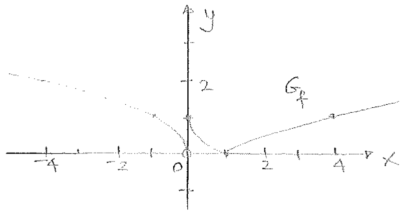
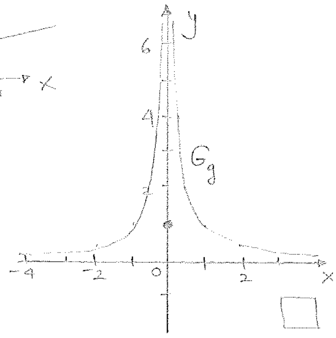
Università degli Studi di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2014/15 – Rovereto, 3 – 7 novembre 2014 (N. 7)

1) i) $f,g:\mathbb{R}\to\mathbb{R}$




$$f(x)=\begin{cases}\sqrt{|x|} & \text{se } x<0\\ |\sqrt{x}-1| & \text{se } x\geq 0;\end{cases}$$

$$g(x)=\begin{cases}\dfrac{1}{|x|} & \text{se } x\neq 0\\ 1 & \text{se } x=0.\end{cases}$$




ii) $\min\limits_{[-1,4]} f = 0$ $\quad x=1$ pt. di min. di $f$ su $[-1,4]$;

$\max\limits_{[-1,4]} f = 1$ $\quad x\in\{-1,0,4\}$ pt. di mass. di $f$ su $[-1,4]$.

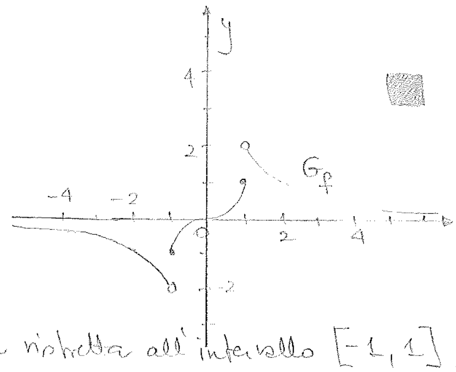
iii) $g$ è definita su un insieme simmetrico rispetto all'origine. Inoltre $\forall x\in\mathbb{R}\setminus\{0\}$ si ha $-x\in\mathbb{R}\setminus\{0\}$ e $g(-x)=\dfrac{1}{|-x|}=\dfrac{1}{|x|}=g(x)$.

Per $x=0$ si ha $g(0)=1$.

iv) $f$ è decrescente su $]-\infty,0[$; è decrescente ristretta all'intervallo $[0,1]$, $f$ è crescente ristretta all'intervallo $[1,+\infty[$.

v) $(f\circ g)(0) \doteq f(g(0)) = f(1) = \underline{\underline{0}}$;

$(g\circ f)(1) \doteq g(f(1)) = g(0) = \underline{\underline{1}}$.




2) i) $f:\mathbb{R}\to\mathbb{R}$ $\quad f(x)=\begin{cases}x^3 & \text{se } x\in[-1,1]\\ \dfrac{2}{x} & \text{altrimenti}\end{cases}$

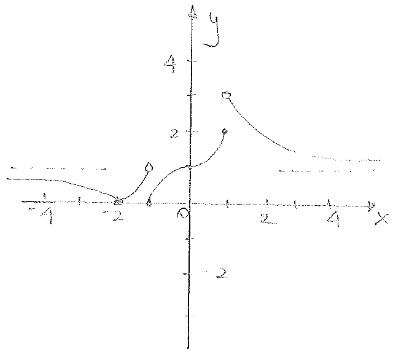
ii) $f$ è decrescente su $]-\infty,-1[$; è crescente ristretta all'intervallo $[-1,1]$, $f$ è decrescente ristretta all'intervallo $]1,+\infty[$.

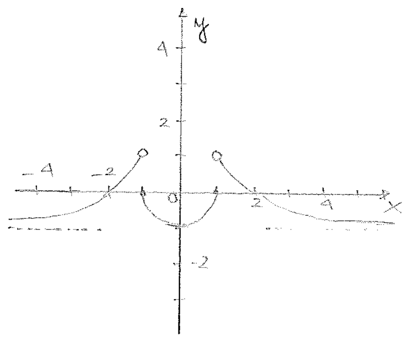
iii) $f$ è definita su un insieme simmetrico rispetto all'origine. Inoltre, $\forall x\in[-1,1]$ si ha $-x\in[-1,1]$ e $f(-x)=(-x)^3=-x^3=-f(x)$

$\forall x\notin[-1,1]$ si ha $-x\notin[-1,1]$ e $f(-x)=\dfrac{2}{-x}=-\left(\dfrac{2}{x}\right)=-f(x)$

provando che $f$ è dispari.

iv) $x \mapsto |f(x)+1|$ $\qquad$ $x \mapsto |f(x)|-1$ .




3) • $3|x-2| \leq 12 \iff |x-2| \leq 4 \iff -4 \leq x-2 \leq 4 \Rightarrow \underline{S=[-2,6]}$ ;

• $|-x|+|x|=4 \iff 2|x|=4 \iff |x|=2 \Rightarrow \underline{S=\{-2,2\}}$ ;

• $-|-2x|+|x|=2 \iff -|x|=2 \Rightarrow \underline{S=\emptyset}$ ;

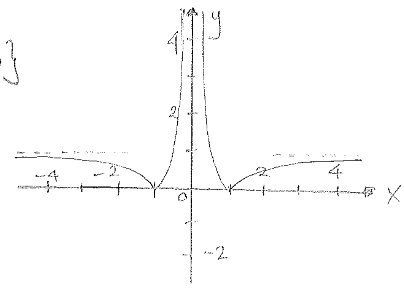
• $|x^3-x^2|=0 \iff x^3-x^2=0 \iff x^2(x-1)=0 \Rightarrow \underline{S=\{0,1\}}$ ;

• $|x^2-3| \leq 1 \iff -1 \leq x^2-3 \leq 1 \iff 2 \leq x^2 \leq 4 \Rightarrow \underline{S=[-2,-\sqrt{2}] \cup [\sqrt{2},2]}$ ;

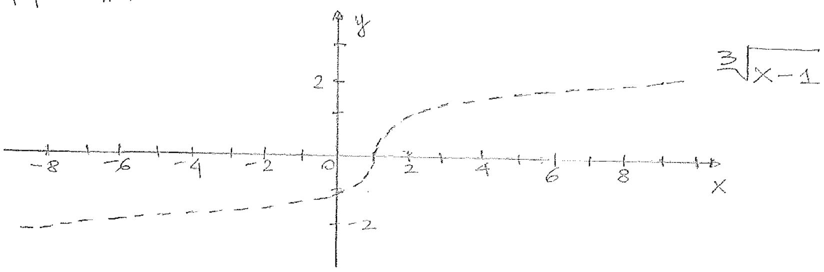
• $||x-2|-4| \leq 0 \iff ||x-2|-4|=0 \iff |x-2|-4=0 \iff |x-2|=4$

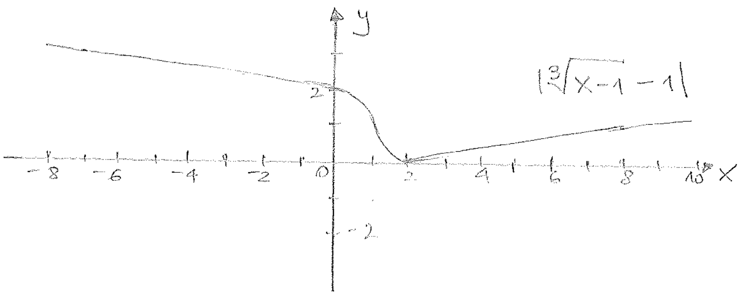
$\iff x-2=-4$ o $x-2=4 \Rightarrow \underline{S=\{-2,6\}}$.

4) • $\left|\frac{1}{x^2}-1\right|$ $\qquad$ $\mathbb{R}\setminus\{0\}$

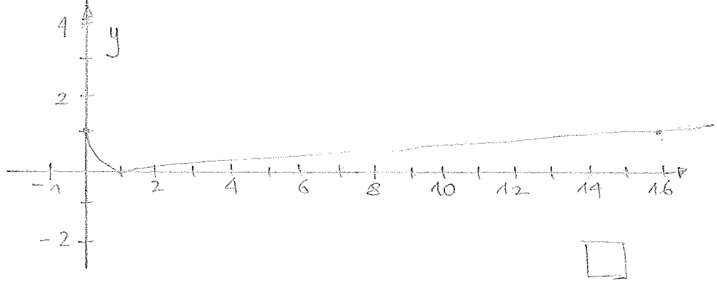



• $|\sqrt[3]{x-1}-1|$ $\qquad$ $\mathbb{R}$

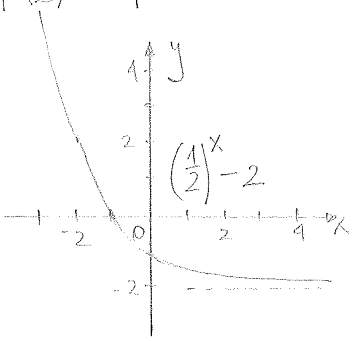


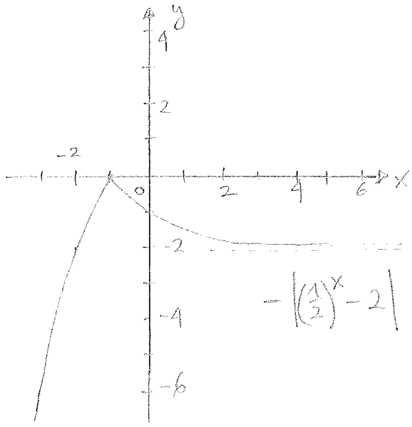



• $|1-\sqrt[4]{x}|$ $\quad [0,+\infty[$



• $-\left|\left(\frac{1}{2}\right)^x-2\right|$ $\quad \mathbb{R}$




5) i) • $x|x+2|\le 3 \iff \begin{cases} x+2\ge 0\\ x(x+2)\le 3\end{cases}$ o $\begin{cases} x+2<0\\ -x(x+2)\le 3\end{cases}$

$\iff \begin{cases} x\ge -2\\ x^2+2x-3\le 0\end{cases}$ o $\begin{cases} x<-2\\ x^2+2x+3\ge 0\end{cases}$

$\iff \begin{cases} x\ge -2\\ (x+3)(x-1)\le 0\end{cases}$ o $\begin{cases} x<-2\\ \mathbb{R}\end{cases}$

$\iff \begin{cases} x\ge -2\\ x\in[-3,1]\end{cases}$ o $\begin{cases} x<-2\\ \mathbb{R}\end{cases}$ $\quad \underline{S=]-\infty,1].}$

• $3x^2+2x|x|<1 \iff \begin{cases} x\ge 0\\ 3x^2+2x^2<1\end{cases}$ o $\begin{cases} x<0\\ 3x^2-2x^2<1\end{cases}$

$\iff \begin{cases} x\ge 0\\ x^2<1/5\end{cases}$ o $\begin{cases} x<0\\ x^2<1\end{cases}$

$\iff \begin{cases} x\ge 0\\ -\frac{1}{\sqrt5}<x<\frac{1}{\sqrt5}\end{cases}$ o $\begin{cases} x<0\\ -1<x<1\end{cases}$ $\quad \underline{S=]-1,\frac{1}{\sqrt5}[.}$

• $|x|(x^2-2|x|)<0 \iff (x^2-2|x|)<0$

$\iff \begin{cases} x\ge 0\\ x^2-2x<0\end{cases}$ o $\begin{cases} x<0\\ x^2+2x<0\end{cases}$

$\iff \begin{cases} x\ge 0\\ x\in]0,2[\end{cases}$ o $\begin{cases} x<0\\ x\in]-2,0[\end{cases}$ $\quad \underline{S=]-2,2[\setminus\{0\}.}$

- $\dfrac{x+3}{|x|+1} \geq 2 \iff \dfrac{x+3-2|x|-2}{|x|+1} \geq 0 \iff \dfrac{x-2|x|+1}{|x|+1} \geq 0$

$$\iff \begin{cases} x \geq 0 \\ -x+1 \geq 0 \end{cases} \text{ o } \begin{cases} x<0 \\ 3x+1 \geq 0 \end{cases}$$

$$\iff \begin{cases} x \geq 0 \\ x \leq 1 \end{cases} \text{ o } \begin{cases} x<0 \\ x \geq -1/3 \end{cases} \Rightarrow \underline{S=[-1/3, 1]}$$

ii) • $3^x = 81 \iff 3^x = 3^4 \Rightarrow \underline{S=\{4\}}$;

- $9^x = -\frac{1}{81}$ $\quad \underline{S=\emptyset}$, poiché $9^x > 0 \ \forall x \in \mathbb{R}$ e quindi $9^x = -\frac{1}{81}$ è impossibile;

- $3^x > -\frac{1}{27}$ $\quad \underline{S=\mathbb{R}}$, poiché $3^x > 0 \ \forall x \in \mathbb{R}$;

- $2^{-|x+1|} < \frac{1}{2} \iff 2^{-|x+1|} < 2^{-1} \underset{2^x \uparrow}{\iff} -|x+1| < -1 \iff |x+1| > 1$

$$\iff x+1 < -1 \text{ o } x+1 > 1 \Rightarrow \underline{S = ]-\infty, -2[ \cup ]0, +\infty[};$$

- $\left(\frac{1}{4}\right)^{|x-1|} = 1 \iff \left(\frac{1}{4}\right)^{|x-1|} = \left(\frac{1}{4}\right)^0 \iff |x-1| = 0 \Rightarrow \underline{S=\{1\}}$;

- $3^{|x^2-4|} \geq 1 \iff 3^{|x^2-4|} \geq 3^0 \underset{3^x \uparrow}{\iff} |x^2-4| \geq 0$, $\quad \underline{S=\mathbb{R}}$;

sempre, perché $|\cdot| \geq 0$ su $\mathbb{R}$

- $3^{|x^2-4|} > 1 \iff |x^2-4| > 0 \iff \underline{S = \mathbb{R} \setminus \{-2, 2\}}$;

- $2^{-|x|+1} = 1 \iff 2^{-|x|+1} = 2^0 \iff -|x|+1 = 0 \iff |x| = 1 \Rightarrow \underline{S=\{-1,1\}}$.

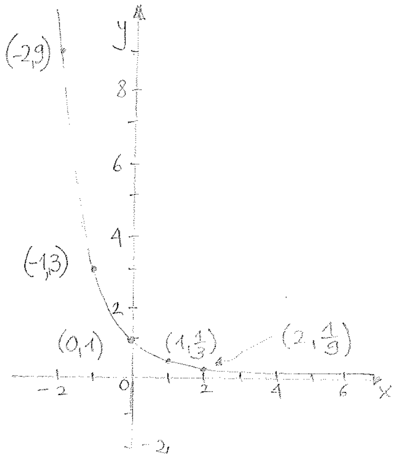
iii) • $-x^2 \log_{1/3} 3 + x \log_4 4 + 4 \log_{1/2} 8 \leq 0 \iff x^2 + x - 12 \leq 0$, poiché $\log_{1/3} 3 = -1$, $\log_4 4 = 1$, $\log_{1/2} 8 = -3$.

Ora $x^2+x-12 \leq 0 \iff (x+4)(x-3) \leq 0 \Rightarrow \underline{S=[-4,3]}$;

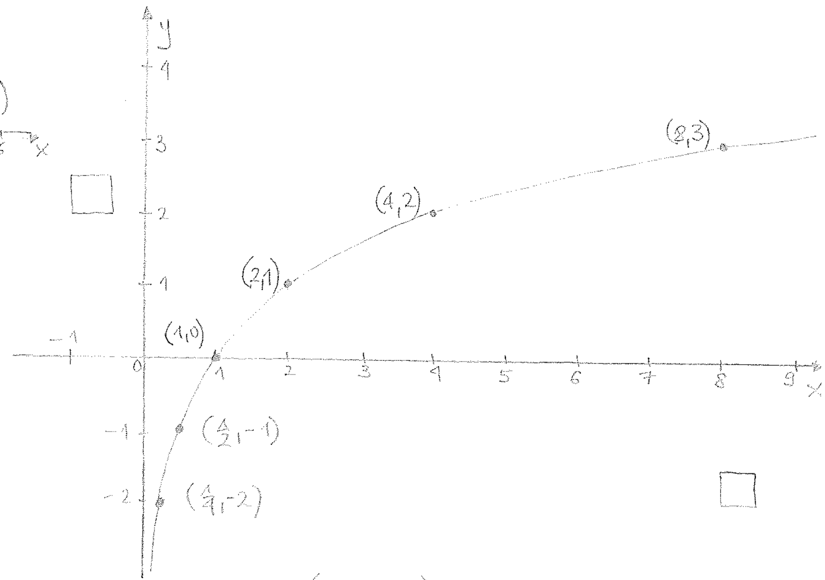
- $\log e^x - \log_{1/4} 4^{-x^2} - \log_3 1 = 0 \iff x - x^2 = 0$, poiché $\log e^x = x \log e = x$, $\log_{1/4} 4^{-x^2} = -x^2 \log_{1/4} 4 = -x^2 \cdot (-1) = x^2$ e $\log_3 1 = 0$.

Ora $x - x^2 = 0 \Rightarrow \underline{S = \{0, 1\}}$.

6) i) $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = \left(\frac{1}{3}\right)^x$




ii) $g : ]0,+\infty[ \to \mathbb{R}$ $\quad g(x) = \log_2 x$




iii) $x \longmapsto -\log_2(x+1)$ $\qquad x \longmapsto (\log_2 |x|) - 1$

$\Uparrow$ $\qquad\qquad\qquad\qquad \Uparrow$

$]-1,+\infty[$ $\qquad\qquad\qquad \mathbb{R}\setminus\{0\}$