Università degli Studi di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2014 – 2015 – Rovereto, 17 – 21 novembre (N. 9)

1) $\lim_{x\to -2^+} f(x) = 3$: $\forall \varepsilon>0\ \exists \delta>0 : \forall x\in \mathrm{dom} f,\ 0< x-(-2)<\delta$
$\Rightarrow |f(x)-3|<\varepsilon$. □

$\lim_{x\to +\infty} f(x) = -\infty$: $\forall M>0, \exists K>0 : \forall x\in \mathrm{dom} f,\ x>K \Rightarrow f(x)<-M$. ■

2) a) $\lim_{x\to 1^-} (x^2-1) = 1-1 = \underline{\underline{0}}$;

$\lim_{x\to 4^+} (\sqrt{x} + \log_2 x) = \sqrt{4} + \log_2 4 = 2+2 = \underline{\underline{4}}$;

$\lim_{x\to 1^+} \frac{1}{\underbrace{(x^3-1)}_{\text{infinitesimo positivo}}} = \underline{\underline{+\infty}}$; $\lim_{x\to 1^-} \frac{1}{\underbrace{(x^3-1)}_{\text{infinitesimo negativo}}} = \underline{\underline{-\infty}}$; □

b) $\lim_{x\to -2^-} \frac{1}{\underbrace{(4-x^2)}_{\text{infinitesimo negativo}}} = \underline{\underline{-\infty}}$; $\lim_{x\to -2^+} \frac{1}{\underbrace{(4-x^2)}_{\text{infinitesimo positivo}}} = \underline{\underline{+\infty}}$;

$\lim_{x\to 2^-} \frac{1}{\underbrace{(|x|-2)}_{\text{infinitesimo negativo}}} = \underline{\underline{-\infty}}$; $\lim_{x\to 2^+} \frac{1}{\underbrace{(|x|-2)}_{\text{infinitesimo positivo}}} = \underline{\underline{+\infty}}$; □

c) $\lim_{x\to 0^+} \frac{1}{x+2x^2} = \lim_{x\to 0^+} \frac{1}{\underbrace{x}_{\text{infinitesimo}>0}\underbrace{(1+2x)}_{\to 1}} = \underline{\underline{+\infty}}$;

$\lim_{x\to 0^-} \frac{1}{x+2x^2} = \lim_{x\to 0^-} \frac{1}{\underbrace{x}_{\text{infinitesimo}<0}\underbrace{(1+2x)}_{\to 1}} = \underline{\underline{-\infty}}$;

$\lim_{x\to +\infty} \frac{3x^3-2x}{4x-4x^3} = \lim_{x\to +\infty} \frac{\cancel{x^3}(3-\overbrace{\frac{2}{x^2}}^{\to 0})}{\cancel{x^3}(\underbrace{\frac{4}{x^2}}_{\to 0}-4)} = \underline{\underline{-\frac{3}{4}}}$;

$\lim_{x\to +\infty} \frac{x^3-2x}{1+x^4} = \lim_{x\to +\infty} \frac{\cancel{x^3}(1-\overbrace{\frac{2}{x^2}}^{\to 0})}{\cancel{x^4}(\underbrace{\frac{1}{x^4}}_{\to 0}+1)} = \underline{\underline{0}}$. ■

3) $f:\mathbb{R}\to\mathbb{R}$,

$$f(x)=\begin{cases} -\frac{3}{(1+x)^2} & \text{se } x<-1 \\ x & \text{se } -1\le x\le 1 \\ 2\log_{\frac{1}{3}} x + 1 & \text{se } x>1 \end{cases}$$

i) $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} -\frac{3}{(1+x)^2} = \underline{0}$,

$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-} -\frac{3}{(1+x)^2} = \underline{\underline{-\infty}}$,

$\lim_{x\to-1^+} f(x) = \lim_{x\to-1^+} x = \underline{\underline{-1}}$, $\quad \lim_{x\to1^-} f(x) = \lim_{x\to1^-} x = 1$;

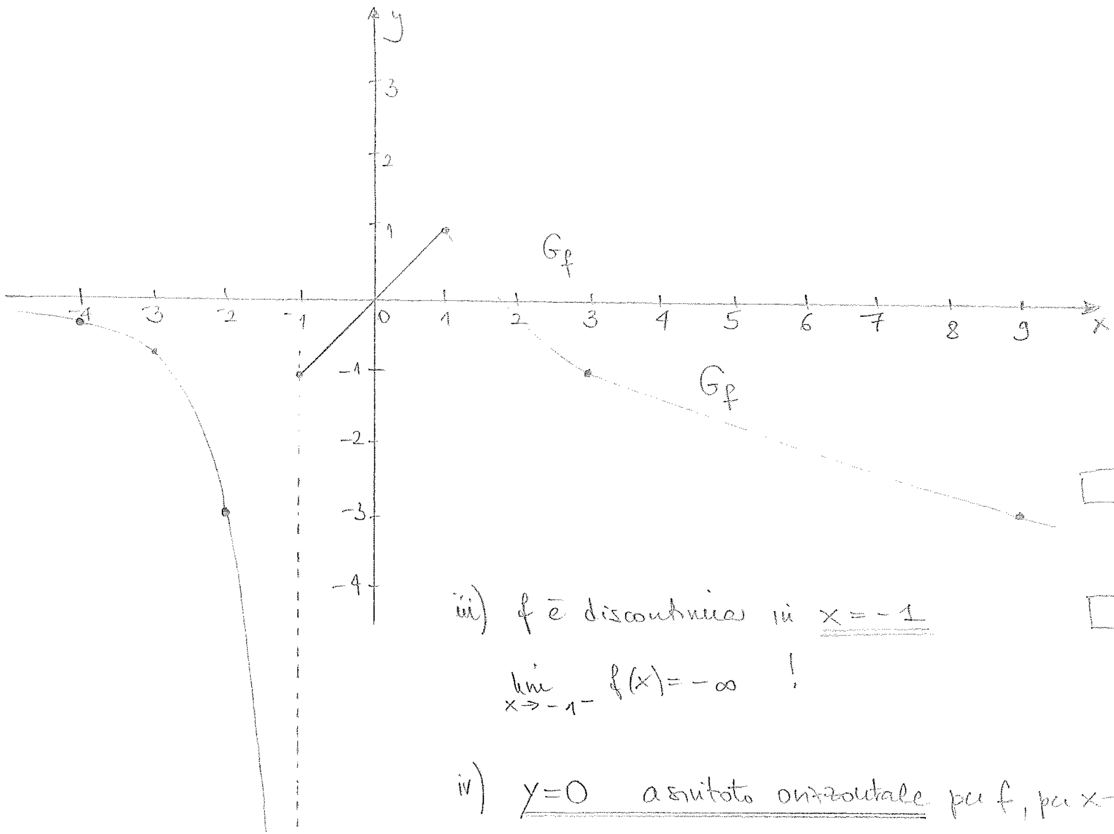
$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} (2\log_{1/3} x + 1) = 1$; $\quad \lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \left[2\log_{1/3} x + 1\right] = \underline{\underline{-\infty}}$. □

ii) $(-4, f(-4)) = (-4, -\frac{3}{9}) = (-4, -1/3)$;

$(-2, f(-2)) = (-2, -3)$;

$(3, f(3)) = (3, -2+1) = (3, -1)$;

$(9, f(9)) = (9, -4+1) = (9, -3)$



□

iii) f è discontinua in $\underline{x=-1}$ □

$\lim_{x\to-1^-} f(x) = -\infty$!

iv) $\underline{y=0}$ $\underline{\text{asintoto orizzontale}}$ per f, per $x\to-\infty$;

$\underline{x=-1}$ $\underline{\text{asintoto verticale}}$ per f. ■

4) i) $\lim_{x\to-\infty} \frac{x}{|x|+1} = \lim_{x\to-\infty} \frac{x}{-x+1} = \lim_{x\to-\infty} \frac{\cancel{x}^{1}}{\cancel{x}(-1+\underbrace{\frac{1}{x}}_{\to 0})} = \underline{\underline{-1}}$;

$\lim_{x\to-\infty} \frac{1-x^4}{x^3} = \lim_{x\to-\infty} \frac{x^{\cancel{4}}(\overbrace{\frac{1}{x^4}}^{\to 0}-1)}{\cancel{x^3}} = \underline{\underline{+\infty}}$;

$\lim_{x\to-\infty} \frac{4}{1-\underbrace{3^x}_{\to 0}} = \underline{\underline{4}}$;

$$\lim_{x\to-\infty} \frac{e^{-x}-1}{e^{-2x}} = \lim_{x\to-\infty} \frac{\cancel{e^{-x}}\left(1-\overbrace{\frac{1}{e^{-x}}}^{\to 0}\right)}{(e^{-x})^{\cancel{2}}} = \underline{0} \ ;$$

ii) $$\lim_{x\to-\infty} \frac{x^2+2}{3\cdot \underbrace{2^x}_{\to 0}-2x^2} = \lim_{x\to-\infty} \frac{\cancel{x^2}\left(1+\overbrace{\frac{2}{x^2}}^{\to 0}\right)}{\cancel{x^2}\left(\underbrace{\frac{3\cdot 2^x}{x^2}}_{\to 0}-2\right)} = \underline{-\frac{1}{2}} \ ;$$

$$\lim_{x\to+\infty} \frac{x^2+2}{3\cdot 2^x-2x^2} = \lim_{x\to+\infty} \frac{x^2\left(1+\overbrace{\frac{2}{x^2}}^{\to 0}\right)}{2^x\left(3-\underbrace{\frac{2x^2}{2^x}}_{\to 0}\right)} = \underline{0} \ ;$$

$$\lim_{x\to+\infty} \frac{x^2+2^x}{2\cdot 3^x-2x^2} = \lim_{x\to+\infty} \frac{2^x\left(\overbrace{\frac{x^2}{2^x}}^{\to 0}+1\right)}{3^x\left(2-\underbrace{\frac{2x^2}{3^x}}_{\to 0}\right)} = \underline{0} \ ;$$

$$\lim_{x\to+\infty} \frac{x^2-3^x}{3\cdot 2^x-2x^2} = \lim_{x\to+\infty} \frac{3^x\left(\overbrace{\frac{x^2}{3^x}}^{\to 0}-1\right)}{2^x\left(3-\underbrace{\frac{2x^2}{2^x}}_{\to 0}\right)} = \underline{-\infty} \ ;$$

iii) $$\lim_{x\to0^-} \frac{e^{4x}-1}{2x} = \lim_{x\to0^-} \underbrace{\frac{e^{4x}-1}{4x}}_{\to 1}\cdot 2 = \underline{2} \ ;$$

$$\lim_{x\to0^+} \frac{e^{-x}-1}{x} = \lim_{x\to0^+} \underbrace{\frac{e^{-x}-1}{-x}}_{\to 1}\cdot(-1) = \underline{-1} \ ;$$

$$\lim_{x\to1^+} \frac{e^{x-1}-1}{x^2-1} = \lim_{x\to1^+} \underbrace{\frac{e^{x-1}-1}{x-1}}_{\to 1}\cdot\frac{1}{x+1} = \underline{\frac{1}{2}} \ ;$$

iv) $$\lim_{x\to+\infty} \frac{\log(1+x^2)}{2x^2} = \underline{0} \ ;$$

$$\lim_{x\to0} \frac{\log(1+x^2)}{2x^2} = \lim_{x\to0} \overbrace{\frac{\log(1+x^2)}{x^2}}^{\to 1}\cdot\frac{1}{2} = \underline{\frac{1}{2}} \ ;$$

$$\lim_{x\to-\infty} \frac{\log(1+x^2)}{x^3} = \underline{0} \ ;$$

$$\lim_{x\to0^+} \frac{e^{2x}-1}{e^{\log(2x)}} = \lim_{x\to0^+} \frac{e^{2x}-1}{2x} = 1.$$

5) i) a) $f(x)=\frac{1}{x^3}$ $\quad f'(x) = -\frac{3}{x^4}$ $\quad y = f(x_0)+f'(x_0)(x-x_0)$ eq. retta tg. al grafico di f in $(x_0, f(x_0))$;

quindi $y=-1-3(x+1)$ eq. retta tg. al grafico di f nel pt $(1,1)$

$\underline{y=-3x-4}$;

b) $g(x)=\sqrt[3]{x}$ $\qquad g'(x)=\frac{1}{3\sqrt[3]{x^2}}$ $\qquad : \quad y=2+\frac{1}{12}(x-8)$

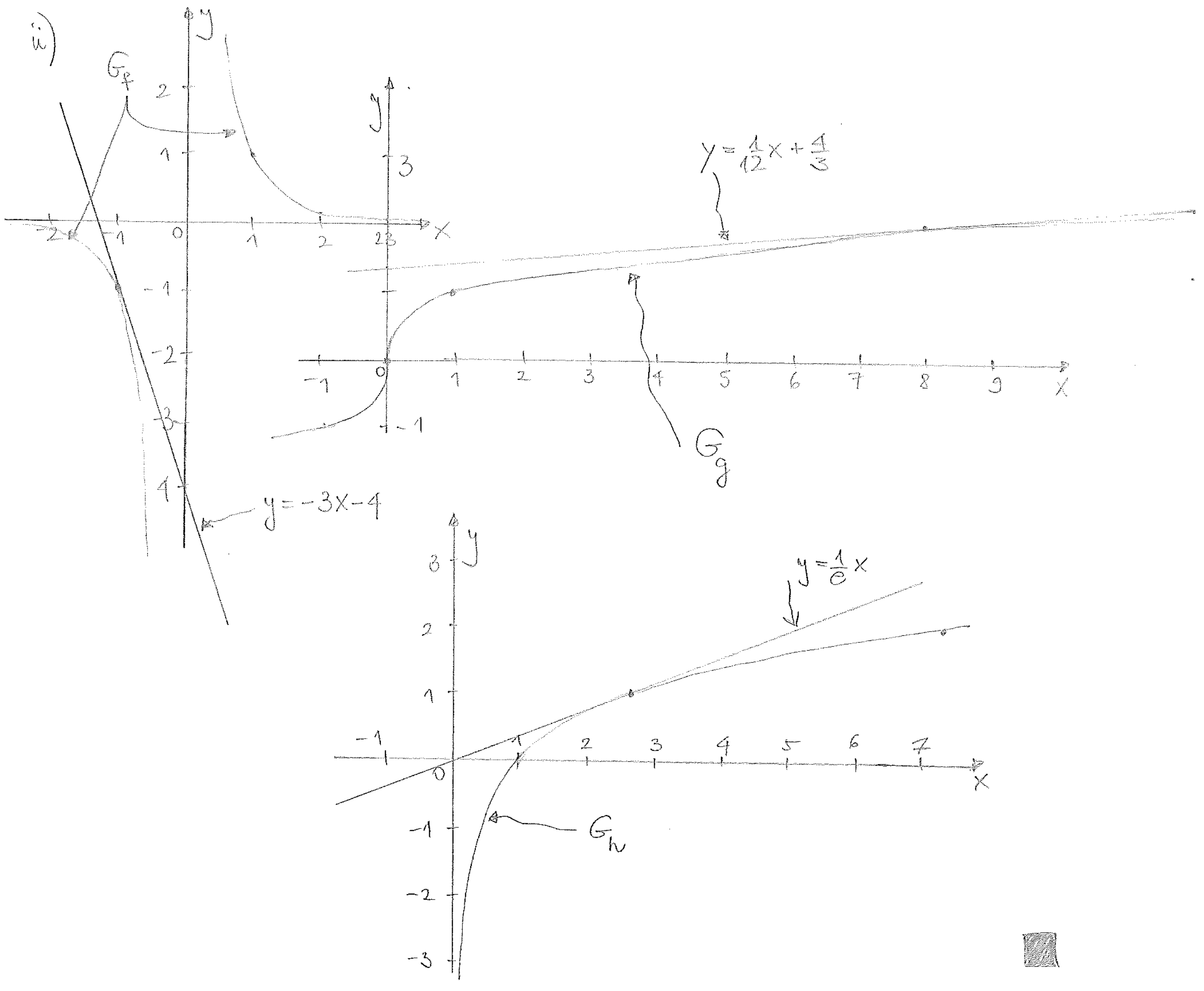
$y=\frac{1}{12}x+2-\frac{2}{3}$, ossia

$\underline{y=\frac{1}{12}x+\frac{4}{3}}$ eq. retta tg. al grafico di g nel pt. $(8,2)$;

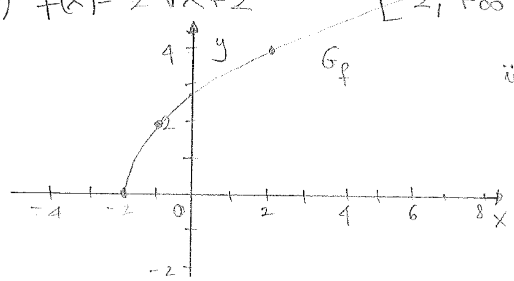
c) $h(x)=\log x$ $\qquad h'(x)=\frac{1}{x}$ $\qquad : \quad y=1+\frac{1}{e}(x-e)$

$\underline{y=\frac{1}{e}x}$ eq. retta tg. al grafico di h nel pt. $(e,1)$. □

ii)

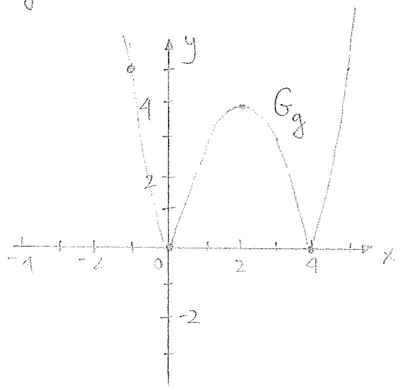


6) i) a) $f(x)=2\sqrt{x+2}$ $\qquad [-2,+\infty[$



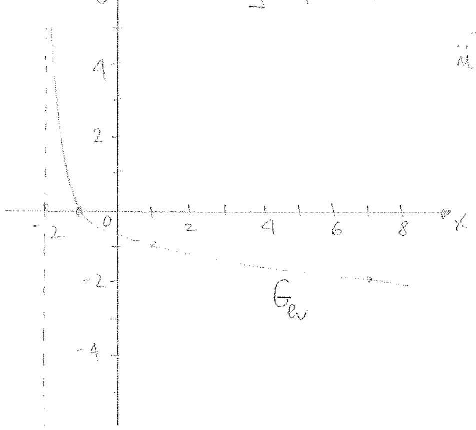
ii) Il segno della derivata di f in $x=-1$ ed in $x=2$ è positivo. □

b) $g(x)=|x^2-4x|$ $\mathbb{R}$,



ii) Il segno della derivata di g in $x=-1$ è negativo; in $x=2$, la derivata vale 0. $\square$

c) $\log_{1/3}(x+2)$ $]-2,+\infty[$



ii) Il segno della derivata di h in $x=-1$ ed in $x=2$, è negativo. $\blacksquare$