Università degli Studi di Trento - Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2014 – 2015 – Rovereto, 8 – 12 dicembre (N. 12)

1) i) $\boxed{f(x) = \frac{1}{x(x+3)}}$

- dom $f = \mathbb{R} \setminus \{-3, 0\} =]-\infty, -3[\cup]-3, 0[\cup]0, +\infty[$
- segno di f

- $\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(x) = 0$ $y = 0$ asintoto orizzontale per f, per $x \to -\infty$ $(+\infty)$;

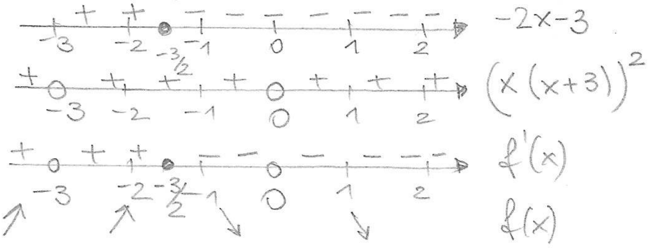
$\lim_{x \to -3^-} f(x) = +\infty$ $x = -3$ asintoto verticale per f

$\lim_{x \to -3^+} f(x) = -\infty$ "

$\lim_{x \to 0^-} f(x) = -\infty$ $x = 0$ asintoto verticale per f

$\lim_{x \to 0^+} f(x) = +\infty$ "

- continuità di f su $\mathbb{R} \setminus \{-3, 0\}$
- dom $f' = \mathbb{R} \setminus \{-3, 0\}$ $f'(x) = \frac{-2x - 3}{(x(x+3))^2}$



$x = -\frac{3}{2}$ pt. critico di f

$f\left(-\frac{3}{2}\right) = \frac{1}{-\frac{3}{2}\left(\frac{3}{2}\right)} = -\frac{4}{9}$

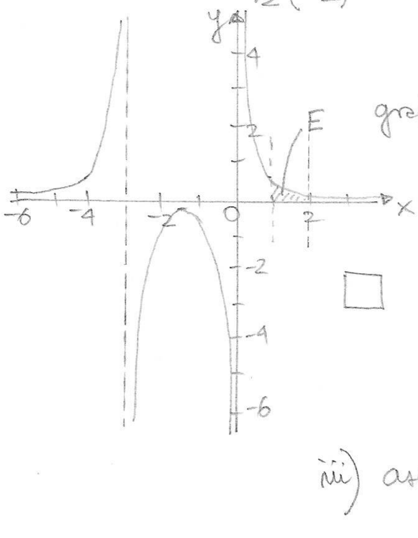


grafico qualitativo di f ☐

☐

ii) $F(x) = \frac{1}{3}[\log x - \log(x+3)]$ è derivabile su $]0, +\infty[$. Inoltre

$F'(x) = \frac{1}{3}\left[\frac{1}{x} - \frac{1}{x+3}\right] = \frac{1}{3}\left[\frac{\cancel{x}+3-\cancel{x}}{x(x+3)}\right] = f(x)$

su $]0, +\infty[$. ☐

iii) $\text{area}(E) = \int_1^2 f(x)\,dx = \left[F(x)\right]_1^2 =$

$= F(2) - F(1) = \frac{1}{3}\left[(\log 2 - \log 5) - (\cancel{\log} 1 - \log 4)\right]$

$= \underline{\underline{\frac{1}{3} \log \frac{8}{5}}}$.

■

2) i) $\boxed{g(x) = (x^2+2)\,e^{-x}} = \dfrac{(x^2+2)}{e^x}$
- dom $g = \mathbb{R}$
- segno di g : sempre positiva

- $\lim\limits_{x\to-\infty} g(x) = +\infty$ $\quad \lim\limits_{x\to+\infty} g(x) = 0$ $\quad y = 0$ asintoto orizzontale per f, per $x \to +\infty$.

- continuità di g su tutto $\mathbb{R}$!

- dom $g' = \mathbb{R}$ $\quad g'(x) = 2xe^{-x} - (x^2+2)e^{-x} = e^{-x}(-x^2+2x-2)$

$g'(x) < 0 \quad \forall x \in \mathbb{R}$, quindi $g(x) \searrow$ su $\mathbb{R}$

dom $g'' = \mathbb{R}$ $\quad g''(x) = \dfrac{(x-2)^2}{e^x} \geq 0 \Rightarrow g$ convessa su $\mathbb{R}$.

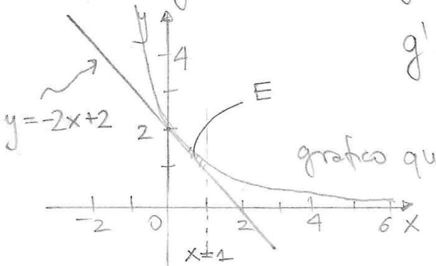


grafico qualitativo di g

$\square$

ii) $G(x) = (-x^2-2x-4)\,e^{-x}$ è derivabile su $\mathbb{R}$ e

$$G'(x) = (-2x-2)\,e^{-x} - (-x^2-2x-4)\,e^{-x} = e^{-x}(x^2+\cancel{2x}+4-\cancel{2x}-2)$$
$$= g(x) \quad \forall x \in \mathbb{R}.$$

$\square$

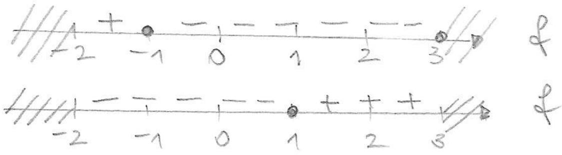
iii) retta tg al grafico di g nel pt $(0,2)$: $\quad y = 2 - 2(x-0)$

$\underline{y = -2x+2}$

$$\text{area}(E) = \int_0^1 \left(g(x) + 2x - 2\right) dx = \int_0^1 g(x)\,dx + \int_0^1 (2x-2)\,dx =$$
$$= \Big[G(x)\Big]_0^1 + \Big[x^2 - 2x\Big]_0^1 = G(1) - G(0) + (1-2) = -\frac{7}{e} + 4 - 1$$
$$= \underline{\underline{3 - \frac{7}{e}}}.$$

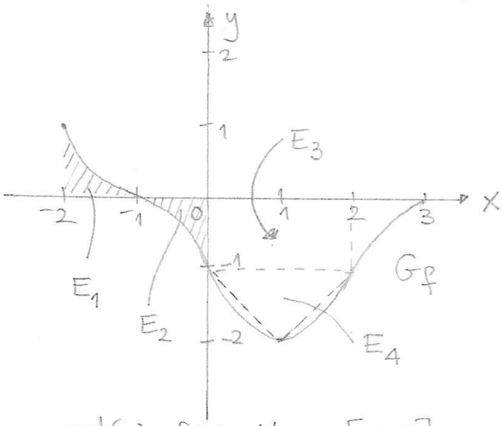
■

3) i)



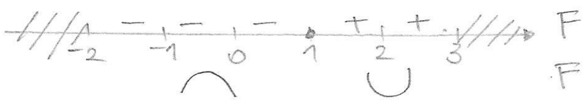
ii) Dal teorema fondamentale del calcolo si ha $F'(x) = f(x) \ \forall x \in [-2,3]$; dal segno di f si ottiene il segno di F' e quindi la monotonia di F : risulta che F è crescente su $[-2,-1]$, mentre è decrescente ristretta all'intervallo $[-1,3]$.

iii)



$$\min_{x\in[-2,3]} F(x) < \cancel{area}(E_1) - \cancel{area}(E_2) - area(E_3) - area(E)_4 = -2-1=-3 \quad \square$$

iv) Da $F'(x)=f(x)$ $\forall x\in[-2,3]$ si ottiene (essendo f derivabile) $F''(x)=f'(x)$ $\forall x\in[-2,3]$. Quindi dal segno di f' si ottiene il segno di $F''(x)$ e quindi la convessità/concavità di $F(x)$:



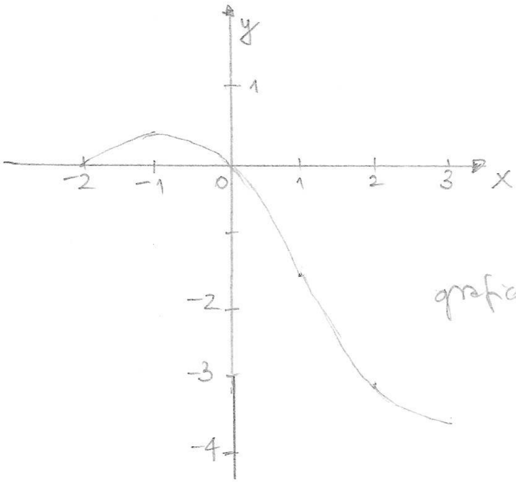
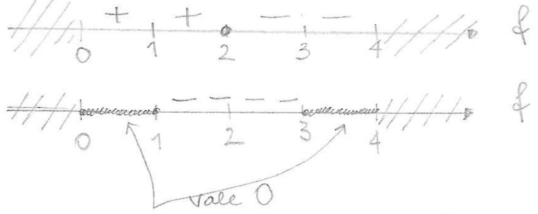


grafico qualitativo di F

$\blacksquare$

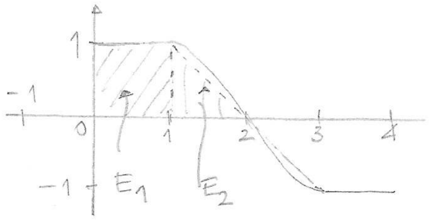
4) i)



ii) Dal teorema fondamentale del calcolo si ha $F'(x)=f(x)$ $\forall x\in[0,4]$

Da questo possiamo dedurre che $x=2$ è un pt. critico per F. $\square$

iii)



$$\max_{x\in[0,4]} F(x) = F(2) > \underset{area(E_1)}{\overset{\parallel}{1}} + \underset{area(E_2)}{\overset{\parallel}{\frac{1}{2}}} = \frac{3}{2}$$

$$\max_{x\in[0,4]} F(x) < 2\,area(E_1) = 2. \quad \square$$

iv) Da $F''(x)=f'(x)$ segue che F è concava in $[0,4]$ (lineare su $[0,1]$ e lineare su $[3,4]$)

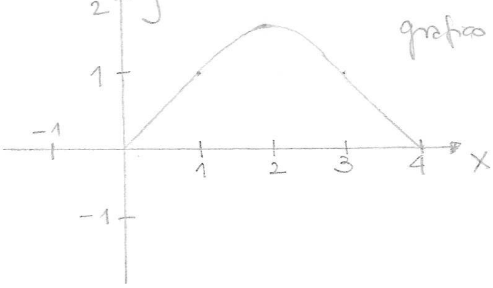


grafico qualitativo di F

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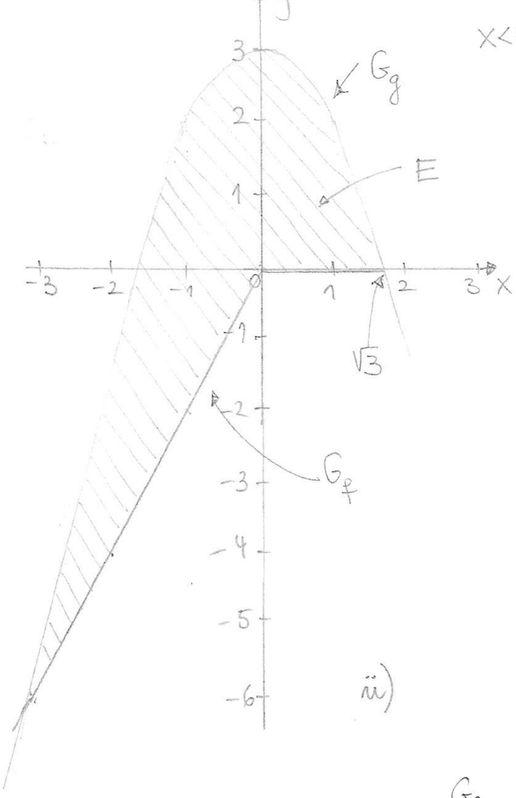
5) i) $f(x)=x-|x| \qquad g(x)=-x^2+3$



$x<0 \qquad 2x=-x^2+3 \iff x^2+2x-3=0$

$(x+3)(x-1)=0$

$$\text{area}(E)=\int_{-3}^{0}(-x^2+3-2x)\,dx+\int_{0}^{\sqrt{3}}(-x^2+3)\,dx$$

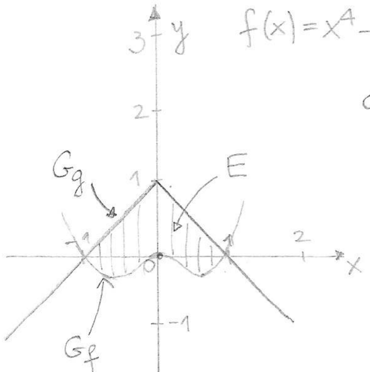
$$=\left[-\frac{x^3}{3}+3x-x^2\right]_{-3}^{0}+\left[-\frac{x^3}{3}+3x\right]_{0}^{\sqrt{3}}$$

$$=-\left[\cancel{9}-\cancel{9}-9\right]+\left[-\sqrt{3}+3\sqrt{3}\right]$$

$$=\underline{\underline{9+2\sqrt{3}}}$$

□

ii) $f(x)=x^4-x^2 \qquad g(x)\underset{1}{=}-x^2+1$



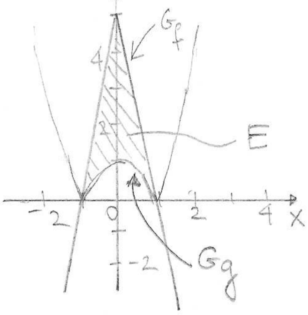
$$\text{area}(E)=2\int_{0}^{1}(1-x-x^4+x^2)\,dx$$

$$=2\left[x-\frac{x^2}{2}-\frac{x^5}{5}+\frac{x^3}{3}\right]_{0}^{1}$$

$$=2\left[1-\frac{1}{2}-\frac{1}{5}+\frac{1}{3}\right]$$

$$=\overset{1}{\cancel{2}}\left(\frac{30-15-6+10}{\cancel{30}_{15}}\right)=\underline{\underline{\frac{19}{15}}}.$$

iii) $f(x)=-5|x|+5 \qquad g(x)=|x^2-1|$



$$\text{area}(E)=2\int_{0}^{1}\left(-5x+5-(-x^2+1)\right)dx=2\int_{0}^{1}(-5x+4+x^2)\,dx$$

$$=2\left[-5\frac{x^2}{2}+4x+\frac{x^3}{3}\right]_{0}^{1}=2\left[-\frac{5}{2}+4+\frac{1}{3}\right]=\frac{-15+24+2}{3}$$

$$=\underline{\underline{\frac{11}{3}}}.$$

□

6) i) $P_6^2 = \frac{6!}{2!} = \frac{6\cdot5\cdot4\cdot3\cdot\cancel{2}}{\cancel{2}} = 30\cdot12 = \underline{\underline{360}}.$ □

ii) $C_{n,2} = 6 \iff \frac{n!}{(n-2)!\,2} = 6 \iff n(n-1) = 12$

$\iff n^2 - n - 12 = 0 \iff n_{1/2} = \frac{1\pm\sqrt{1+48}}{2} = \frac{1\pm7}{2} \begin{array}{l} \nearrow 4 \\ \cancel{\searrow -3} \end{array}$ □

iii) $P_5 = 5! = 5\cdot4\cdot3\cdot2 = 20\cdot6 = \underline{\underline{120}};$

$P_{11}^{2,2,4,3} = \frac{11!}{2!\,2!\,4!\,3!} = \frac{11\cdot10\cdot9\cdot\overset{2}{\cancel{8}}\cdot7\cdot\cancel{6}\cdot5\cdot\cancel{4}\cdot\cancel{3\cdot2}}{\cancel{2}\cdot\cancel{2}\cdot\cancel{4}\cdot\cancel{3\cdot2}\cdot\cancel{3\cdot2}} = 990\cdot70$

$= \underline{\underline{69.300}};$

$C_{8,5} = \frac{8!}{3!\,5!} = \frac{8\cdot7\cdot\cancel{6}\cdot\cancel{5!}}{\cancel{3\cdot2}\cdot\cancel{5!}} = \underline{\underline{56}};$

$D_{4,4} = \frac{4!}{0!} = 4! = P_4$ □

iv) $D_{n,n} = \frac{n!}{0!} = n! = P_n$

$C_{n,n} = \frac{\cancel{n!}}{0!\,\cancel{n!}} = 1.$ ■