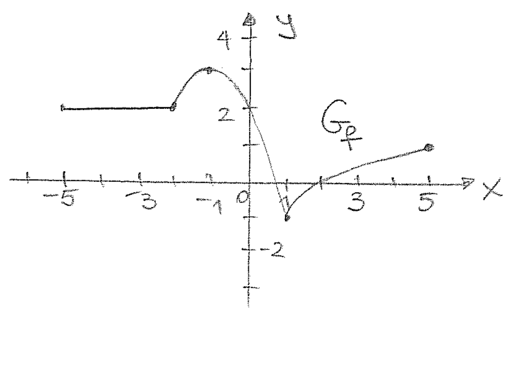
iii) $\underline{\underline{\nexists \min B}}$, $\underline{\underline{\max B = \frac{1}{3}}}$.

iv) $A\times B$



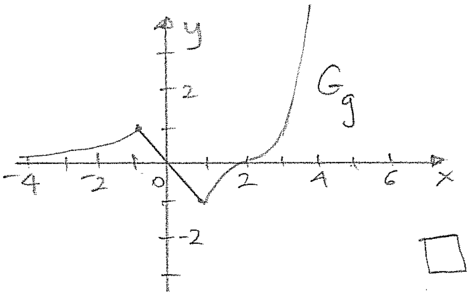
b5) i) $f:[-5,5]\to\mathbb{R}$

$$f(x)=\begin{cases}2 & \text{se } -5\le x\le -2\\ -(x+1)^2+3 & \text{se } -2<x<1\\ \sqrt{x-1}-1 & \text{se } 1\le x\le 5\end{cases}$$



$g : \mathbb{R} \to \mathbb{R}$

$$g(x) = \begin{cases} -1/x & \text{se } x < -1 \\ -x & \text{se } -1 \le x \le 1 \\ (x-2)^3 & \text{se } x > 1 \end{cases}$$



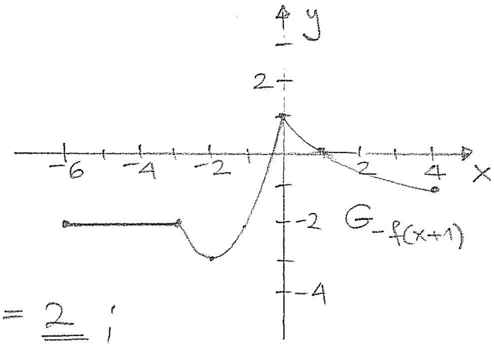
ii) $f([-5,5]) = \underline{\underline{[-1, 3]}}$.

$\min_{[-5,5]} f = \underline{\underline{-1}}$ $\quad x = 1$ pt. di minimo di f su $[-5,5]$.

$\max_{[-5,5]} f = \underline{\underline{3}}$ $\quad x = -1$ pt. di massimo di f su $[-5,5]$.

iii) g non è iniettiva: basta osservare che $x_1 = 0 \neq x_2 = 2$, ma $g(0) = g(2)$.

iv) $[-6, 4] \ni x \longmapsto -f(x+1)$



v) $(f+g)(0) \doteq f(0) + g(0) = 2 + 0 = \underline{\underline{2}}$;

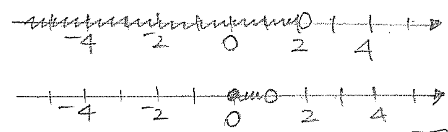
$(fg)(2) \doteq f(2) \cdot g(2) = 0 \cdot 0 = \underline{\underline{0}}$;

$(g \circ f)(1) \doteq g(f(1)) = g(-1) = \underline{\underline{1}}$.

vi) Per $x \in [1,5]$ si ha $-1 \le f(x) \le 1$, quindi $(g \circ f)(x) \doteq$
$g(f(x)) = g(\sqrt{x-1} - 1) = -[\sqrt{x-1} - 1] = \underline{\underline{1 - \sqrt{x-1}}}$.

FILA (B)

a1) $A =]-\infty, 2[\qquad B = [0, 1[$ A



$\underline{\underline{A \setminus B =]-\infty, 0[\cup [1, 2[}}$.

a2) $A = \{x \in \mathbb{R} : x^2 > 1\} = \underline{]-\infty, -1[\cup]1, +\infty[}$.

(i) F (ii) V (iii) V (iv) F.

a3) "Tutti gli studenti iscritti nell'a.a. corrente al primo anno del CdL in STPC si sono iscritti alla PPI di AM e si sono presentati alla prova."

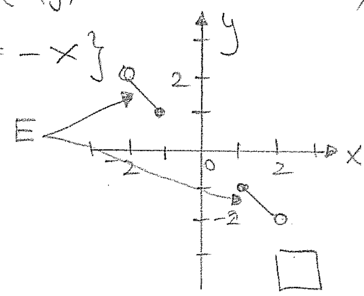
a4) $a x^2 - 3x + 1 = 0$ ha due soluzioni se $a \neq 0$ e

$$x_{1/2} = \frac{3 \pm \sqrt{9 - 4a}}{2a} \iff 9 - 4a > 0$$

$$a < 9/4$$

$\underline{\underline{a \in]-\infty, \tfrac{9}{4}[\setminus \{0\}}}$.

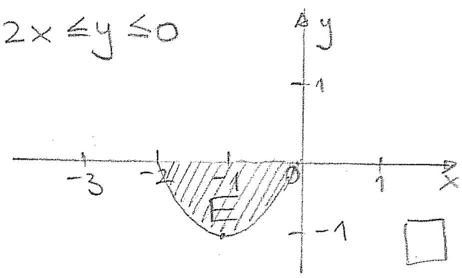
a5) $E = \{(x,y) \in \mathbb{R}^2 : 1 \leq x^2 < 4,\ y = -x\} = \{(x,y) \in \mathbb{R}^2 : -2 < x \leq -1,\ y = -x\} \cup \{(x,y) \in \mathbb{R}^2 : 1 \leq x < 2,\ y = -x\}$



a6) r passante per $P = (-1, 2)$ e $Q = (3, 2)$

ha eq. $\underline{\underline{y = 2}}$.

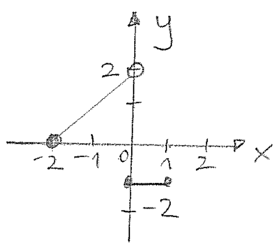
a7) L'insieme delle coppie $(x,y) \in \mathbb{R}^2 : x^2 + 2x \leq y \leq 0$



a8) $\dfrac{x^2+1}{x+2} \geq 0 \iff x + 2 > 0$

$\underline{\underline{S =]-2, +\infty[}}$.

a9)



$f([-2, 1]) = \underline{\underline{\{-1\} \cup [0, 2[}}$.

a10) $(f-g)(-1) = f(-1) - g(-1) = -2 + 1 = \underline{\underline{-1}}$;

$(fg)(1) = f(1)\,g(1) = \underline{\underline{3}}$.

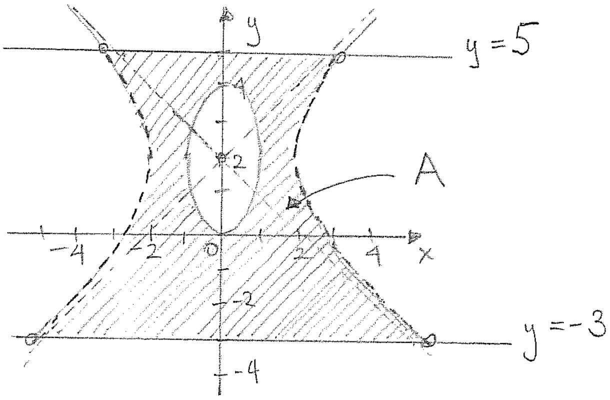
b1) i) non $\{\forall x_1, x_2 \in \mathbb{Z}, \ [(x_1 \neq x_2) \Rightarrow (x_2 - x_1 > 0)]\} \Leftrightarrow$

$\exists x_1, x_2 \in \mathbb{Z}$: non $[(x_1 \neq x_2) \Rightarrow (x_2 - x_1 > 0)] \Leftrightarrow$

$\exists x_1, x_2 \in \mathbb{Z}$: $(x_1 \neq x_2)$ e $(x_2 - x_1 \leq 0)$. ☐

ii) Osserviamo che non A è vera: basta prendere $x_1 = 0 \neq x_2 = -1$ e $x_2 - x_1 = -1 \leq 0$. ■

b2) i) $\begin{cases} x^2 - y^2 + 4y - 8 < 0 \\ 4x^2 + (y-2)^2 \geq 4 \\ -3 \leq y \leq 5 \end{cases} \Leftrightarrow \begin{cases} x^2 - (y-2)^2 < 4 \\ x^2 + \frac{(y-2)^2}{4} \geq 1 \\ -3 \leq y \leq 5 \end{cases} \Leftrightarrow \begin{cases} \frac{x^2}{4} - \frac{(y-2)^2}{4} < 1 \\ x^2 + \frac{(y-2)^2}{4} \geq 1 \\ -3 \leq y \leq 5 \end{cases}$



ii) $(-3, 2) \notin A$; infatti $(-3, 2)$ non soddisfa $x^2 - y^2 + 4y - 8 < 0$ $(9 - 4 \not< 0)$ ☐

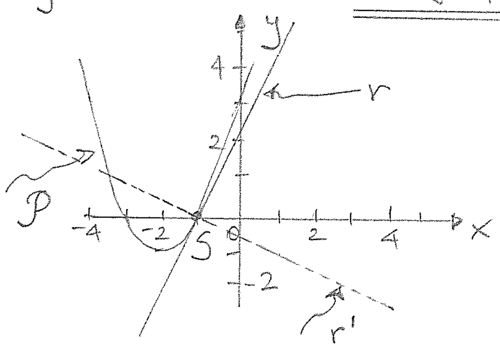
iii) $y = k$ con $k < -3$ o $k > 5$. ■

b3) i) $y = 2x + q$ tocca la parabola $\mathcal{P}$ di eq. $y = x^2 + 4x + 3$ in un solo punto se e solo se $2x + q = x^2 + 4x + 3$ ha un'unica soluzione. Si ha $0 = x^2 + 2x + 3 - q$ da cui

$x_{1/2} = \dfrac{-2 \pm \sqrt{4 - 4(3-q)}}{2}$; ne segue che $0 = x^2 + 2x + 3 - q$ ha un' unica soluzione se $4 - 4(3-q) = 0$, da cui $\underline{\underline{q = 2}}$. ☐

ii) Si ha $x = -1$ e $y = 0$. Quindi $\underline{\underline{S = (-1, 0)}}$

$y = x^2 + 4x + 3$
$= (x+2)^2 - 1$



☐

iii) r' : $y = 0 - \frac{1}{2}(x+1)$ $\Rightarrow$ $\underline{\underline{y = -\frac{1}{2}x - \frac{1}{2}}}$.

iv) $V = (-2,-1)$ $S = (-1,0)$ $d(V,S) = \sqrt{2}$

Eq. della circonferenza di centro il vertice e raggio $\sqrt{2}$:

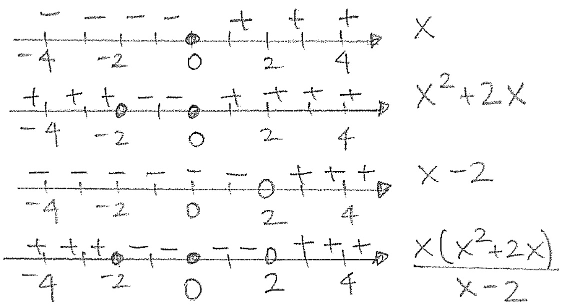
$\underline{\underline{(x+2)^2 + (y+1)^2 = 2}}$.

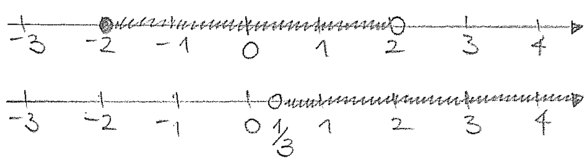
b4) i) $A = \left\{ x \in \mathbb{R} : \frac{x(x^2+2x)}{x-2} \leq 0 \right\}$

$= \underline{\underline{[-2,2[}}$;

$B = \left\{ x \in \mathbb{R} : \frac{\cancel{x^2}+3x-\cancel{x^2}-1}{x^2+1} > 0 \right\}$

$= \underline{\underline{]\frac{1}{3},+\infty[}}$.



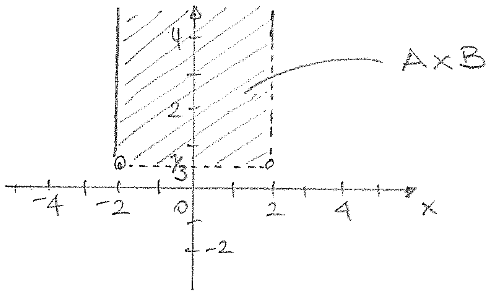


A e B non sono insiemi disgiunti, poiché $A \cap B \neq \emptyset$.

ii) $A \cup B = \underline{\underline{[-2,+\infty[}}$; $\mathbb{R} \setminus A = \underline{\underline{]-\infty,-2[\cup [2,+\infty[}}$.

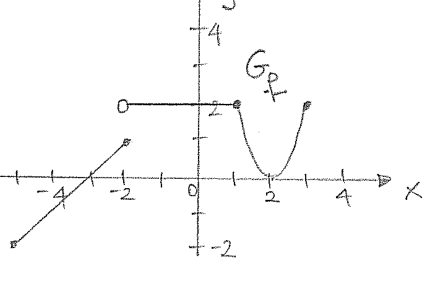
iii) $\underline{\underline{\min A = -2}}$, $\underline{\underline{\nexists \max A}}$.

iv)



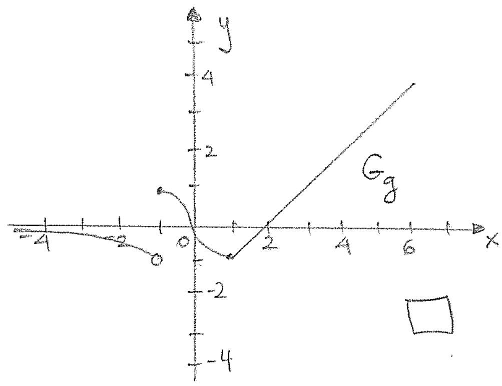
b5) i) $f : [-5,3] \rightarrow \mathbb{R}$

$$f(x) = \begin{cases} x+3 & \text{se } -5 \leq x \leq -2 \\ 2 & \text{se } -2 < x \leq 1 \\ 2(x-2)^2 & \text{se } 1 < x \leq 3 \end{cases}$$



$g: \mathbb{R} \to \mathbb{R}$

$$g(x) = \begin{cases} -\frac{1}{x^2} & \text{se } x < -1 \\ -\sqrt[3]{x} & \text{se } -1 \leq x \leq 1 \\ x - 2 & \text{se } x > 1 \end{cases}$$



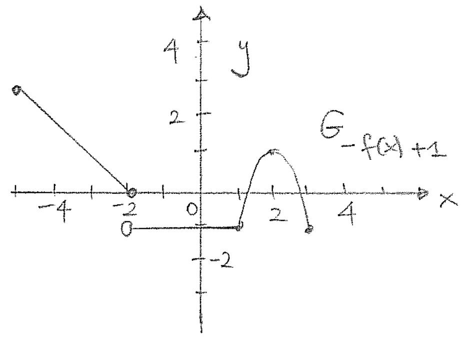
ii) $f([-5,3]) = \underline{\underline{[-2,2]}}$.

$\min_{[-5,3]} f = -2$ $x = -5$ pt. di minimo di f su $[-5,3]$.

$\max_{[-5,3]} f = 2$ $x \in]-2,1] \cup \{3\}$ pt. di massimo di f su $[-5,3]$.

iii) g non è iniettiva: basta osservare che $x_1 = 0 \neq x_2 = 2$, ma $g(0) = g(2)$.

iv) $[-5,3] \ni x \longmapsto -f(x)+1$

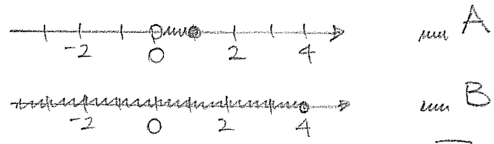


v) $(f+g)(0) \doteq f(0)+g(0) = 2+0 = \underline{\underline{2}}$;
$(fg)(3) \doteq f(3)\cdot g(3) = 2\cdot 1 = \underline{\underline{2}}$;
$(g\circ f)(1) \doteq g(f(1)) = g(2) = \underline{\underline{0}}$.

vi) Per $x \in]-2,1]$ si ha $f(x) = 2$, quindi $(g\circ f)(x) \doteq g(f(x)) = g(2) = 0 \quad \forall x \in]-2,1]$.

FILA C

a1) $A =]0,1]$ $\quad B =]-\infty, 4]$



$\underline{\underline{B \setminus A =]-\infty, 0] \cup]1, 4].}}$

a2) $A = \{x \in \mathbb{R} : x^2 \leq 4\} = \underline{\underline{[-2,2].}}$

(i) V $\qquad$ (ii) F $\qquad$ (iii) V $\qquad$ (iv) F

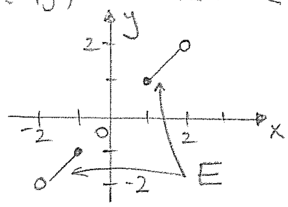
a3) "Tutti gli studenti iscritti nell'a.a. corrente al primo anno del CdL in STPC non si sono iscritti alla PPI di AM o si sono presentati alla prova."

a4) $a x^2 - 2x - 2 = 0$ ha due soluzioni se $a \neq 0$ e $x_{1/2} = \dfrac{2 \pm \sqrt{4+8a}}{2a}$

$\Leftrightarrow$ $4 + 8a > 0$, ossia $a > -\frac{1}{2}$.

Quindi $\underline{\underline{a \in]-\frac{1}{2}, +\infty[\setminus \{0\}.}}$

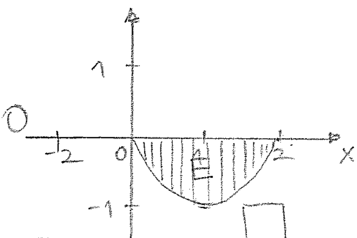
a5) $E = \{(x,y) \in \mathbb{R}^2 : 1 \leq x^2 < 4,\ y = x\} = \{(x,y) \in \mathbb{R}^2 : -2 < x \leq -1,\ y = x\}$
$\cup \{(x,y) \in \mathbb{R}^2 : 1 \leq x < 2,\ y = x\}$



a6) r passante per $P = (-2,2)$, $Q = (-2,3)$

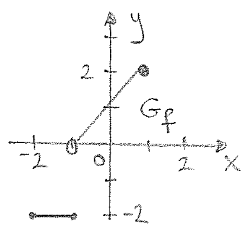
ha eq. $\underline{\underline{x = -2.}}$

a7) L'insieme delle coppie $(x,y) \in \mathbb{R}^2$: $x^2 - 2x \leq y \leq 0$



a8) $\dfrac{x^2+1}{x-2} \leq 0 \iff x - 2 < 0 \qquad \underline{\underline{S =]-\infty, 2[.}}$

a9)



$f([-2,1]) = \{-2\} \cup]0,2].$

a10) $(f+g)(-1) = -2 + 1 = \underline{\underline{-1}}$; $\qquad \left(\dfrac{f}{g}\right)(1) = \dfrac{1}{1} = \underline{\underline{1.}}$

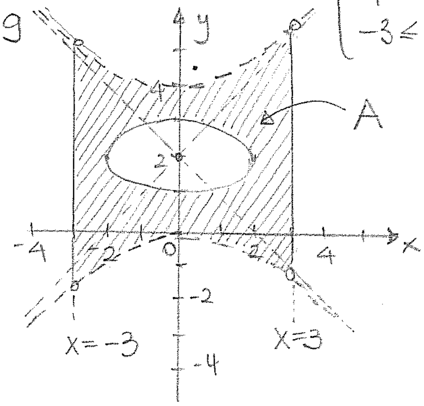
b1) i) non$\{\forall x_1, x_2 \in \mathbb{Z},\ [(x_1 \neq x_2) \Rightarrow (x_2 - x_1 < 0)]\} \Leftrightarrow$

$\exists x_1, x_2 \in \mathbb{Z}$: non$[(x_1 \neq x_2) \Rightarrow (x_2 - x_1 < 0)] \Leftrightarrow$

$\exists x_1, x_2 \in \mathbb{Z}$: $(x_1 \neq x_2)$ e $(x_2 - x_1 \geq 0)$. □

ii) Osserviamo che non $\mathcal{A}$ è vera: basta prendere $x_1 = 0 \neq x_2 = 2$ e $x_2 - x_1 \geq 0$. ■

b2) i) $\begin{cases} -x^2 + y^2 - 4y < 0 \\ x^2 + 4(y-2)^2 \geq 4 \\ x^2 \leq 9 \end{cases} \Leftrightarrow \begin{cases} -x^2 + (y-2)^2 < 4 \\ \frac{x^2}{4} + (y-2)^2 \geq 1 \\ -3 \leq x \leq 3 \end{cases} \Leftrightarrow \begin{cases} -\frac{x^2}{4} + \frac{(y-2)^2}{4} < 1 \\ \frac{x^2}{4} + (y-2)^2 \geq 1 \\ -3 \leq x \leq 3 \end{cases}$



ii) $(0,3) \in A$; infatti soddisfa $-x^2 + y^2 - 4y < 0$ poiché $0 + 9 - 12 < 0$; inoltre soddisfa $x^2 + 4(y-2)^2 \geq 4$ poiché $0 + 4 \geq 4$;

Infine $x = 0 \in [-3, 3]$. □

iii) $x = k$ con $k < -3$ o $k > 3$. ■

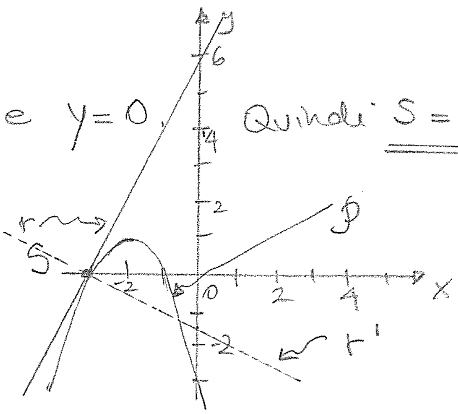
b3) i) $y = 2x + q$ tocca la parabola $\mathcal{P}$ di eq. $y = -x^2 - 4x - 3$ in un solo punto se e solo se $2x + q = -x^2 - 4x - 3$ ha un'unica soluzione. Si ha $0 = -x^2 - 6x - 3 - q$, da cui

$x_{1/2} = \dfrac{6 \pm \sqrt{36 + 4(-3-q)}}{-2}$; ne segue che $0 = -x^2 - 6x - 3 - q$ ha un'unica soluzione se $36 + 4(-3-q) = 0$, da cui

$\underline{\underline{q = 6}}$. □

ii) Si ha $x = -3$ e $y = 0$. Quindi $\underline{\underline{S = (-3, 0)}}$.

$y = -x^2 - 4x - 3$
$= -(x+2)^2 + 1$



iii) r' : $y = 0 - \frac{1}{2}(x+3) \Rightarrow \underline{y = -\frac{1}{2}x - \frac{3}{2}}$.

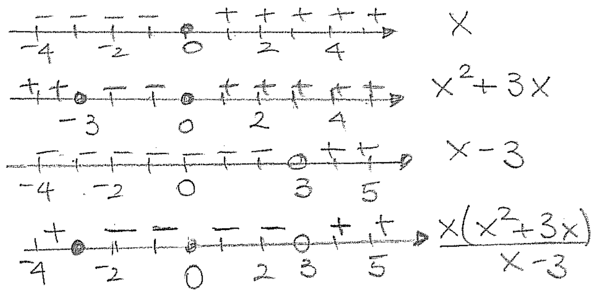
iv) $V = (-2, 1)$ $S = (-3, 0)$ $d(V, S) = \sqrt{2}$.

Eq. della circonferenza di centro il vertice e raggio $\sqrt{2}$:

$$\underline{(x+2)^2 + (y-1)^2 = 2.}$$

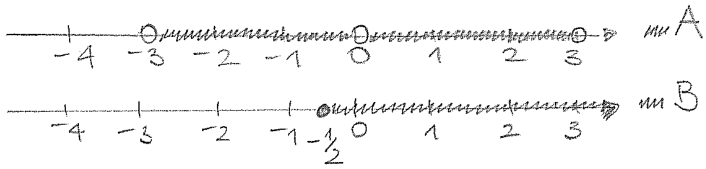
b4) i) $A = \left\{ x \in \mathbb{R} : \frac{x(x^2+3x)}{x-3} < 0 \right\} =$

$= \underline{]-3, 3[\setminus \{0\}}$;



$B = \left\{ x \in \mathbb{R} : \frac{\cancel{x^2} - 2x - \cancel{x^2} - 1}{x^2+1} \le 0 \right\}$

$= \underline{\left[-\frac{1}{2}, +\infty\right[}$.

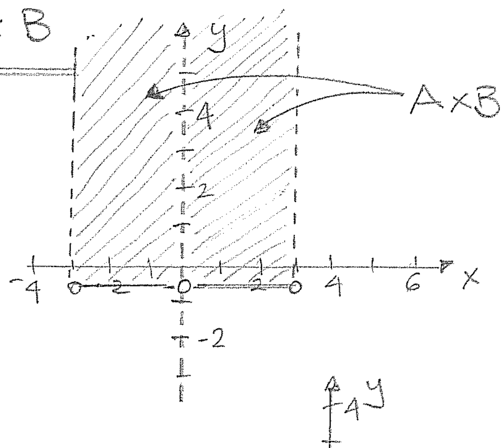


A e B non sono disgiunti, poiché $A \cap B \neq \emptyset$

ii) $A \cup B = \underline{]-3, +\infty[}$; $\mathbb{R} \setminus B = \underline{\left]-\infty, -\frac{1}{2}\right[}$.

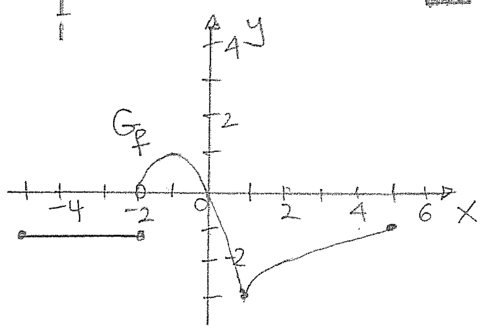
iii) $\underline{\min B = -\frac{1}{2}}$, $\underline{\nexists \max B}$

iv) $A \times B$



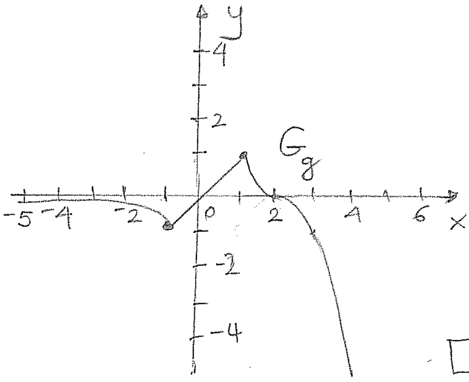
b5) i) $f : [-5, 5] \to \mathbb{R}$

$$f(x) = \begin{cases} -1 & \text{se } -5 \le x \le -2 \\ -(x+1)^2 + 1 & \text{se } -2 < x < 1 \\ \sqrt{x-1} - 3 & \text{se } 1 \le x \le 5 \end{cases}$$



$g : \mathbb{R} \to \mathbb{R}$

$$g(x) = \begin{cases} \frac{1}{x} & \text{se } x \le -1 \\ x & \text{se } -1 < x \le 1 \\ -(x-2)^3 & \text{se } x > 1 \end{cases}$$



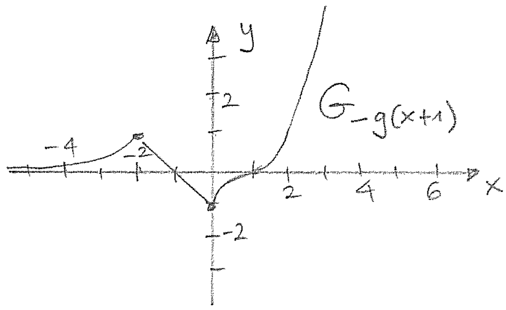
ii) $f([-5,5]) = \underline{\underline{[-3, 1]}}$.

$\min\limits_{[-5,5]} f = \underline{\underline{-3}}$ $\quad x = 1$ pt. di minimo di f su $[-5,5]$.

$\max\limits_{[-5,5]} f = \underline{\underline{1}}$ $\quad x = -1$ pt. di massimo di f su $[-5,5]$.

iii) g non è iniettiva: basta osservare che $x_1 = 0 \neq x_2 = 2$, ma $g(0) = g(2)$.

iv) $\mathbb{R} \ni x \mapsto -g(x+1)$



v) $(f+g)(0) \doteq f(0) + g(0) = 0 + 0 = \underline{0}$;

$(fg)(5) \doteq f(5) \cdot g(5) = (-1)(-27) = \underline{\underline{27}}$

$(g \circ f)(1) \doteq g(f(1)) = g(-3) = \underline{\underline{-\frac{1}{3}}}$.

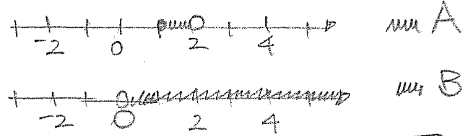
vi) Per $x \in [1,5]$ si ha $-3 \le f(x) \le -1$ e si ha

$$g(f(x)) \doteq g\left(\sqrt{x-1} - 3\right) = \underline{\underline{\frac{1}{\sqrt{x-1} - 3}}}.$$

FILA (D)

a1) $A = [1,2[\qquad B =]0,+\infty[$

$B \setminus A =]0,1[\cup [2,+\infty[$.



a2) $A = \{x \in \mathbb{R} : x^2 > 4\} =]-\infty,-2[\cup]2,+\infty[$

(i) F (ii) V (iii) V (iii) F.

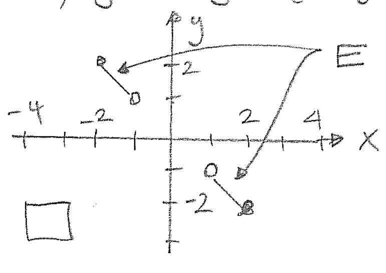
a3) "C'è almeno uno studente presente alla PPI di AM che non è iscritto nell'a.a. corrente al CdL in STPC e non è iscritto al CdL in ITC."

a4) $ax^2 - x + 3 = 0$ ha due soluzioni se $a \neq 0$ e

$$x_{1/2} = \frac{1 \pm \sqrt{1-12a}}{2a} \iff 1-12a > 0$$
$$a < 1/12$$

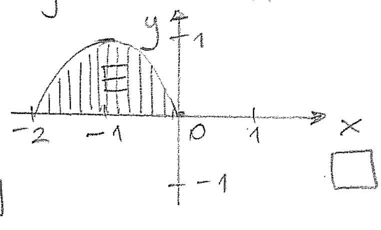
$a \in]-\infty, \frac{1}{12}[\setminus \{0\}$.

a5) $E = \{(x,y) \in \mathbb{R}^2 : 1 < x^2 \leq 4 \ , y = -x\} =$
$= \{(x,y) \in \mathbb{R}^2 : -2 \leq x < -1, \ y = -x\} \cup \{(x,y) \in \mathbb{R}^2 : 1 < x \leq 2, \ y = -x\}$



a6) r passante per $P = (-2,-4)$ e $Q = (-1,-4)$ ha eq. $y = -4$.

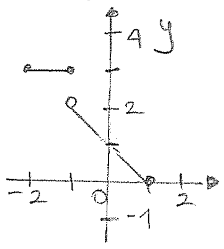
a7) L'insieme delle coppie $(x,y) \in \mathbb{R}^2$: $0 \leq y \leq -x^2 - 2x$



a8) $\frac{x^2+1}{x-3} \geqslant 0 \iff x - 3 > 0$

$S =]3,+\infty[$.

a9)



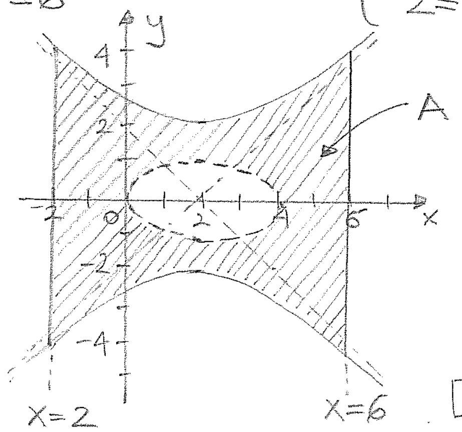
$f([-2,1]) = [0,2[\cup \{3\}$.

a10) $(f-g)(-1) = 1 - 1 = \underline{\underline{0}}$; $\left(\frac{f}{g}\right)(1) = \frac{4}{1} = \underline{\underline{4}}$.

b1) i) $\text{non}\{\forall x_1, x_2 \in \mathbb{R},\ [(x_1 \neq x_2) \Rightarrow (x_1^4 \neq x_2^4)]\} \Leftrightarrow$
$\exists x_1, x_2 \in \mathbb{R} : \text{non}\,[(x_1 \neq x_2) \Rightarrow (x_1^4 \neq x_2^4)] \Leftrightarrow$
$\exists x_1, x_2 \in \mathbb{R} : (x_1 \neq x_2)$ e $(x_1^4 = x_2^4)$. $\square$

ii) Osserviamo che non $\mathcal{A}$ è vera: basta prendere $x_1 = -1 \neq x_2 = 1$ e $x_1^4 = x_2^4$. $\blacksquare$

b2) i) $\begin{cases} -x^2+y^2+4x-8 \leq 0 \\ (x-2)^2+4y^2>4 \\ -2 \leq x \leq 6 \end{cases} \Leftrightarrow \begin{cases} -(x-2)^2+y^2 \leq 4 \\ \frac{(x-2)^2}{4}+y^2>1 \\ -2 \leq x \leq 6 \end{cases} \Leftrightarrow \begin{cases} -\frac{(x-2)^2}{4}+\frac{y^2}{4} \leq 1 \\ \frac{(x-2)^2}{4}+y^2>1 \\ -2 \leq x \leq 6 \end{cases}$



$\square$

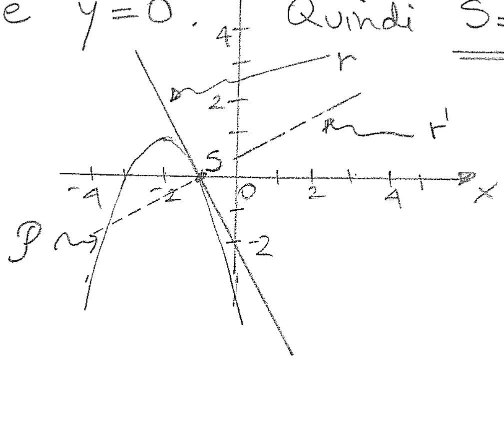
ii) $(2,3) \notin A$; infatti $(2,3)$ non soddisfa $-x^2+y^2+4x-8 \leq 0$ (poiché $-4+9 \not\leq 0$). $\square$

iii) $x=k$ con $k<-2$ o $k>6$. $\blacksquare$

b3) i) $y=-2x+q$ tocca la parabola $\mathcal{P}$ di eq. $y=-x^2-4x-3$ in un solo punto se e solo se $-2x+q=-x^2-4x-3$ ha un'unica soluzione. Si ha $0=-x^2-2x-3-q$, da cui $x_{1/2} = \dfrac{2 \pm \sqrt{4+4(-3-q)}}{-2}$; ne segue che $0=-x^2-2x-3-q$ ha un'unica soluzione se $4+4(-3-q)=0$, da cui $\underline{q=-2}$.

ii) Si ha $x=-1$ e $y=0$. Quindi $\underline{S=(-1,0)}$.

$y=-x^2-4x-3$
$=-(x+2)^2+1$



iii) $r' \ : \ y = 0 + \frac{1}{2}(x+1) \ \Rightarrow \ \underline{\underline{y = \frac{1}{2}x + \frac{1}{2}}}$.

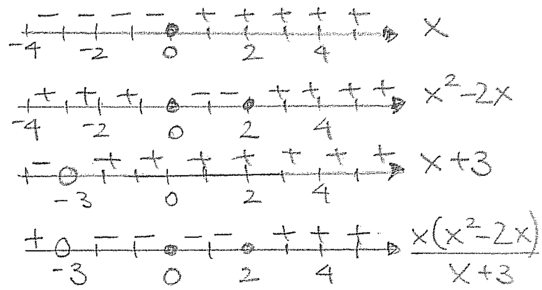
iv) $V = (-2,1)$, $S = (-1,0)$ $\quad d(V,S) = \sqrt{2}$.

Eq. della circonferenza di centro il vertice e raggio $\sqrt{2}$:

$$\underline{\underline{(x+2)^2 + (y-1)^2 = 2}} .$$

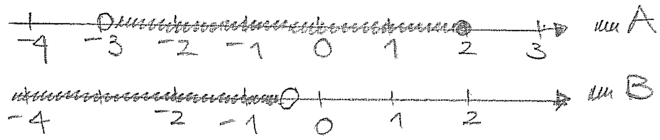
b4) i) $A = \left\{ x \in \mathbb{R} : \frac{x(x^2-2x)}{x+3} \leq 0 \right\}$

$= \underline{\underline{\left]-3, 2\right]}}$;



$B = \left\{ x \in \mathbb{R} : \frac{\cancel{x^2} - 2x - \cancel{x^2} - 1}{x^2+1} > 0 \right\}$

$= \underline{\underline{\left]-\infty, -\frac{1}{2}\right[}}$.

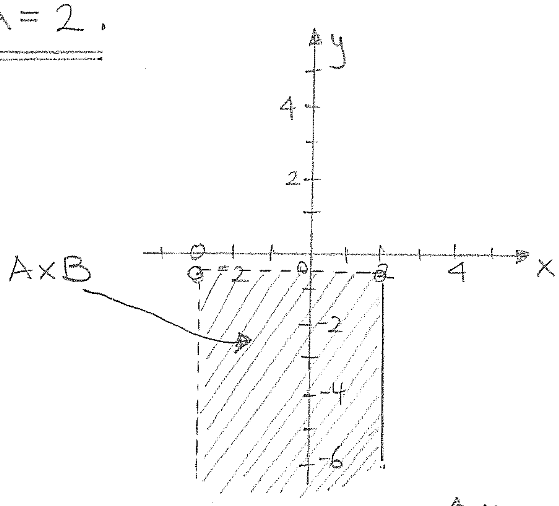


A e B non sono insiemi disgiunti, poiché $A \cap B \neq \emptyset$

ii) $A \cup B = \underline{\underline{\left]-\infty, 2\right]}}$; $\quad \mathbb{R} \setminus A = \underline{\underline{\left]-\infty, -3\right] \cup \left]2, +\infty\right[}}$.

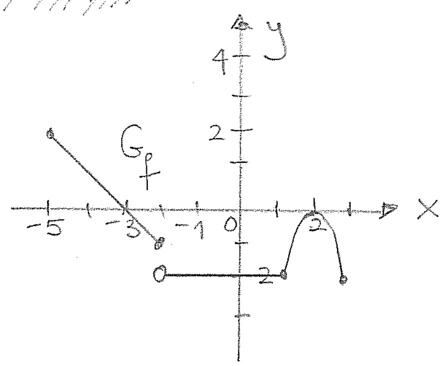
iii) $\underline{\underline{\nexists \min A}}$, $\quad \underline{\underline{\max A = 2}}$.

iv) $A \times B$



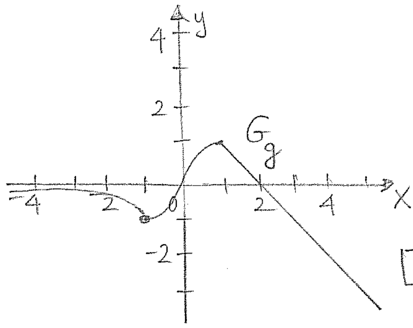
b5) i) $f : [-5, 3] \to \mathbb{R}$,

$$f(x) = \begin{cases} -x-3 & \text{se } -5 \leq x \leq -2 \\ -2 & \text{se } -2 < x \leq 1 \\ -2(x-2)^2 & \text{se } 1 < x \leq 3 \end{cases}$$



$g : \mathbb{R} \to \mathbb{R}$

$$g(x) = \begin{cases} -\dfrac{1}{x^2} & \text{se } x < -1 \\ \sqrt[3]{x} & \text{se } -1 \le x \le 1 \\ -x+2 & \text{se } x > 1. \end{cases}$$



☐

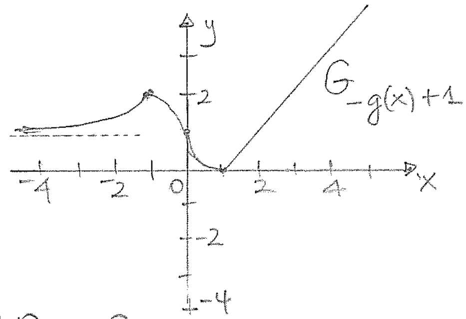
ii) $f([-5,3]) = \underline{\underline{[-2,2]}}.$

$\min\limits_{[-5,3]} f = \underline{\underline{-2}}$ $\quad x \in]-2,1] \cup \{3\}$ sono pt. di minimo di f su $[-5,3]$.

$\max\limits_{[-5,3]} f = \underline{\underline{2}}$ $\quad x = -5$ pt. di massimo di f su $[-5,3]$. ☐

iii) g non è iniettiva: basta osservare che $x_1 = 0 \neq x_2 = 2$, ma $g(0) = g(2)$. ☐

iv) $\mathbb{R} \ni x \longmapsto -g(x)+1$



☐

v) $(f+g)(0) \doteq f(0)+g(0) = -2+0 = \underline{\underline{-2}}$;

$(fg)(3) \doteq f(3)\cdot g(3) = (-2)(-1) = \underline{\underline{2}}$;

$(g\circ f)(1) \doteq g(f(1)) = g(-2) = \underline{\underline{-\frac{1}{4}}}$. ☐

vi) Per $x \in]-2,1]$ si ha $f(x) \equiv -2$ e si ha

$g(f(x)) \doteq g(-2) = -1/4 \quad \forall x \in]-2,1]$. ■