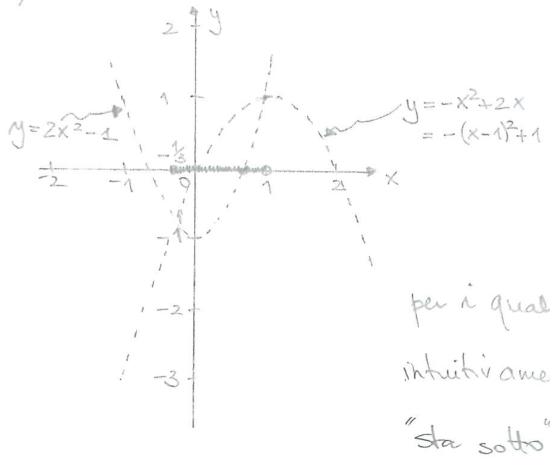
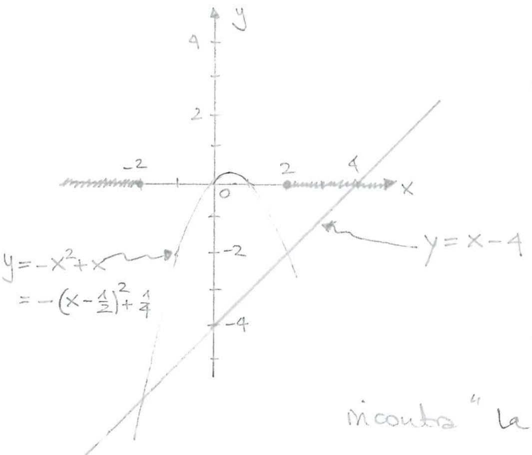
Università degli Studi di Trento − Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2014/15 − Rovereto, 13−17 ottobre 2014 (N. 4)

1) i) $2x^2-1 < -x^2+2x \iff 3x^2-2x-1<0 \Rightarrow \underline{x \in \left]-\tfrac{1}{3}, 1\right[}$.




Interpretazione geometrica della diseq. $2x^2-1<-x^2+2x$: la soluzione corrisponde alle coordinate x dei pt. $(x,y)$ appartenenti alla parabola $y=2x^2-1$ per i quali $y=2x^2-1$ è minore di $y=-x^2+2x$ che intuitivamente significa le x per cui la parabola $y=2x^2-1$ "sta sotto" la parabola $y=-x^2+2x$. □

ii) $-x^2+x \le x-4 \iff -x^2+4\le 0 \iff x^2-4\ge 0 \Rightarrow \underline{x\le -2 \text{ o } x\ge 2}$.

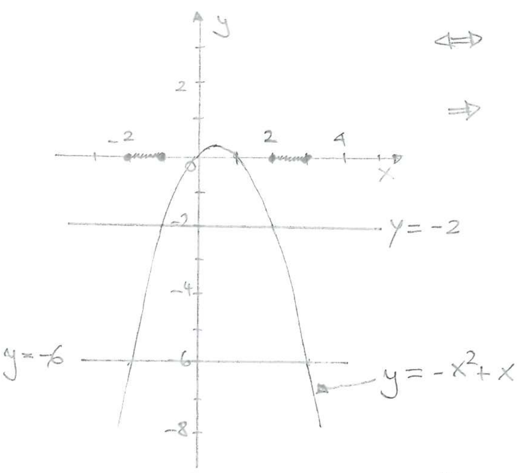



Interpret. geom. della diseq. $-x^2+x\le x-4$: la soluzione corrisponde alle x dei pt. $(x,y)$ appartenenti alla parabola $y=-x^2+x$, per i quali $y=-x^2+x$ è minore o uguale di $y=x-4$ che intuitiv. significa le x per cui la parabola $y=-x^2+x$ "sta sotto o incontra" la retta $y=x-4$. □

iii) $-6 \le -x^2+x \le -2 \iff \begin{cases} -x^2+x+6\ge 0 \\ -x^2+x+2\le 0 \end{cases} \iff \begin{cases} (x-3)(x+2)\le 0 \\ (x-2)(x+1)\ge 0 \end{cases}$

$\iff x\in[-2,3]\cap\left(\left]-\infty,-1\right]\cup\left[2,+\infty\right[\right)$

$\Rightarrow \underline{x\in[-2,-1]\cup[2,3]}$.



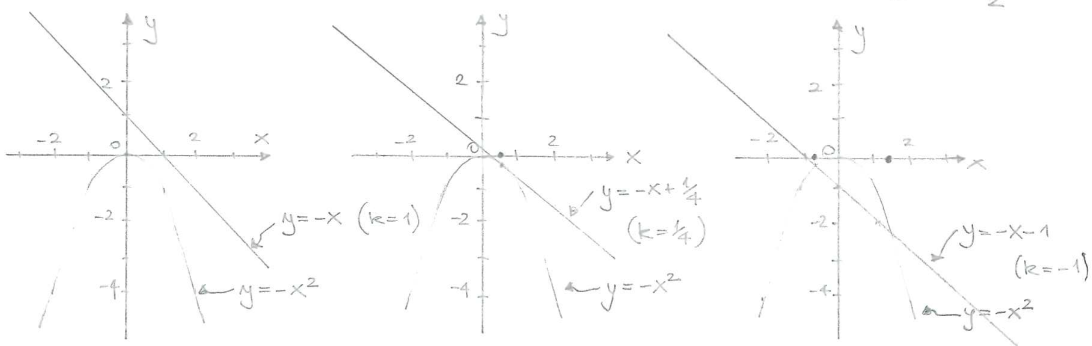

Interpret. geom. della diseq. $-6\le -x^2+x\le -2$: la soluzione corrisponde alle x dei pt. $(x,y)$ appartenenti alla parabola $y=-x^2+x$, per i quali $y=-x^2+x$ è maggiore o uguale di $-6$ e minore o uguale di $-2$ che intuitiv. significa le x per cui la parabola $y=-x^2+x$ "sta sopra o incontra" la retta $y=-6$ e "sta sotto o incontra" la retta $y=-2$. □

iv) $-x^2 = -x + k \iff -x^2 + x - k = 0 \iff x_{1/2} = \dfrac{-1 \pm \sqrt{1-4k}}{-2}$

Ora se $1-4k < 0$, ossia $k > \frac{1}{4}$, $\nexists$ soluzione di $-x^2 = -x + k$.

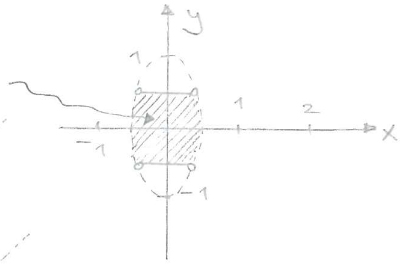
se $1-4k = 0$, ossia $k = \frac{1}{4}$, $\exists!$ soluzione $x = \frac{1}{2}$;

se $1-4k > 0$, ossia $k < \frac{1}{4}$, sono due soluzioni $x_{1/2} = \dfrac{1 \pm \sqrt{1-4k}}{2}$

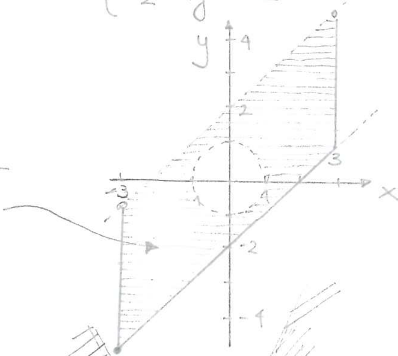



<u>Interpret. geom.</u> La soluzione corrisponde alla coordinate $x$ dei pt. $(x, y)$ appartenenti alla retta $y = -x + k$ per i quali $y = -x^2$ è uguale a $y = -x + k$ che intuitiv. significa le $x$ per cui la retta $y = -x + k$ interseca la parabola $y = -x^2$; se $k > \frac{1}{4}$ non c'è intersezione, se $k = \frac{1}{4}$ c'è solo un pt. di intersezione, se $k < \frac{1}{4}$ ce ne sono 2 ∎

2) i) $\begin{cases} 4x^2 + y^2 < 1 \\ y^2 \le \frac{1}{4} \end{cases} \iff \begin{cases} \dfrac{x^2}{1/4} + y^2 < 1 \\ -\frac{1}{2} \le y \le \frac{1}{2} \end{cases}$



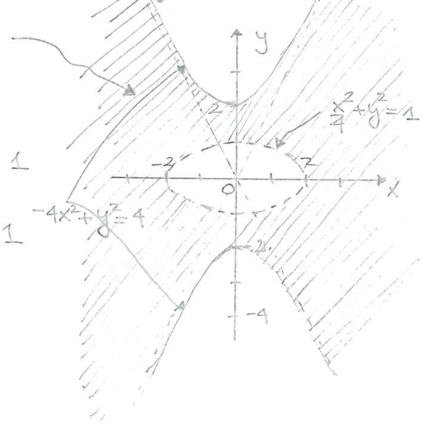
ii) $\begin{cases} x^2 + y^2 > 1 \\ x - 2 \le y < x + 2 \\ -3 \le x \le 3 \end{cases}$



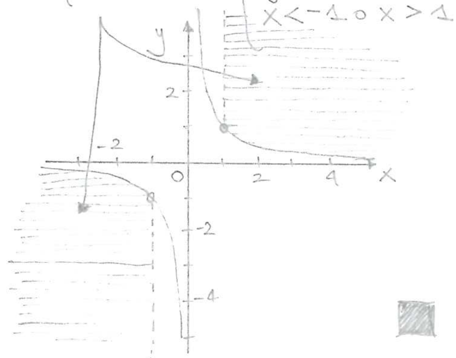
iii) $\begin{cases} -4x^2 + y^2 \le 4 \\ x^2 + 4y^2 > 4 \end{cases}$

$\Updownarrow$

$\begin{cases} -x^2 + \dfrac{y^2}{4} \le 1 \\ \dfrac{x^2}{4} + y^2 > 1 \end{cases}$

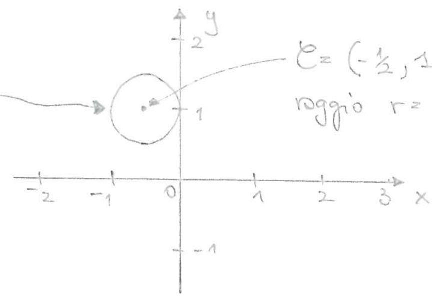



iv) $\begin{cases} xy \ge 1 \\ x^2 > 1 \end{cases} \iff \begin{cases} y \ge \frac{1}{x} & \text{se } x > 0 \\ y \le \frac{1}{x} & \text{se } x < 0 \\ x < -1 \text{ o } x > 1 \end{cases}$



∎

3) i) $\left(x+\frac{1}{2}\right)^2+(y-1)^2=\frac{1}{4}$

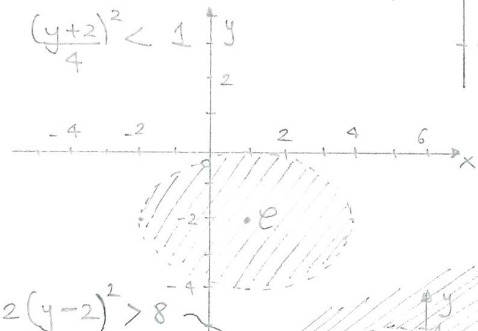


$\mathcal{C}=\left(-\frac{1}{2},1\right)$

raggio $r=\frac{1}{2}$

ii) $4(x-1)^2+9(y+2)^2<36$

$\Leftrightarrow \quad \frac{(x-1)^2}{9}+\frac{(y+2)^2}{4}<1$

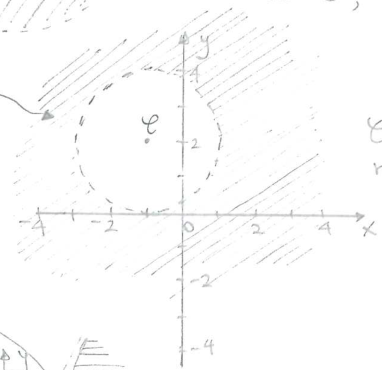


$\mathcal{C}=(1,-2)$

$a=3$, $b=2$

iii) $2(x+1)^2+2(y-2)^2>8$

$\Leftrightarrow \quad (x+1)^2+(y-2)^2>4$

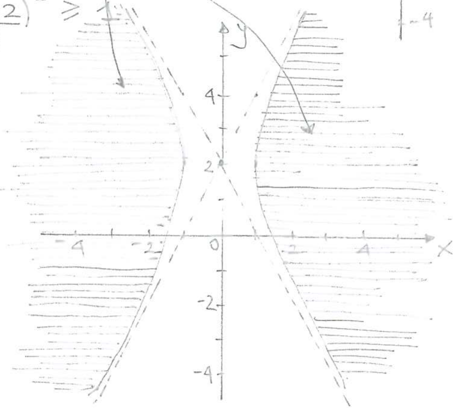


$\mathcal{C}=(-1,2)$

$r=2$

iv) $4x^2-(y-2)^2\geq 4$

$\Leftrightarrow \quad x^2-\frac{(y-2)^2}{4}\geq 1$

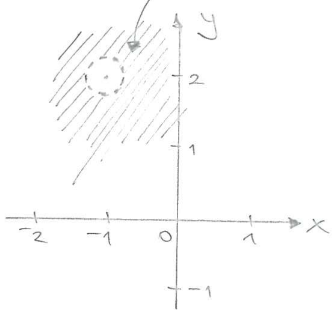


4) i) $4x^2+4y^2+8x-16y+19>0 \quad \Leftrightarrow \quad 4(x^2+2x)+4(y^2-4y)+19>0$

$\Leftrightarrow \quad 4(x+1)^2-4+4(y-2)^2-16+19>0$

$\Leftrightarrow \quad 4(x+1)^2+4(y-2)^2>1$

$\Leftrightarrow \quad (x+1)^2+(y-2)^2>\frac{1}{4}$



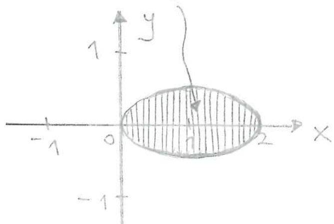
$\mathcal{C}=(-1,2)$

$r=\frac{1}{2}$

ii) $x^2-2x+4y^2\leq 0 \quad \Leftrightarrow \quad (x-1)^2-1+4y^2\leq 0$

$\Leftrightarrow \quad (x-1)^2+4y^2\leq 1$

$\Leftrightarrow \quad (x-1)^2+\frac{y^2}{1/4}\leq 1$



$\mathcal{C}=(1,0)$

$a=1$, $b=\frac{1}{2}$

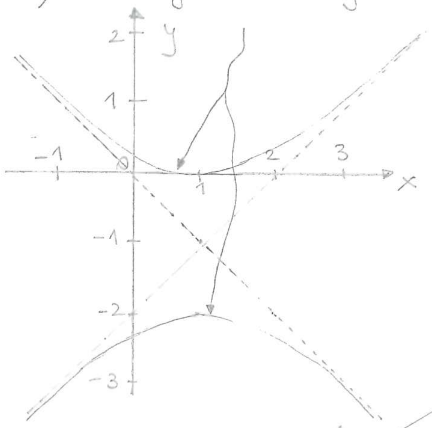
iii) $-x^2+y^2+2x+2y=1 \Leftrightarrow -x^2+2x+y^2+2y=1$

$\Leftrightarrow -(x^2-2x)+(y+1)^2-1=1$

$\Leftrightarrow -(x-1)^2+\cancel{1}+(y+1)^2-\cancel{1}=1$

$\Rightarrow -(x-1)^2+(y+1)^2=1$ $\qquad C=(1,-1)$

$a=b=1$



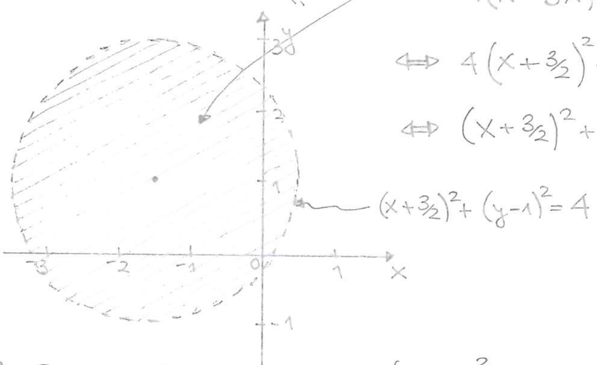
iv) $4x^2+4y^2+12x-8y-3<0$

$\Leftrightarrow 4(x^2+3x)+4(y^2-2y)-3<0$

$\Leftrightarrow 4\left(x+\frac{3}{2}\right)^2-9+4(y-1)^2-4-3<0$

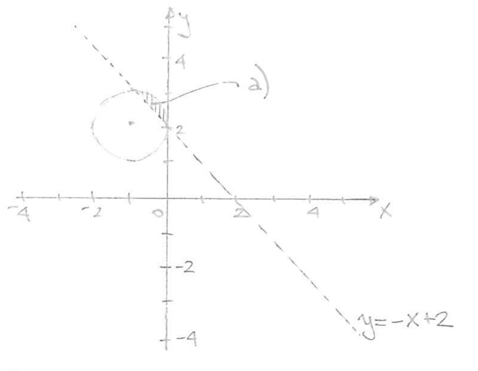
$\Leftrightarrow \left(x+\frac{3}{2}\right)^2+(y-1)^2<4$ $\qquad C=\left(-\frac{3}{2},1\right)$

$r=2$

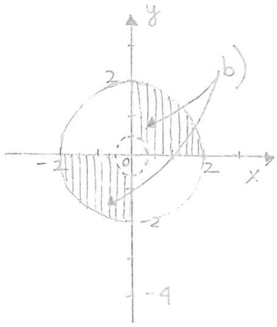



5) a) $\begin{cases} x^2+y^2+2x-4y+4\leq 0 \\ y<-x+2 \end{cases} \Leftrightarrow \begin{cases} (x+1)^2-1+(y-2)^2-\cancel{4}+\cancel{4}\leq 0 \\ y<-x+2 \end{cases}$

$\Leftrightarrow \begin{cases} (x+1)^2+(y-2)^2\leq 1 \\ y<-x+2 \end{cases}$

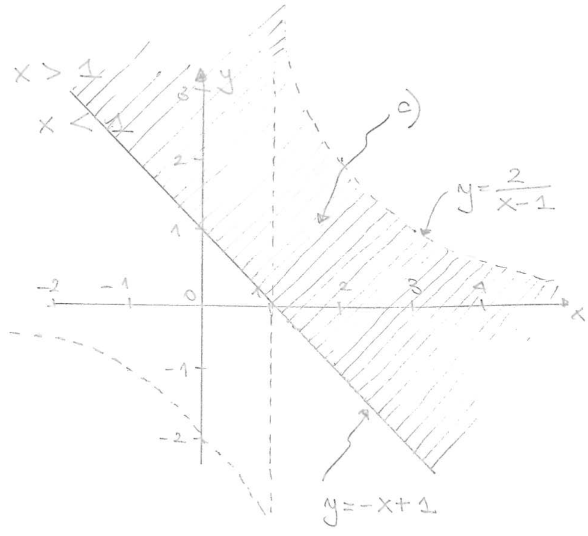



b) $\begin{cases} \frac{1}{4} < x^2+y^2 \leq 4 \\ xy \geq 0 \end{cases}$

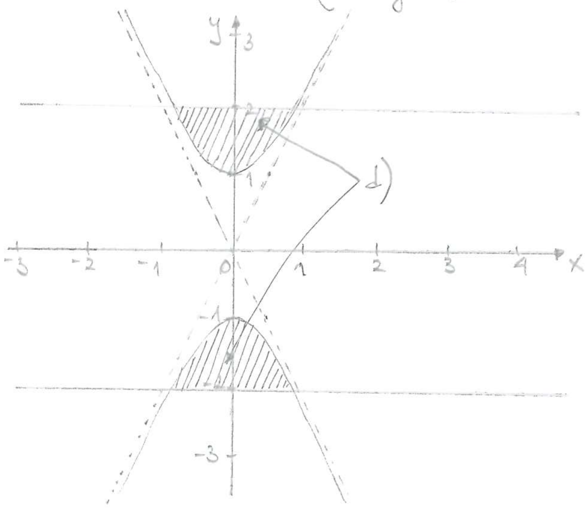



c) $\begin{cases} (x-1)y<2 \\ y\geq -x+1 \end{cases}$

$\Leftrightarrow \begin{cases} y<\frac{2}{x-1} & \text{se } x>1 \\ y>\frac{2}{x-1} & \text{se } x<1 \\ y\geq -x+1 \end{cases}$



d) $\begin{cases} 4x^2 - y^2 \le -1 \\ y^2 \le 4 \end{cases} \iff \begin{cases} -\frac{x^2}{1/4} + y^2 \ge 1 \\ -2 \le y \le 2 \end{cases}$




6) $A = \{x \in \mathbb{R} : x^3 + 3x^2 + 4x \ge 0\} = \{x \in \mathbb{R} : x(x^2 + 3x + 4) \ge 0\} = [0, +\infty[$

- è limitato inferiormente, ma non limitato superiormente
- $\min A = 0$; $\nexists \max A$;

$B = \left\{ x \in \mathbb{R} : \frac{x^2 + x - 2}{x^2} \le 1 \right\} = \left\{ x \in \mathbb{R} : \frac{x-2}{x^2} \le 0 \right\} = ]-\infty, 2] \setminus \{0\}$

- è limitato superiormente, ma non limitato inferiormente
- $\nexists \min B$; $\max B = 2$;

$C = \left\{ x \in \mathbb{R} : \frac{3x}{x^2+2} = -1 \right\} = \left\{ x \in \mathbb{R} : \frac{x^2 + 3x + 2}{x^2+2} = 0 \right\} = \left\{ x \in \mathbb{R} : \frac{(x+1)(x+2)}{x^2+2} = 0 \right\} = \{-2, -1\}$

- è limitato inferiormente e limitato superiormente
- $\min C = -2$, $\max C = -1$;

$D = \left\{-3, -\frac{15}{7}\right\} \cup ]1, 2[$;

- è limitato inferiormente e limitato superiormente
- $\min D = -3$; $\nexists \max D$;

$E = ]-5, 8[ \setminus [-4, 6[ \; = ]-5, -4[ \cup [6, 8[$

- è limitato inferiormente e limitato superiormente
- $\nexists \min E$, $\nexists \max E$.