

1) Calcolate, se esistono, i seguenti limiti (usando, se possibile, il teorema di de l'Hopital, e non solo ...):

$$\begin{array}{ll} \text{a)} \quad \lim_{x \rightarrow 0} \frac{x - \sin x \cos x}{x^3}; & \text{b)} \quad \lim_{x \rightarrow +\infty} \frac{\log(1 + \sqrt{x})}{\log x}; \\ \text{c)} \quad \lim_{x \rightarrow 0^+} \frac{\log x}{\log(\sin x)}; & \text{d)} \quad \lim_{x \rightarrow 0^+} \left(\frac{1}{\sin x} - \frac{1}{x} \right); \\ \text{e)} \quad \lim_{x \rightarrow 0^+} (\arcsin x) \log x; & \text{f)} \quad \lim_{x \rightarrow 1} \frac{\log x}{\tan(\pi x)}; \quad \text{g)} \quad \lim_{x \rightarrow +\infty} x(\arctan x - \frac{\pi}{2}). \end{array}$$

2) i) Calcolate, se esistono, i seguenti limiti (usando il teorema di de l'Hopital, e non solo ...):

$$\text{a)} \quad \lim_{x \rightarrow 0^+} \frac{\cos \sqrt{x} - \cos x}{2x}; \quad \text{b)} \quad \lim_{x \rightarrow +\infty} \left(x \cos \frac{1}{\sqrt{x}} - x^2 \sin \frac{1}{x} \right).$$

ii) Calcolate, se esistono, i seguenti limiti (usando il teorema di de l'Hopital, e non solo ...):

$$\text{a)} \quad \lim_{x \rightarrow +\infty} \frac{e^{(\sin \frac{1}{x})^{-1}}}{e^x}; \quad \text{b)} \quad \lim_{x \rightarrow +\infty} \frac{e^{(\sin \frac{1}{x})^{-2}}}{e^{x^2}}.$$

3) Calcolate, se esistono, i seguenti limiti (usando, se possibile, la formula di Taylor):

$$\begin{array}{lll} \text{a)} \quad \lim_{x \rightarrow 0} \frac{\log^2(1 + \sin x)}{x \tan x}; & \text{b)} \quad \lim_{x \rightarrow 0} \frac{x - \log(1 + x)}{1 - \cos x}; & \text{c)} \quad \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}; \\ \text{d)} \quad \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x}}; & \text{e)} \quad \lim_{x \rightarrow 0} \left(\frac{1}{x \tan x} - \frac{1}{x^2} \right); & \text{f)} \quad \lim_{x \rightarrow 0} \frac{1 - \cos x^5}{(\sin x - x \cos(x/\sqrt{3}))^2}; \\ \text{g)} \quad \lim_{x \rightarrow 0} \frac{1}{x^2} \left(\frac{\sin x}{x} - \frac{x}{\sin x} \right); & \text{h)} \quad \lim_{x \rightarrow 0} \frac{x^2(\sin x^2 - \sin^2 x)}{\sin x^2 - \tan x^2}; & \\ \text{i)} \quad \lim_{x \rightarrow 0^+} \frac{x - \sin^2 \sqrt{x} - \sin^2 x}{x^2}; & \text{j)} \quad \lim_{x \rightarrow 0} \frac{[(1-x)^{-1} + e^x]^2 - 4e^{2x} - 2x^2}{x^3}; & \\ \text{k)} \quad \lim_{x \rightarrow 0^+} \frac{\sin^2 x + 3(x - \sin^2 \sqrt{x})}{x^2}; & \text{l)} \quad \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{1 + x^2 - e^{x^2} + \sin^3 x}; & \\ \text{m)} \quad \lim_{x \rightarrow 0} \frac{(e^x - 1)^3 - \log^2(1 - x)}{1 - \cos^2 x}; & \text{n)} \quad \lim_{x \rightarrow 0} \frac{e^{x^2} - 1 - \log(1 + x \arctan x)}{\sqrt{1 + 2x^4} - 1}. & \end{array}$$