Università degli Studi di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2014/15 – Rovereto, 6 – 10 ottobre 2014

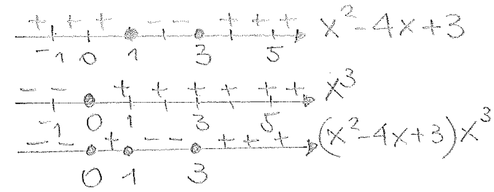
N. 3

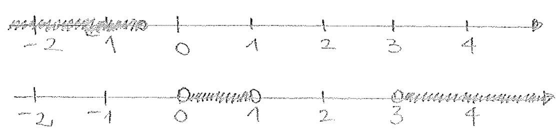
1) i) $A = \left\{x \in \mathbb{R} : \frac{2x^2-4x}{x^2+1} \geq 2\right\} = \left\{x \in \mathbb{R} : \frac{-4x-2}{x^2+1} \geq 0\right\} = \left\{x \in \mathbb{R} : \frac{4x+2}{x^2+1} \leq 0\right\}$

$= \left]-\infty, -\frac{1}{2}\right]$;

$B = \left\{x \in \mathbb{R} : \frac{(x^2-4x+3)x^3}{4} > 0\right\}$

$= \left]0,1\right[\cup \left]3,+\infty\right[$.





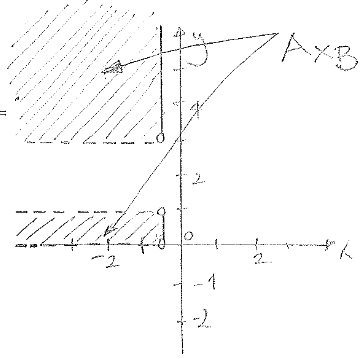
A è un intervallo di $\mathbb{R}$;

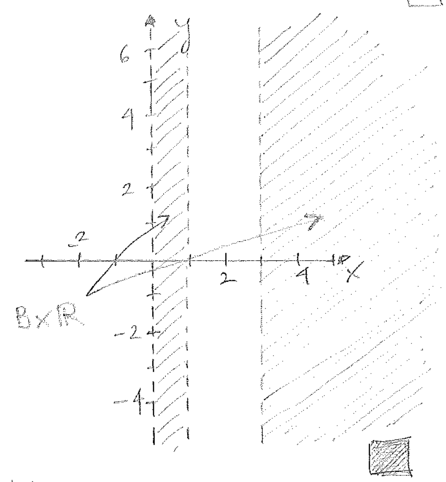
B non è un intervallo di $\mathbb{R}$.

ii) $A \cup B = \left]-\infty, -\frac{1}{2}\right] \cup \left]0,1\right[\cup \left]3,+\infty\right[$;

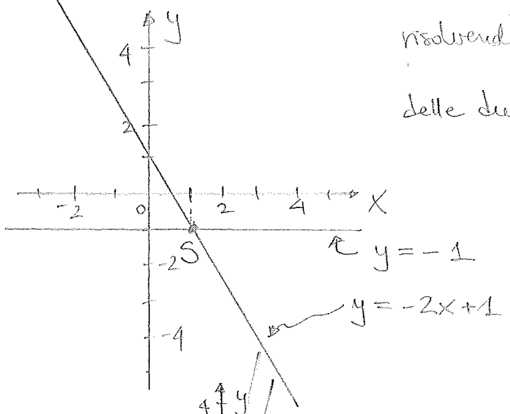
$A \cap B = \emptyset$

$B \setminus A = B$



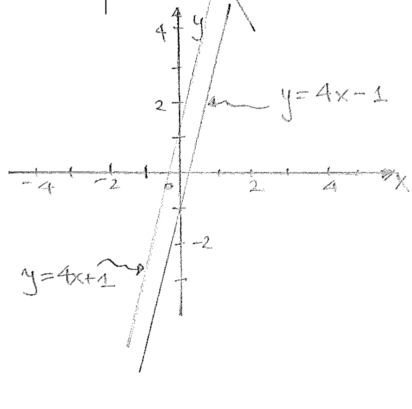


2) i) • $-2x+1 = -1 \iff -2x = -2 \iff x=1$. Interpret. geometrica dell'eq. $-2x+1=-1$: risolvendo l'eq. si trova la coordinata x del pt. di intersezione S delle due rette di eq. $y=-2x+1$ e $y=-1$.



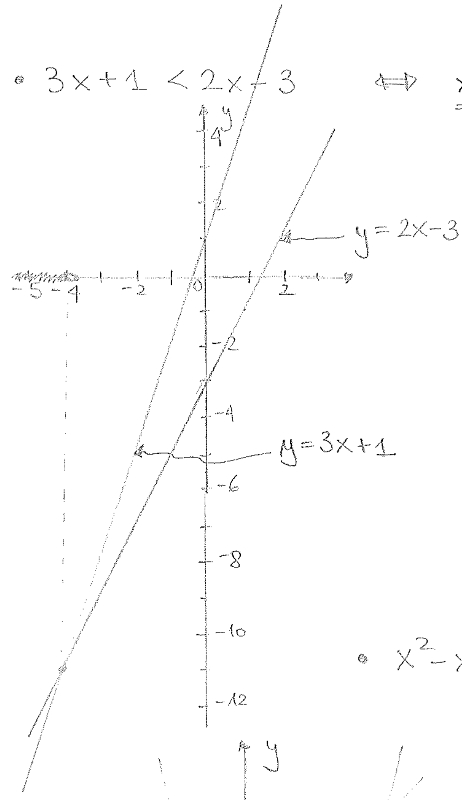
• $4x+1 > 4x-1 \iff 1 > -1$

$\forall x \in \mathbb{R}$

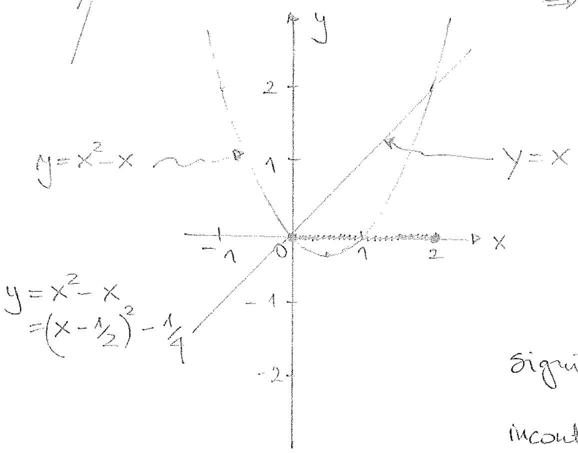


Interpretazione geom. della diseq. $4x+1 > 4x-1$: La soluzione corrisponde alle coordinate x dei pt. (x,y) appartenenti alla retta $y=4x+1$, per i quali $y=4x+1$ è maggiore di $y=4x-1$ che intuitivamente significa le x per cui la retta $y=4x+1$ "sta sopra" la retta $y=4x-1$.

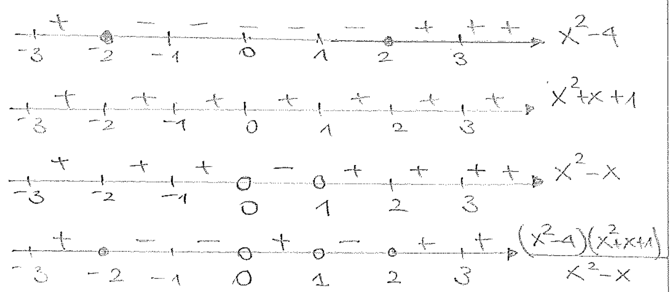
- $3x+1 < 2x-3 \iff \underline{x < -4}$. Interpretazione geom. della diseq. $3x+1<2x-3$: la soluzione corrisponde alle coordinate x dei pt. (x,y) appartenenti alla retta $y=3x+1$, per i quali $y=3x+1$ è minore di $y=2x-3$ che intuitivamente significa le x per cui la retta $y=3x+1$ "sta sotto" la retta $y=2x-3$.



- $x^2-x \le x \iff x^2-2x \le 0 \iff x(x-2)\le 0$
$\Rightarrow \underline{x \in [0,2]}$. Interpretazione geom. della diseq. $x^2-x \le x$: la soluzione corrisponde alle coordinate x dei pt. (x,y) appartenenti alla parabola $y=x^2-x$, per i quali $y=x^2-x$ è minore o uguale di $y=x$ che intuitivamente significa le x per cui la parabola "sta sotto o incontra" la retta $y=x$. $\square$



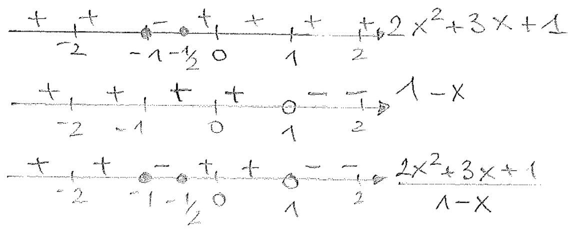
ii) - $\dfrac{(x^2-4)(x^2+x+1)}{x^2-x} \ge 0$



$\underline{S =]-\infty,-2] \cup]0,1[\cup [2,+\infty[}$.

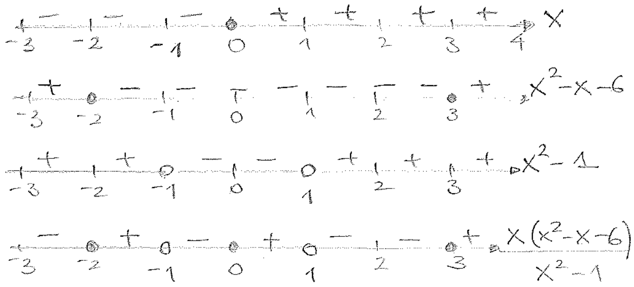
- $\dfrac{x^2+5x}{1-x} < x-1 \iff \dfrac{x^2+5x}{1-x} - x + 1 < 0 \iff \dfrac{x^2+5x-x+x^2+1-x}{1-x} < 0$

$\iff \dfrac{2x^2+3x+1}{1-x} < 0$



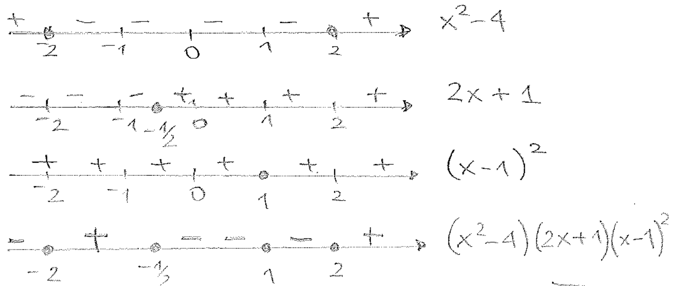
$\underline{S =]-1,-\tfrac{1}{2}[\cup]1,+\infty[}$.

- $\dfrac{x^3-6x-x^2}{x^2-1}>0 \iff \dfrac{x(x^2-x-6)}{x^2-1}>0$



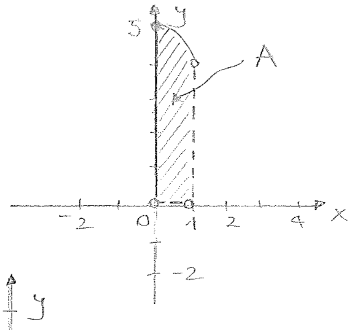
$S =]-2,-1[\cup]0,1[\cup]3,+\infty[$

- $(x^2-4)(2x+1)(x-1)^2 \leq 0$



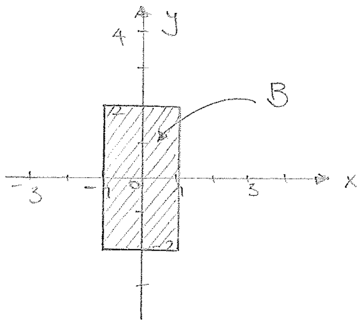
$S =]-\infty,-2] \cup [-\frac{1}{2}, 2]$

3) i) $A = \{(x,y)\in\mathbb{R}^2 : 0 \leq x < 1,\ 0 < y \leq -x^2+5\}$

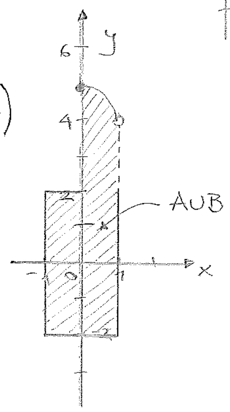


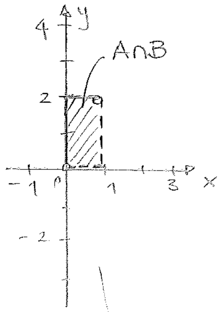
$B = \{(x,y)\in\mathbb{R}^2 : x^2 \leq 1,\ y^2 \leq 4\} =$

$= \{(x,y)\in\mathbb{R}^2 : -1 \leq x \leq 1,\ -2 \leq y \leq 2\}$



ii)





4) i) $y = x^2+4x+1$

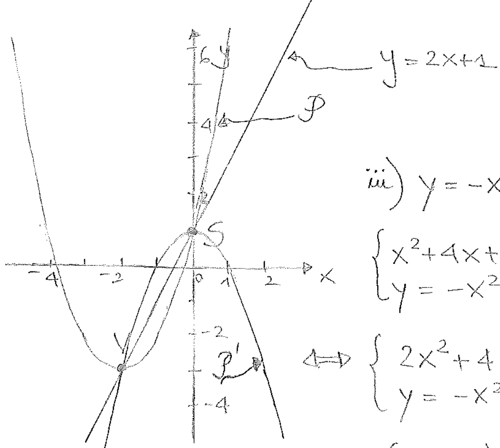
$= (x+2)^2-3$

ii) $V = (-2,-3)$

$S = (0,1)$

$r: \quad \dfrac{y-1}{-3-1} = \dfrac{x-0}{-2-0}$

$\iff y-1 = 2x \qquad y = 2x+1$

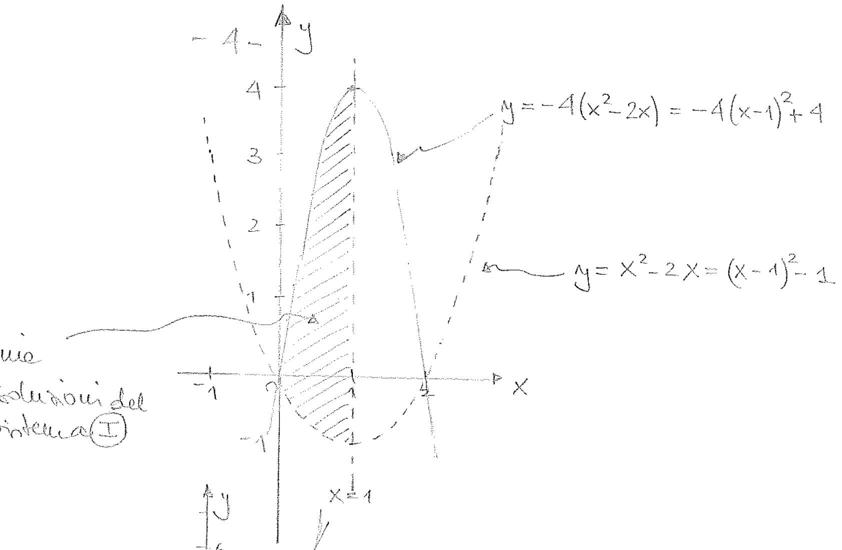


iii) $y = -x^2+1$

$\begin{cases} x^2+4x+1 = -x^2+1 \\ y = -x^2+1 \end{cases} \iff \begin{cases} 2x^2+4x = 0 \\ y = -x^2+1 \end{cases}$

$\Rightarrow S_1 = (-2,-3) = V \qquad S_2 = (0,1) = S$

5) I
$$\begin{cases} y > x^2 - 2x \\ y \leq -4x^2 + 8x \\ x < 1 \end{cases}$$



II
$$\begin{cases} y \geq x^2 + x - 6 \\ -4 < y \leq 2x \end{cases}$$