Università degli Studi di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2014 – 2015 – Rovereto, 1–5 dicembre

(N. 11)

1) a) $$\left(\frac{x^2+\log(2x)}{e^x}\right)' = \frac{\left(2x+\frac{\cancel{2}}{\cancel{2}x}\right)\cancel{e^x} - \left(x^2+\log(2x)\right)\cancel{e^x}}{e^{\cancel{2}x}} = \frac{\left(2x+\frac{1}{x}\right)-\left(x^2+\log(2x)\right)}{e^x};$$

$$\left(\frac{e^{4x}}{\sqrt{2x}+1}\right)' = \frac{4e^{4x}\left(\sqrt{2x}+1\right) - e^{4x}\frac{1}{\cancel{2}}(2x)^{-1/2}\cdot\cancel{2}}{\left(\sqrt{2x}+1\right)^2} = \frac{e^{4x}\left[4\left(\sqrt{2x}+1\right)-\frac{1}{\sqrt{2x}}\right]}{\left(\sqrt{2x}+1\right)^2};$$

$$\left(e^{x^2-2x}\right)' = e^{x^2-2x}\cdot(2x-2);$$

$$\left[(3x-4)^{-2}\right]' = -2(3x-4)^{-3}\cdot 3 = -6(3x-4)^{-3}; \qquad \square$$

b) $$\left[e^{(2x-1)^3}\right]' = e^{(2x-1)^3}\cdot 3(2x-1)^2\cdot 2 = 6(2x-1)^2 e^{(2x-1)^3};$$

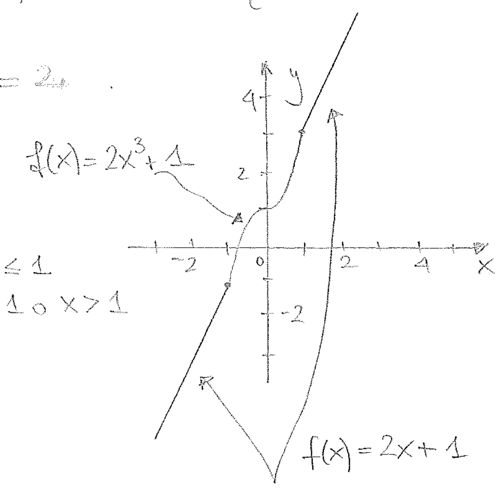
$$\left[\left(\log x^2 - e^x\right)^3\right]' = 3\left(\log x^2-e^x\right)^2\cdot\left(\frac{1}{x^{\cancel{2}}}\cdot 2\cancel{x} - e^x\right) = 3\left(\frac{2}{x}-e^x\right)\left(\log x^2-e^x\right)^2;$$

$$\left[\log\left(5x+2^x\right)^4\right]' = \frac{1}{(5x+2^x)^{\cancel{4}}}\cdot 4\cancel{(5x+2^x)^3}\cdot\left(5+2^x\log 2\right) =$$

$$= \frac{4\left(5+2^x\log 2\right)}{5x+2^x} \qquad \blacksquare$$

2) i) $$\begin{cases} \lim_{x\to -1^-}(ax+b) = f(-1) \\ \lim_{x\to 1^+}(ax+b) = f(1) \end{cases} \iff \begin{cases} -a+b = -2+1 \\ a+b = 2+1 \end{cases}$$

$\iff b = 1, \quad a = 2.$



ii) $$f(x) = \begin{cases} 2x^3+1 & \text{se } |x|\le 1 \\ 2x+1 & \text{se } x<-1 \text{ o } x>1 \end{cases}$$

a) $$\lim_{h\to 0^-}\frac{f(-1+h)-f(-1)}{h} =$$

$$= \lim_{h\to 0^-}\frac{\left[2(-1+h)+1\right]-(-1)}{h} = \lim_{h\to 0^-}\frac{\cancel{-2}+2h\cancel{+1+1}}{h} = 2$$

$$\lim_{h\to 0^+}\frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0^+}\frac{\left[2(-1+h)^3+1\right]-(-1)}{h} = \lim_{h\to 0^+}\frac{\cancel{-2}+6h-6h^2+2h^3\cancel{+2}}{h}$$

$$= 6.$$

Poiché i due limiti sono diversi, la funzione f non è derivabile in $x=-1$. $\square$

b) $$\lim_{h\to 0^-}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0^-}\frac{[2(1+h)^3+1]-3}{h}=\lim_{h\to 0^-}\frac{\cancel{2}+6h+6h^2+2h^3-\cancel{2}}{h}=6$$

$$\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0^+}\frac{[2(1+h)+1]-3}{h}=\lim_{h\to 0^+}\frac{\cancel{2}+2h-\cancel{2}}{h}=2.$$

Poiché i due limiti sono diversi, la funzione f non è derivabile in $x=1$. $\blacksquare$

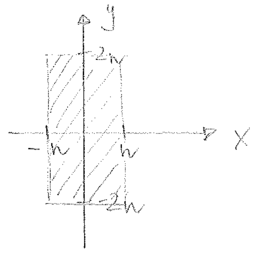
3) i) $$1-\frac{1}{3}+\frac{1}{5}-\cdots+\frac{1}{29}=\underline{\underline{\sum_{n=0}^{14}(-1)^n\frac{1}{2n+1}}};$$

$$a^2-\frac{a^4}{2}+\frac{a^6}{4}-\frac{a^8}{8}+\cdots+\frac{a^{18}}{256}=\underline{\underline{\sum_{n=0}^{8}(-1)^n\frac{a^{2n+2}}{2^n}}};$$

$$\frac{1}{3}-\frac{1}{2}+\frac{1}{5}-\frac{1}{4}+\cdots+\frac{1}{21}-\frac{1}{22}=\underline{\underline{\sum_{n=1}^{10}\frac{1}{2n+1}-\sum_{n=1}^{11}\frac{1}{2n}}};$$

$$\int_0^1 x^2\,dx+\int_1^2 x^3\,dx+\cdots+\int_{13}^{14}x^{15}\,dx=\underline{\underline{\sum_{n=0}^{13}\int_n^{n+1}x^{n+2}\,dx}}.$$ $\square$

ii) a) $$\sum_{n=1}^{6}\text{perimetro}(R_n)=\qquad R_n=\{(x,y)\in\mathbb{R}^2;\ |x|\le n,\ |y|\le 2n\}$$



$$\sum_{n=1}^{6}[2\cdot 2n+2\cdot 4n]=$$
$$\sum_{n=1}^{6}12n=12\sum_{n=1}^{6}n=$$
$$=12(1+2+3+4+5+6)=12\cdot 21=\underline{\underline{252}};$$

$$\sum_{n=1}^{6}\text{area}(R_n)=\sum_{n=1}^{6}2n\cdot 4n=8\sum_{n=1}^{6}n^2=8(1+4+9+16+25+36)$$
$$=8\cdot 91=\underline{\underline{728}};$$ $\square$

b) $$\sum_{j=1}^{4}f((-1)^j j)\qquad f(x)=\begin{cases}-2 & \text{se } x\le 0\\ 1 & \text{se } x>0\end{cases}$$

$$\|$$

$$f(-1)+f(2)+f(-3)+f(4)=-2+\cancel{1}-\cancel{2}+\cancel{1}=\underline{\underline{-2}};$$ $\square$

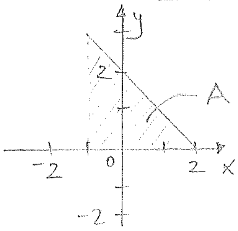
c) $$\sum_{m=0}^{40}(m^2-(m+1)^2)=(-\cancel{1})+(\cancel{1}-\cancel{2^2})+(\cancel{2^2}-\cancel{3^2})+\cdots+(\cancel{39^2}-\cancel{40^2})+(\cancel{40^2}-41^2)=$$
$$=\underline{\underline{-41^2}}.$$

$$\sum_{k=0}^{3}\int_{k}^{k+1}\frac{x}{k+1}\,dx=\sum_{k=0}^{3}\left[\frac{1}{k+1}\cdot\frac{x^2}{2}\right]_{k}^{k+1}=\sum_{k=0}^{3}\frac{1}{k+1}\left(\frac{(k+1)^2}{2}-\frac{k^2}{2}\right)=$$

$$=\left(\frac{1}{2}\right)+\frac{1}{2}\left(\frac{2^2}{2}-\frac{1}{2}\right)+\frac{1}{3}\left(\frac{3^2}{2}-\frac{2^2}{2}\right)+\frac{1}{4}\left(\frac{4^2}{2}-\frac{3^2}{2}\right)=$$

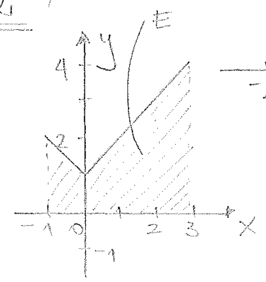
$$=\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}=\frac{12+18+20+21}{24}=\underline{\underline{\frac{71}{24}}}\,.$$

4) $\displaystyle\int_{-1}^{2}(-x+2)\,dx=\text{area}(A)=\underline{\underline{\frac{9}{2}}}\,;$



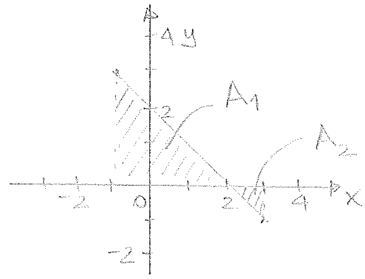
$\displaystyle\int_{-1}^{3}(|x|+1)\,dx=\text{area}(E)=$

$\displaystyle=\frac{3}{2}+\frac{15}{2}=\underline{9}\,;$



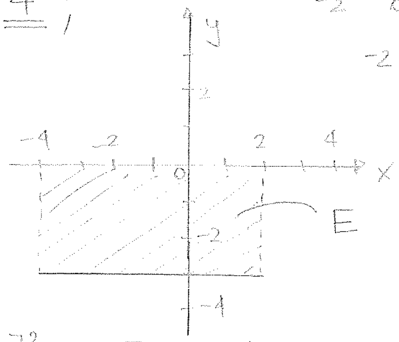
$\displaystyle\int_{-1}^{3}(-|x+1|+3)\,dx=\text{area}(A_1)-\text{area}(A_2)$

$\displaystyle=\frac{9}{2}-\frac{1}{2}=\underline{4}\,;$



$\displaystyle\int_{-4}^{2}(-3)\,dx=-\text{area}\,E$

$=\underline{\underline{-18}}$



5) i) $\displaystyle\int_{1}^{2}(x^4+3x)\,dx=\left[\frac{x^5}{5}+3\frac{x^2}{2}\right]_{1}^{2}=\left(\frac{2^5}{5}+3\cdot\frac{2^{\cancel{2}}}{\cancel{2}}\right)-\left(\frac{1}{5}+\frac{3}{2}\right)=$

$\displaystyle=\frac{32}{5}+6-\frac{1}{5}-\frac{3}{2}=\frac{31}{5}+\frac{9}{2}=\underline{\underline{\frac{107}{10}}}\,;$

$\displaystyle\int_{-2}^{-1}\left(e^x-\frac{2}{x}\right)dx=\left[e^x-2\log|x|\right]_{-2}^{-1}=\left(e^{-1}-2\cancel{\log 1}\right)-\left(e^{-2}-2\log 2\right)$

$\displaystyle=\underline{\underline{\frac{1}{e}-\frac{1}{e^2}+2\log 2}}\,;$

$\displaystyle\int_{0}^{1}\left(\sqrt[3]{x}+\sqrt{x}\right)dx=\left[\frac{x^{4/3}}{4/3}+\frac{x^{3/2}}{3/2}\right]_{0}^{1}=\frac{3}{4}+\frac{2}{3}=\frac{9+8}{12}=\underline{\underline{\frac{17}{12}}}\,;$

ii) $\displaystyle\int_{1}^{2}\left(\frac{3x^2+4x^3}{x}\right)dx=\int_{1}^{2}(3x+4x^2)\,dx=\left[3\frac{x^2}{2}+4\frac{x^3}{3}\right]_{1}^{2}=\left(3\cdot\frac{4}{2}+4\cdot\frac{8}{3}\right)-\left(\frac{3}{2}+\frac{4}{3}\right)$

$\displaystyle=\frac{9}{2}+\frac{28}{3}=\frac{27+56}{6}=\underline{\frac{83}{6}}\,;$

$\displaystyle\int_{0}^{1}\left(2^x-\frac{x^2-1}{x+1}\right)dx=\int_{0}^{1}\left(2^x-\frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}}\right)dx=\int_{0}^{1}\left(2^x-x+1\right)dx=$

$$= \left[\frac{2^x}{\log 2} - \frac{x^2}{2} + x\right]_0^1 = \left(\frac{2}{\log 2} - \frac{1}{2} + 1\right) - \left(\frac{1}{\log 2}\right) = \frac{1}{\log 2} + \frac{1}{2};$$

vedi fine pag. 5 (*)

$\square$

iii) $$\int_0^4 \frac{3x}{x^2+1}\,dx = \frac{3}{2}\int_0^4 \frac{2x}{x^2+1}\,dx = \frac{3}{2}\left[\log(x^2+1)\right]_0^4 =$$

$$= \frac{3}{2}\left[\log 17 - \cancel{\log 1}\right] = \frac{3}{2}\log 17;$$

$$\int_0^1 x e^{x^2+3}\,dx = \frac{1}{2}\int_0^1 2x\, e^{x^2+3}\,dx = \frac{1}{2}\left[e^{x^2+3}\right]_0^1 = \frac{1}{2}\left[e^4 - e^3\right];$$

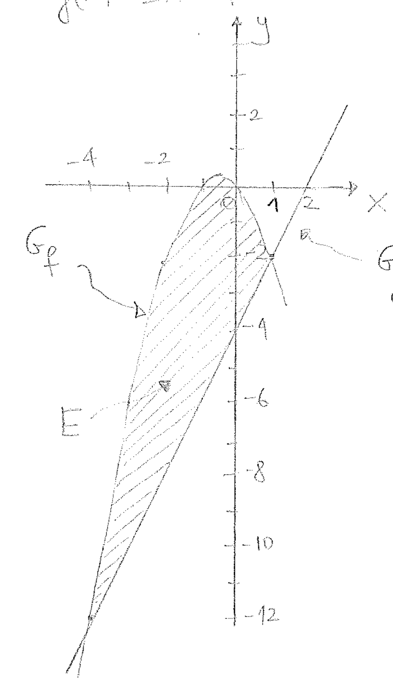
$$\int_0^1 (3x-1)^4\,dx = \frac{1}{3}\int_0^1 3(3x-1)^4\,dx = \frac{1}{3}\left[\frac{(3x-1)^5}{5}\right]_0^1 = \frac{1}{15}\left[2^5 - (-1)^5\right]$$

$$= \frac{1}{15}(32+1) = \frac{33}{15} = \frac{11}{5}.$$

6) i) a) $f(x) = -(x^2+x) = -\left(x+\frac{1}{2}\right)^2 + \frac{1}{4}$

$g(x) = 2x - 4$

$-x^2 - x = 2x - 4 \iff x^2 + 3x - 4 = 0 \iff (x+4)(x-1) = 0$



$$\text{area}(E) = \int_{-4}^{1}\left[f(x) - g(x)\right]dx =$$

$$= \int_{-4}^{1}\left(-x^2 - x - 2x + 4\right)dx = \int_{-4}^{1}\left(-x^2 - 3x + 4\right)dx$$

$$= \left[-\frac{x^3}{3} - \frac{3x^2}{2} + 4x\right]_{-4}^{1} = \left(-\frac{1}{3} - \frac{3}{2} + 4\right) - \left(+\frac{4^3}{3} - \frac{3\cdot 4^2}{2} - 16\right)$$

$$= -\frac{1}{3} - \frac{3}{2} + 4 - \frac{4^3}{3} + \frac{3}{2}\cdot 4^2 + 16 =$$

$$= 20 - \frac{65}{3} + \frac{45}{2} = \frac{120 - 130 + 135}{6} = \frac{125}{6}.$$

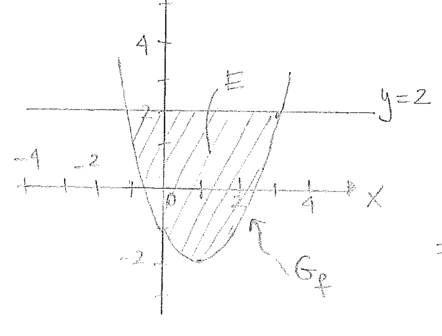
$\square$

b) $f(x) = (x-1)^2 - 2 \qquad y = 2$

$(x-1)^2 - 2 = 2 \iff (x-1)^2 = 4$

$x_{1/2} - 1 = \pm 2 \qquad x_{1/2} = \pm 2 + 1$

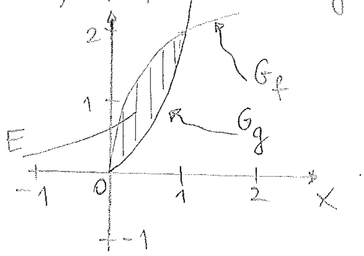
$x_1 = -1, \; x_2 = 3$



$$\text{area}(E) = \int_{-1}^{3}\left(2 - (x-1)^2 + 2\right)dx =$$

$$= \int_{-1}^{3}\left(-x^2 + 2x + 3\right)dx = \left[-\frac{x^3}{3} + x^2 + 3x\right]_{-1}^{3} = \frac{32}{3};$$

c) $f(x) = 2\sqrt{x}$ $\quad g(x) = 3^x - 1$



$$\text{area}(E) = \int_0^1 \left(2\sqrt{x} - 3^x + 1\right) dx = \left[2\,\frac{x^{3/2}}{3/2} - \frac{3^x}{\log 3} + x\right]_0^1$$

$$= \left(\frac{4}{3} - \frac{3}{\log 3} + 1\right) - \left(-\frac{1}{\log 3}\right) = \underline{\underline{\frac{7}{3} - \frac{2}{\log 3}}}$$ $\square$

ii) a) $h(x) = (x+1)e^{2x}$

- $\operatorname{dom} h = \mathbb{R}$
- segno di h $\quad$ $- \; \bullet_{-1} \; + \; + \; + \to h(x)$
- $\lim_{x \to -\infty} h(x) = 0$ $\quad$ $y = 0$ asintoto orizzontale per h, per $x \to -\infty$,

  $\lim_{x \to +\infty} h(x) = +\infty$

- continua su tutto $\mathbb{R}$!
- $\operatorname{dom} h'(x) = \mathbb{R}$ $\quad h'(x) = e^{2x} + (x+1)e^{2x} \cdot 2 = e^{2x}(2x+3)$

$x = -\frac{3}{2}$ pt. critico per h

$h\left(-\frac{3}{2}\right) = -\frac{1}{2}e^{-3} \sim -0.02$

$- \; - \; \bullet_{-3/2} \; + \; + \; + \; + \to h'(x)$ $\quad \searrow \; \nearrow \; h(x)$

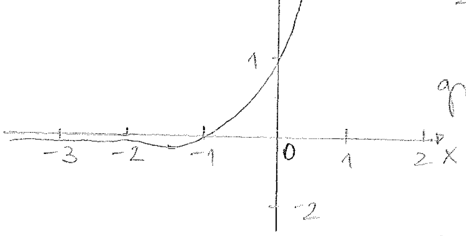


grafico qualitativo di h

b) $H(x) = \left(\frac{x}{2} + \frac{1}{4}\right)e^{2x}$ è derivabile e

$$H'(x) = \frac{1}{2}e^{2x} + \left(\frac{x}{2} + \frac{1}{4}\right)e^{2x} \cdot 2 = (x+1)e^{2x} = h(x)$$

$\forall x \in \mathbb{R}$.

c) $\int_0^1 h(x)\,dx = \left[H(x)\right]_0^1 = H(1) - H(0) =$

$$= \underline{\underline{\left[\frac{3}{4}e^2 - \frac{1}{4}\right]}}.$$

■

(*) Es. 5 ii) $\int_1^2 \left(\frac{1}{x^3} - x^{-3}\right) dx = 0$ poiché la funzione integranda è $\equiv 0$.