Università di Trento — Dip. di Psicologia e Scienze Cognitive
CDL in Scienze e Tecniche di Psicologia Cognitiva
a.a. 2013 – 2014 — Rovereto, 25 – 29 novembre 2013

1) i) $f(x) = -\frac{1}{(x+1)^2}$: $f(x) = -(x+1)^{-2}$

$f'(x) = 2(x+1)^{-3} = \frac{2}{(x+1)^3}$

Eq. retta tg. al grafico di f nel pt. $(-2,-1)$ è

$y = -1 - 2(x+2)$, ossia $\underline{\underline{y = -2x - 5}}$.

ii) $g(x) = (x-1)^4$: $g'(x) = 4(x-1)^3$

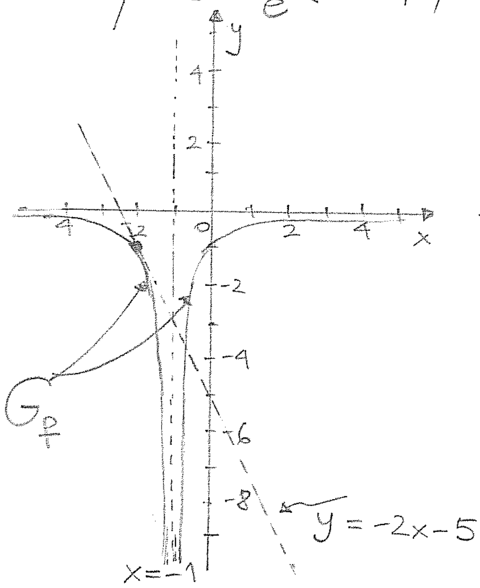
Eq. retta tg. al grafico di g nel pt. $(0,1)$

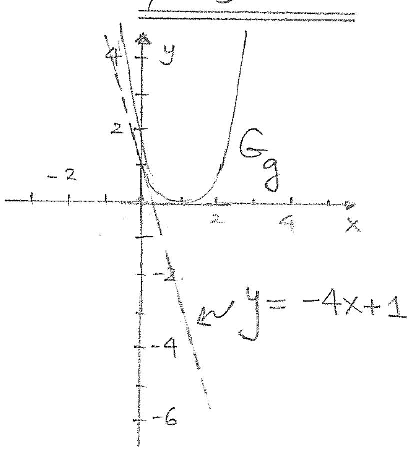
$y = 1 - 4(x-0)$, ossia $\underline{\underline{y = -4x + 1}}$.

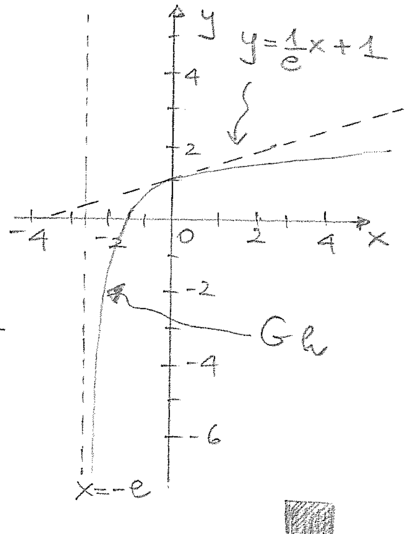
iii) $h(x) = \log(x+e)$: $h'(x) = \frac{1}{x+e}$

Eq. retta tg. al grafico di h nel pt. $(0,1)$ è

$y = 1 + \frac{1}{e}(x-0)$, ossia $\underline{\underline{y = \frac{1}{e}x + 1}}$.







2) $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} -3x & \text{se } x<0 \\ -x^2+2x & \text{se } 0 \le x < 2 \\ x^2-6x+8 & \text{se } x \ge 2 \end{cases} \iff f(x) = \begin{cases} -3x & \text{se } x<0 \\ -(x-1)^2+1 & \text{se } 0 \le x < 2 \\ (x-3)^2-1 & \text{se } x \ge 2 \end{cases}$$

i) • $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(-3x) = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(-x^2+2x) = 0$

$\Rightarrow \lim_{x\to 0} f(x) = 0$; poichè $f(0)=0$ si ha che

$\lim_{x\to 0} f(x) = 0 = f(0)$ e quindi f è continua in $x=0$;

• $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(-x^2+2x) = -4+4=0$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x^2-6x+8) = 4-12+8=0$

$\Rightarrow \lim_{x\to 2} f(x) = 0$; poichè $f(2)=0$,

si ha che $\lim_{x\to 2} f(x) = 0 = f(2)$ e quindi f è continua in $x=2$. □

ii) $f'_-(0)$: $\lim_{h\to 0^-} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^-} \frac{-3h-0}{h} = -3$

$f'_+(0)$: $\lim_{h\to 0^+} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^+} \frac{-h^2+2h-0}{h} = \lim_{h\to 0^+}(-h+2) = 2$.

Possiamo concludere che f non è derivabile in $x=0$, ma $x=0$ è un pt. angoloso per f. □

iii) $f'_-(2)$: $\lim_{h\to 0^-} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0^-} \frac{-(2+h)^2+2(2+h)-0}{h}$

$= \lim_{h\to 0^-} \frac{\cancel{-4}-4h-h^2\cancel{+4}+2h}{h} = \lim_{h\to 0^-}(-h-2) = -2$.

$f'_+(2)$: $\lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0^+} \frac{(2+h)^2-6(2+h)+8-0}{h} =$

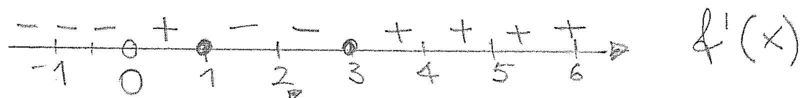
$= \lim_{h\to 0^+} \frac{\cancel{4}+4h+h^2\cancel{-12}-6h\cancel{+8}}{h} = \lim_{h\to 0^+}(h-2) = -2$.

Possiamo concludere che f è derivabile in $x=2$. □

iv)

$$f'(x) = \begin{cases} -3 & \text{se } x<0 \\ -2x+2 & \text{se } 0<x<2 \\ -2 & \text{se } x=2 \\ 2x-6 & \text{se } x>2 \end{cases}$$

$\nexists\, f'(0)$



$f'(0)\ \nexists$ (provato in ii))

$f'(2) = -2\ (<0)$ (provato in iii))

■

3) i) $\left(\dfrac{x^2-\log_2 x}{e^x+x}\right)' = \dfrac{\left(2x-\frac{1}{x\log 2}\right)(e^x+x)-(x^2-\log_2 x)(e^x+1)}{(e^x+x)^2}$;

$\left(\dfrac{x^2e^x}{3x+\log x}\right)' = \dfrac{(2xe^x+x^2e^x)(3x+\log x)-x^2e^x\left(3+\frac{1}{x}\right)}{(3x+\log x)^2}$;

$\left[2^x\left(\sqrt[3]{x}+\frac{1}{x^3}\right)\right]' = \left[2^x\left(x^{1/3}+x^{-3}\right)\right]' = \left(2^x\log 2\right)\left(x^{1/3}+x^{-3}\right)+2^x\left(\frac{1}{3}x^{-\frac{2}{3}}-3x^{-4}\right)$

$= \left(2^x\log 2\right)\left(\sqrt[3]{x}+\frac{1}{x^3}\right)+2^x\left(\frac{1}{3\sqrt[3]{x^2}}-\frac{3}{x^4}\right)$; $\square$

ii) $\left[(4x-x^2)^{-2}\right]' = -2\,(4x-x^2)^{-3}(4-2x)$;

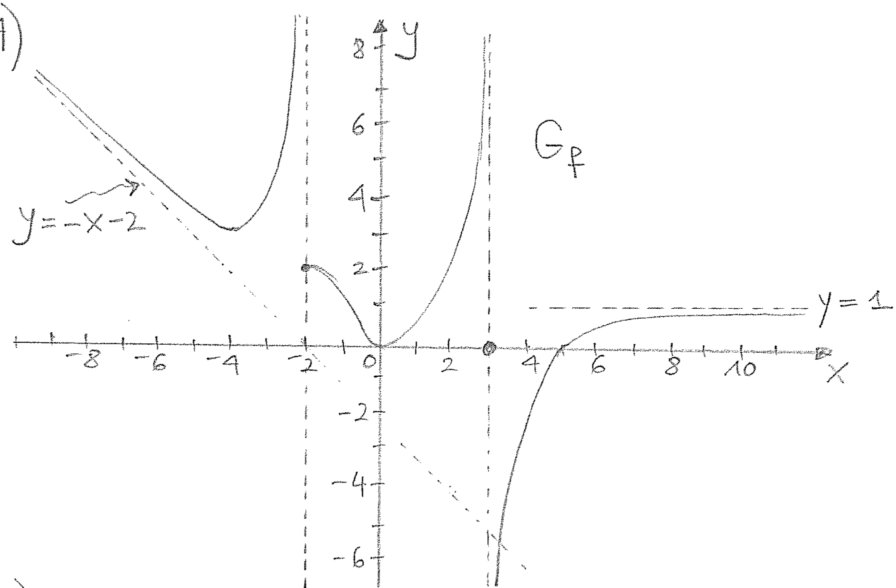
$\left[\left(\log x^2-e^{4x}\right)^3\right]' = 3\left(\log x^2-e^{4x}\right)^2\cdot\left(\frac{2x}{x^2}-4e^{4x}\right)$

$= 3\left(\log x^2-e^{4x}\right)^2\left(\frac{2}{x}-4e^{4x}\right)$;

$\left(e^{x^2+2x}\right)' = e^{x^2+2x}\cdot(2x+2)$;

$\left(e^{(3x-1)^3}\right)' = e^{(3x-1)^3}\cdot 3(3x-1)^2\cdot 3 = 9(3x-1)^2e^{(3x-1)^3}$ $\blacksquare$

4)



i) $y=-x-2$ asintoto obliquo per f, per $x\to-\infty$

$y=1$ asintoto orizzontale per f, per $x\to+\infty$.

$x=-2$, $x=3$ asintoto verticale per f $\square$

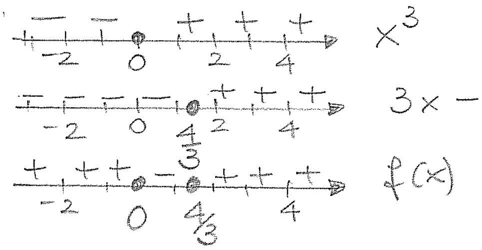
iii) $f(-4)=3$ minimo locale per f $x=-4$ pt. di minimo locale per f ;

$f(0)=0$ minimo locale per f, $x=0$ pt. di minimo locale per f.

iii) $(-4,3)$ e $(0,0)$ sono i punti del grafico di f in cui la retta tangente al grafico risulta essere orizzontale.

5) $\boxed{f(x)=3x^4-4x^3} = x^3(3x-4)$

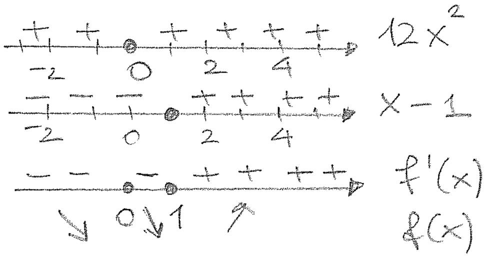
- $\operatorname{dom} f = \mathbb{R} =]-\infty,+\infty[$
- segno di f:



- $\lim\limits_{x\to-\infty} f(x) = +\infty \qquad \lim\limits_{x\to+\infty} f(x) = +\infty$

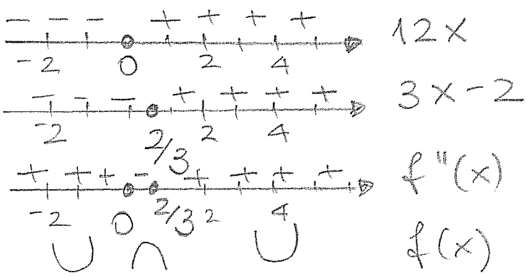
- f è continua su tutto $\mathbb{R}$ (prodotto e somma di funz. continue)
- $\operatorname{dom} f' = \mathbb{R} \qquad f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

$f'(x)=0 \iff x=0,\ x=1$ pt. critici di f



$x=1$ pt. di minimo loc. stretto per f, $f(1)=-1$

- $\operatorname{dom} f'' = \mathbb{R}$, $f''(x) = 36x^2 - 24x = 12x(3x-2)$



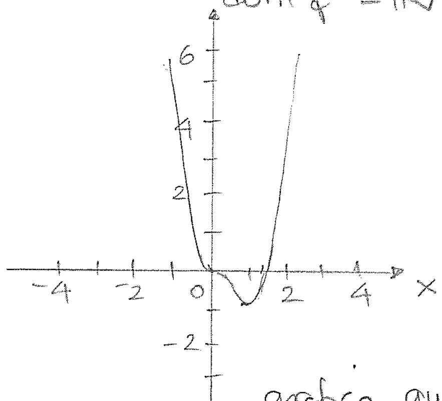
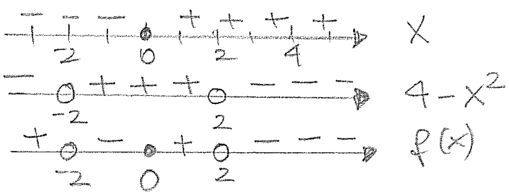


grafico qualitativo di f

$\boxed{f(x) = \dfrac{x}{4-x^2}}$ (f è dispari!)

- $\mathrm{dom}\, f = \mathbb{R} \setminus \{-2, 2\} =]-\infty, -2[\cup]-2, 2[\cup]2, +\infty[$
- segno di f:



- $\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(x) = 0 \Rightarrow y = 0$ è asintoto orizzontale per f per $x \to -\infty$ (per $x \to +\infty$);

$\lim_{x \to -2^-} f(x) = +\infty$

$x = -2$ è asintoto verticale per f;

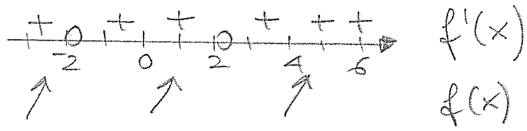
$\lim_{x \to -2^+} f(x) = -\infty$

$\lim_{x \to 2^-} f(x) = +\infty$

$x = 2$ è asintoto verticale per f;

$\lim_{x \to 2^+} f(x) = -\infty$

- f è continua in $\mathbb{R} \setminus \{-2, 2\}$ (rapporto di funz. continue);
- $\mathrm{dom}\, f' = \mathbb{R} \setminus \{-2, 2\}$ $f'(x) = \dfrac{4 - x^2 - x \cdot (-2x)}{(4-x^2)^2} = \dfrac{x^2+4}{(4-x^2)^2}$



f non ha punti critici.

- $\mathrm{dom}\, f'' = \mathbb{R} \setminus \{-2, 2\}$ $f''(x) = \dfrac{2x(4-x^2)^{\cancel{2}} - (x^2+4)\, 2\cancel{(4-x^2)}(-2x)}{(4-x^2)^{\cancel{4}\,3}}$

$= \dfrac{8x - 2x^3 + 4x^3 + 16x}{(4-x^2)^3} =$

$= \dfrac{2x(x^2+12)}{(4-x^2)^3}$

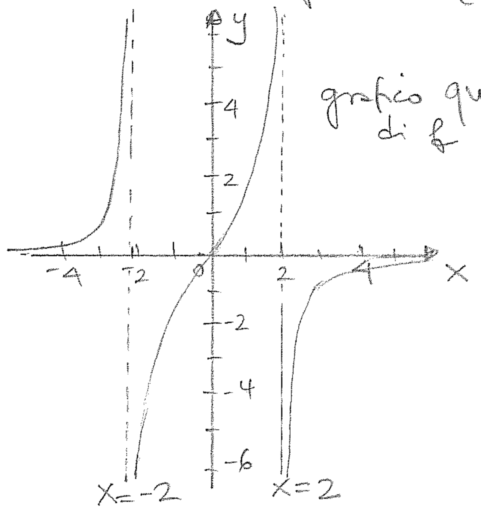
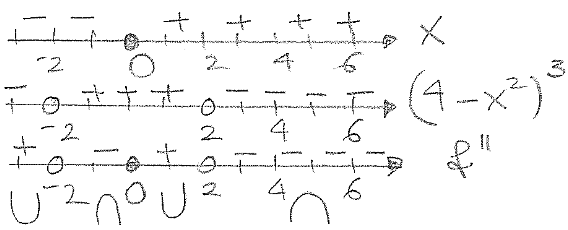


grafico qualitativo di f.



$\square$

$$\boxed{f(x) = x^2 e^{-2x+1}}$$

- $\operatorname{dom} f = \mathbb{R} = \left]-\infty, +\infty\right[$
- segno di f : $+ \ + \ + \ 0 \ + \ + \ +$ $f(x)$ (−2, 0, 2)
- $\lim\limits_{x \to -\infty} f(x) = +\infty$ $\quad \lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \dfrac{x^2}{e^{2x+1}} = 0$;
- f è continua su tutto $\mathbb{R}$ (composizione e prodotto di funz. continue su $\mathbb{R}$) ;
- $\operatorname{dom} f' = \mathbb{R}$ $\quad f'(x) = 2x e^{-2x+1} + x^2 e^{-2x+1}(-2)$

$$= (2x - 2x^2) e^{-2x+1}$$

$f'(x) = 0 \iff x = 0,\ x = 1$ pti. critici di f

$- \ - \ - \ 0 \ + \ 1 \ - \ - \ -$ $f'(x)$ (−4, −2, 0, 2, 4)

$\searrow \ \nearrow \ \searrow$ $f(x)$

$x = 0$ pt. di minimo loc. stretto per f ; $f(0) = 0$

$x = 1$ pt. di massimo loc. stretto per f ; $f(1) = e^{-1}$

- $\operatorname{dom} f'' = \mathbb{R}$ $\quad f''(x) = (2 - 4x) e^{-2x+1} + (2x - 2x^2) e^{-2x+1} \cdot (-2)$

$$= e^{-2x+1}(2 - 4x - 4x + 4x^2)$$

$$= 2e^{-2x+1}(2x^2 - 4x + 1)$$

$+ \ + \ + \ 1-\frac{\sqrt{2}}{2} \ - \ 1+\frac{\sqrt{2}}{2} \ + \ + \ +$ $f''(x)$ (−4, −2, 0, 2, 4, 6)

$\cup \quad \cap \quad \cup$ $f(x)$

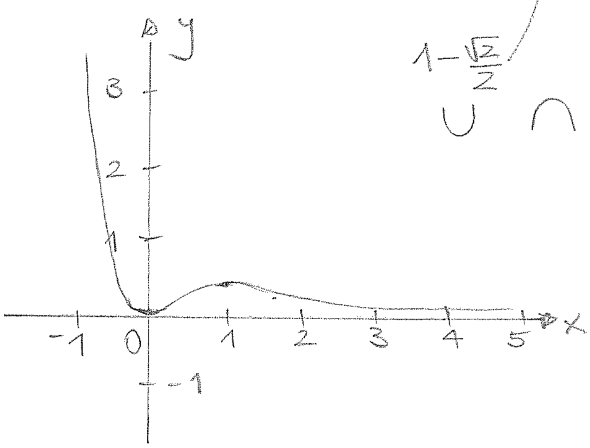
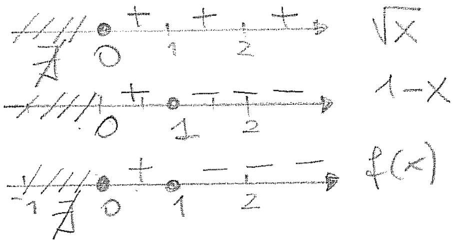


grafico qualitativo di f

$\square$

$$\boxed{f(x)=\sqrt{x}\,(1-|x|)}$$

- $\operatorname{dom} f = [0,+\infty[$ e risulta $f(x)=\sqrt{x}\,(1-x)$
- segno di f

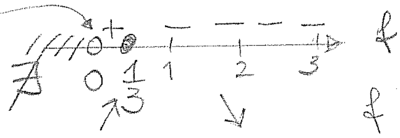


- $\lim\limits_{x\to+\infty} f(x) = -\infty$
- f è continua su $[0,+\infty[$ (prodotto di funz. continue)
- $\operatorname{dom} f' =]0,+\infty[$ $\quad f'(x)=\dfrac{1}{2\sqrt{x}}(1-x)-\sqrt{x}=$

$$=\frac{1-x-2x}{2\sqrt{x}}=\frac{1-3x}{2\sqrt{x}}$$

$f'(x)=0 \iff x=\frac{1}{3}$, pt. critico per f.

oss. che $f'_+(0)$ esiste e vale $+\infty$.



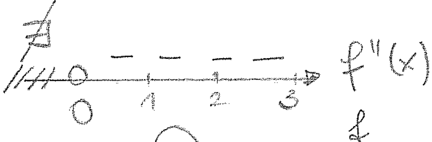
$\left(2\sqrt{x}>0 \;\; \forall x>0\right)$

$x=\frac{1}{3}$ è pt. di massimo locale stretto per f

$$f\left(\tfrac{1}{3}\right)=\frac{1}{\sqrt{3}}\left(1-\frac{1}{3}\right)=\frac{2}{3\sqrt{3}}\sim 0.4$$

- $\operatorname{dom} f'' =]0,+\infty[$ $\quad f''(x)=\dfrac{-3\cdot 2\sqrt{x}-(1-3x)\frac{1}{\sqrt{x}}}{4x}$

$$=\frac{-6x-1+3x}{4x\sqrt{x}}=\frac{-3x-1}{4x\sqrt{x}}$$



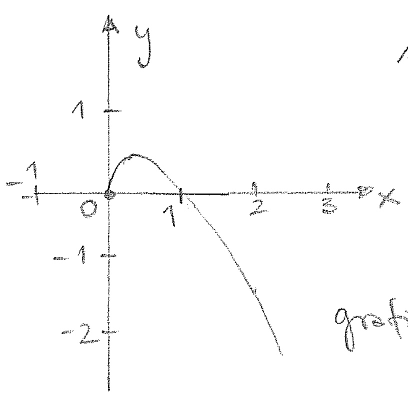


grafico qualitativo di f