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$\boxed{1}$. $\lim\limits_{x\to 3} \dfrac{x^2-6x+10}{x^2-x-3} = \dfrac{9-18+10}{9-3-3} = \underline{\dfrac{1}{3}}$;

$\lim\limits_{x\to 0} \dfrac{12x^2-2}{\frac{1}{5x}} = \lim\limits_{x\to 0} 5x(12x^2-2) = \underline{0}$; infatti $\overset{\nearrow 0}{5x}\,\overset{\nearrow -2}{(12x^2-2)} \to 0$ per $x\to 0$.

$\lim\limits_{x\to -1} \dfrac{x^3+1}{x^2+x+1} = \dfrac{-1+1}{1-\cancel{1}+\cancel{1}} = \underline{0}$;

$\left[\dfrac{\infty}{\infty}\right]$ $\lim\limits_{x\to +\infty} \dfrac{2x^2+x+5}{(x-2)^2} = \underline{\underline{2}}$; infatti $\dfrac{2x^2+2x+5}{(x-2)^2} = \dfrac{\cancel{x^2}\overset{\nearrow 2}{\left(2+\frac{2}{x}+\frac{5}{x^2}\right)}}{\cancel{x^2}\underset{\to 1}{\left(1-\frac{2}{x}\right)^2}} \to 2$ per $x\to+\infty$.

$\left[\dfrac{\infty}{\infty}\right]$ $\lim\limits_{x\to -\infty} \dfrac{x^5-1}{x^2-1} = \underline{\underline{-\infty}}$; infatti $\dfrac{x^5-1}{x^2-1} = \dfrac{\overset{3}{\cancel{x^5}}\overset{\nearrow 1}{\left(1-\frac{1}{x^5}\right)}}{\cancel{x^2}\underset{\to 1}{\left(1-\frac{1}{x^2}\right)}} \xrightarrow[x\to-\infty]{} -\infty$.

$\left[\dfrac{0}{0}\right]$ $\lim\limits_{x\to 2} \dfrac{x^2+x-6}{x^3-x^2-x-2} = \underline{\underline{\dfrac{5}{7}}}$;

infatti $\dfrac{x^2+x-6}{x^3-x^2-x-2} = \dfrac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x^2+x+1)} \xrightarrow[x\to 2]{} \dfrac{5}{7}$.

$\left[\dfrac{0}{0}\right]$ $\lim\limits_{x\to 0^+} \dfrac{x^4+2x^2}{x^7} = \underline{\underline{+\infty}}$; infatti $\dfrac{x^4+2x^2}{x^7} = \dfrac{\overset{\nearrow 2}{x^2+2}}{\underset{\text{infinitesimo positivo}}{x^5}} \to +\infty$.

$\left[\dfrac{0}{0}\right]$ $\lim\limits_{x\to -\frac{1}{5}} \dfrac{5x^2+16x+3}{25x^2+10x+1} = \underline{\underline{-\infty}}$; infatti $\dfrac{5\left(x^2+\frac{16}{5}x+\frac{3}{5}\right)}{25\left(x^2+\frac{2}{5}x+\frac{1}{25}\right)} = \dfrac{\cancel{5}\cancel{\left(x+\frac{1}{5}\right)}\overset{\nearrow \frac{14}{5}}{(x+3)}}{\underset{5}{\cancel{25}}\underset{\text{infinitesimo negativo}}{\left(x+\frac{1}{5}\right)^{\cancel{2}}}} \xrightarrow[x\to-\frac{1}{5}]{} -\infty$.

$\left[\dfrac{\infty}{\infty}\right]$ $\lim\limits_{x\to +\infty} \dfrac{x^5-\frac{1}{2x^3}}{2x^4+5} = \underline{\underline{+\infty}}$; infatti $\dfrac{x^5-\frac{1}{2x^3}}{2x^4+5} = \dfrac{x^5\overset{\nearrow 1}{\left(1-\frac{1}{2x^8}\right)}}{\cancel{x^4}\underset{\to 2}{\left(2+\frac{5}{x^4}\right)}} \xrightarrow[x\to+\infty]{} +\infty$.

$\left[\dfrac{0}{0}\right]$ $\boxed{2}$. $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x-1} = \underline{\underline{\dfrac{1}{2}}}$; infatti $\dfrac{\sqrt{x}-1}{x-1} = \dfrac{\cancel{\sqrt{x}-1}}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)} \xrightarrow{x\to 1} \dfrac{1}{2}$.

$[\infty-\infty]$ $\lim\limits_{x\to +\infty} \sqrt{x+1}-\sqrt{x-2} = \underline{\underline{0}}$; infatti $\dfrac{\left(\sqrt{x+1}-\sqrt{x-2}\right)\left(\sqrt{x+1}+\sqrt{x-2}\right)}{\sqrt{x+1}+\sqrt{x-2}} =$

$= \dfrac{\cancel{x}+1-\cancel{x}+2}{\sqrt{x+1}+\sqrt{x-2}} \xrightarrow[x\to+\infty]{} 0$.

$[\infty-\infty]$ $\lim\limits_{x\to -\infty} |x|-\sqrt{2-x} = \underline{\underline{+\infty}}$; infatti, $|x|-\sqrt{2-x} \underset{x\to-\infty}{=} -x-\sqrt{2-x} =$

$= x\left(-1-\underset{\to 0}{\sqrt{\frac{2}{x^2}-\frac{1}{x}}}\right) \to +\infty$ per $x\to-\infty$.

$\left[0\cdot\infty\right]$ $\displaystyle\lim_{x\to+\infty} \sqrt[3]{\frac{1}{x+2}}\,\sqrt{x^2+x+1} = +\infty$; infatti $\displaystyle\frac{x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{x^{1/3}\sqrt[3]{1+\frac{2}{x}}} = x^{2/3}\left(\frac{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{\sqrt[3]{1+\frac{2}{x}}}\right) \xrightarrow[x\to+\infty]{} +\infty$.

$\left[\frac{0}{0}\right]$ $\displaystyle\lim_{x\to3^+} \frac{\sqrt{x+9}-\sqrt{4x}}{\sqrt{x-3}} = \underline{0}$; infatti $\displaystyle\frac{(\sqrt{x+9}-\sqrt{4x})(\sqrt{x+9}+\sqrt{4x})}{\sqrt{x-3}\,(\sqrt{x+9}+\sqrt{4x})} =$

$\displaystyle= \frac{9-3x}{\sqrt{x-3}\,(\sqrt{x+9}+\sqrt{4x})} = \frac{-3(x-3)}{\sqrt{x-3}\,(\sqrt{x+9}+\sqrt{4x})}$

$\displaystyle= \frac{-3\sqrt{x-3}}{\sqrt{x+9}+\sqrt{4x}} \xrightarrow[x\to3^+]{} 0$.

$\left[\frac{0}{0}\right]$ $\displaystyle\lim_{x\to0^+} \frac{\sqrt{x+16}-4}{|x|} = \underline{\underline{\frac{1}{8}}}$; infatti $\displaystyle\frac{(\sqrt{x+16}-4)(\sqrt{x+16}+4)}{x\,(\sqrt{x+16}+4)} = \frac{\cancel{x}}{\cancel{x}(\sqrt{x+16}+4)} \xrightarrow[x\to0]{} \frac{1}{8}$

$\left[\frac{\infty}{\infty-\infty}\right]$ $\displaystyle\lim_{x\to+\infty} \frac{4x-1}{\sqrt{x+1}-\sqrt{2x-1}} = \underline{\underline{-\infty}}$; infatti $\displaystyle\frac{(4x-1)(\sqrt{x+1}+\sqrt{2x-1})}{x+1-2x+1} =$

$\displaystyle= \frac{(4x-1)(\sqrt{x+1}+\sqrt{2x-1})}{-x+2} =$

$\displaystyle= \frac{\cancel{x}(4-1/x)(\sqrt{x+1}+\sqrt{2x-1})}{\cancel{x}(-1+2/x)} \xrightarrow[\text{per } x\to+\infty]{} -\infty$.

$\left[\frac{\infty}{\infty}\right]$ $\displaystyle\lim_{x\to+\infty} \frac{4x-1}{\sqrt{x+1}+\sqrt{2x-1}} = \underline{\underline{+\infty}}$; $\displaystyle\frac{\overset{\sqrt{x}}{\cancel{x}}(\underbrace{4-1/x}_{\to 4})}{\cancel{\sqrt{x}}(\underbrace{\sqrt{1+\frac{1}{x}}}_{\to1}+\underbrace{\sqrt{2-1/x}}_{\to2})} \xrightarrow[x\to+\infty]{} +\infty$.

$\left[\frac{0}{0}\right]$ $\displaystyle\lim_{x\to-1} \frac{\sqrt[3]{x}+1}{x+1} = \underline{\underline{\frac{1}{3}}}$; infatti $\displaystyle\frac{\sqrt[3]{x}+1}{x+1} = \frac{\cancel{\sqrt[3]{x}+1}}{(\cancel{\sqrt[3]{x}+1})(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}$

$\displaystyle\xrightarrow[x\to-1]{} \frac{1}{1+1+1} = \frac{1}{3}$.

$\boxed{3}$. $\displaystyle\lim_{x\to+\infty} \frac{\arctan x^2+\sin x\cos x}{e^{x^2-2x}} = \underline{0}$; infatti $\displaystyle\left|\frac{\arctan x^2+\sin x\cos x}{e^{x^2-2x}}\right| \le \frac{\frac{\pi}{2}+1}{e^{x^2-2x}} \xrightarrow[\text{per } x\to+\infty]{} 0$.

$\left[\frac{\infty}{\infty}\right]$ $\displaystyle\lim_{x\to+\infty} \frac{\sqrt{x^2+x+4}}{x^2 e^{\left(\frac{5x^3-5}{x^3-2}-5\right)}+2} = \underline{0}$; $\displaystyle\frac{\cancel{x}\,\overbrace{\sqrt{1+\frac{1}{x}+4/x^2}}^{\to1}}{x^{\cancel{2}}\left[\underbrace{e^{\left(\frac{5x^3-5}{x^3-2}-5\right)}}_{\to e^0=1}+\underbrace{\frac{2}{x^2}}_{\to0}\right]} \xrightarrow[\text{per } x\to+\infty]{} 0$.

$\left[\frac{\infty}{\infty}\right]$ $\displaystyle\lim_{x\to+\infty} \frac{x^2+e^{2x}}{x^3+\log x} = \underline{\underline{+\infty}}$; infatti $\displaystyle\frac{\overset{\nearrow +\infty}{e^{2x}}\left(\overset{\nearrow 0}{\frac{x^2}{e^{2x}}}+1\right)}{x^3\left(1+\underset{\downarrow 0}{\frac{\log x}{x^3}}\right)} \xrightarrow[x\to+\infty]{} +\infty$.

$\displaystyle\lim_{x\to+\infty} \arctan\left(\frac{\log(x^2-x)+3^x-x^5}{2^{x-1}+2}\right) = \underline{\underline{\frac{\pi}{2}}}$; poiché $\displaystyle\frac{\log(x^2-x)+3^x-x^5}{2^{x-1}+2} =$

$$= \frac{3^x\left(\overset{\nearrow 0}{\log(x^2-x)/3^x} + 1 - \overset{\nearrow 0}{\frac{x^5}{3^x}}\right)}{2^x\left(\frac{1}{2}+\underset{\downarrow 0}{\frac{2}{2^x}}\right)} \xrightarrow[x\to+\infty]{} +\infty$$

$\left(\frac{3^x}{2^x} \to +\infty\right)$

e $\arctan y \xrightarrow[y\to+\infty]{} \frac{\pi}{2}$.

$[\infty^0]$ $\displaystyle\lim_{x\to+\infty} (x^2+5x)^{\frac{1}{x}} = \underline{\underline{1}}$; infatti $(x^2+5x)^{1/x} = e^{\overset{\nearrow 0}{1/x\,\log(x^2+5x)}_{x\to+\infty}} \to e^0 = 1$.

$[0^0]$ $\displaystyle\lim_{x\to0^+} \left(\frac{x^5}{x+7}\right)^{x^3+x} = \underline{\underline{1}}$; infatti $\displaystyle\left(\frac{x^5}{x+7}\right)^{x^3+x} = e^{(x^3+x)\left[\log x^5-\log(x+7)\right]} \to e^0$

$[0^0]$ $\displaystyle\lim_{x\to1^+} \left(\log^6(x-1)\right)^{x-1} = 1$; infatti $\left(\log^6(x-1)\right)^{x-1} = e^{(x-1)\log\left[\log^6(x-1)\right]} =$

$$\underset{\substack{\uparrow \\ \log(x-1)=y \xrightarrow[x\to1^+]{} -\infty}}{=} e^{\overset{\nearrow 0}{e^y}\,\overset{\nearrow +\infty}{\log y^6}} = e^{\overset{\nearrow +\infty}{[\log y^6]}/\overset{\nearrow +\infty}{e^{-y}}} = 1.$$

$\displaystyle\lim_{x\to+\infty} \frac{\log(3+\sin x)}{x^2} = \underline{\underline{0}}$; $\displaystyle\underset{\downarrow 0}{\frac{\log 2}{x^2}} \le \frac{\log(3+\sin x)}{x^2} \le \underset{\downarrow 0}{\frac{\log 4}{x^2}}$ per $x\to+\infty$.

$\left[\frac{\infty}{\infty}\right]$ $\displaystyle\lim_{x\to+\infty} \frac{\sin 3x+3x}{\arctan 4x - e^{2x}} = \underline{\underline{0}}$; $\displaystyle\frac{x\left(\overset{\nearrow 0}{\frac{\sin 3x}{x}}+3\right)}{e^{2x}\left(\underset{\downarrow 0}{\frac{\arctan 4x}{e^{2x}}}-1\right)} \xrightarrow[x\to+\infty]{} 0$

$\blacksquare$

$[0\cdot\infty]$ $\boxed{4}$ $\displaystyle\lim_{x\to0^+} \sin x \log\frac{1}{x} = \underline{\underline{0}}$; $\displaystyle\overset{x\to0\ \nearrow 1}{\frac{\sin x}{x}} \cdot \overset{\nearrow 0}{\frac{\log\frac{1}{x}}{1/x}}_{x\to0^+} \xrightarrow[x\to0^+]{} 0$.

$\displaystyle\lim_{x\to0^-} \frac{1-\cos x}{\log\left(\frac{x}{x-1}\right)} = \underline{\underline{0}}$; $\displaystyle\underset{\substack{\downarrow x\to0 \\ \frac{1}{2}}}{\frac{1-\cos x}{x^2}} \cdot \overset{\nearrow 0}{x^2} \cdot \frac{1}{\underset{\to -\infty}{\log\left(1+\frac{1}{x-1}\right)}} \to 0$.

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$\left[\frac{0}{0}\right]$ $\lim\limits_{x\to 1} \frac{\sin \pi x}{x-1} =$ $\qquad$ infatti $\frac{\sin \pi x}{x-1} \underset{y=x-1}{=} \frac{\sin \pi (y+1)}{y} = \frac{\sin \pi y \cos \pi + \cancel{\cos \pi y \sin \pi}}{y}$

$$= \lim_{y\to 0} \frac{\sin \pi y}{y} \cos \pi$$

$$= \lim_{y\to 0} \frac{\sin \pi y}{\pi y} \cdot \pi \cdot (-1) = \underline{\underline{-\pi}} ;$$

$\left[\frac{0}{0}\right]$ $\lim\limits_{x\to 0} \frac{\tan(\sin x) + \log(1+x)}{x} = \underline{\underline{2}}$; $\qquad$ infatti

$$\underbrace{\frac{\tan(\sin x)}{\sin x}}_{\to 1 \text{ per } x\to 0} \cdot \overset{\nearrow 1}{\frac{\sin x}{x}} + \overset{\nearrow 1}{\frac{\log(1+x)}{x}}$$

$\left[\frac{0}{0}\right]$ $\lim\limits_{x\to 0} \frac{1-\cos x^4}{\sin x^2} = \underline{\underline{0}}$; $\qquad$ infatti $\frac{1-\cos x^4}{\sin x^2} = \underbrace{\frac{1-\cos x^4}{x^8}}_{\to 1/2} \cdot \overset{\nearrow 1}{\frac{x^2}{\sin x^2}} \cdot \overset{\nearrow 0}{x^6}$ per $x\to 0$

$\left[1^\infty\right]$ $\lim\limits_{x\to +\infty} \left(\frac{x+1}{x+2}\right)^{3x+4} = \underline{\underline{e^{-3}}}$; $\qquad$ infatti

$$\left(\frac{x+1}{x+2}\right)^{3x+4} = e^{(3x+4)\log\left(\frac{x+1}{x+2}\right)} =$$

$$= e^{3x+4 \left[\underbrace{\frac{\log\left(1-\frac{1}{x+2}\right)}{-1/(x+2)}}_{\to 1}\right]\left(-\frac{1}{x+2}\right)} \xrightarrow[x\to+\infty]{} e^{-3} .$$

$\left[1^{-\infty}\right]$ $\lim\limits_{x\to -\infty} \left(1+\frac{1}{x^2}\right)^{x^3} = \underline{\underline{0}}$; $\qquad$ infatti $\left(1+\frac{1}{x^2}\right)^{x^3} = e^{x^3 \overset{\nearrow 1}{\frac{\log(1+1/x^2)}{1/x^2}} \cdot \frac{1}{x^2}} \to 0$

$\left[\frac{0}{0}\right]$ $\lim\limits_{x\to 0} \frac{x e^{\cos x} - x e}{\log(1+x)} = \underline{\underline{0}}$; $\qquad$ infatti $\frac{x\left(e^{\cos x} - e\right)}{\log(1+x)} \xrightarrow[x\to 0]{} 0 .$ $\quad \frac{x}{\log(1+x)} \to 1$

$\left[\infty-\infty\right]$ $\lim\limits_{x\to +\infty} \left(5^{x^2} - 5^{2x}\right) = \underline{\underline{+\infty}}$; $\qquad$ infatti $5^{x^2} - 5^{2x} = 5^{x^2}\left[1 - 5^{\overset{\nearrow -\infty}{2x-x^2}}\right] \xrightarrow[x\to+\infty]{} +\infty$

$\left[\frac{0}{0}\right]$ $\lim\limits_{x\to 0} \frac{\sin\left(\log(1+3x)\right)}{x \sin^2 4x} = \underline{\underline{+\infty}}$; $\qquad$ infatti

$$\overset{\nearrow 1}{\frac{\sin\left(\log(1+3x)\right)}{\log(1+3x)}} \cdot \overset{\nearrow 1}{\frac{\log(1+3x)}{3x}} \cdot \frac{3}{\sin^2 4x} \quad \sin^2 4x \xrightarrow{x\to 0} \text{infinitesimo positivo}$$

$\left[\frac{0}{0}\right]$ $\lim\limits_{x\to 0} \frac{2\arcsin^2 x + \log(1-\sin^2 x)}{\cos x^2 - 1} = \underline{\underline{-\infty}}$;

$$\text{infatti } \left[\overset{\nearrow 2}{\frac{2\arcsin^2 x}{x^2}} + \overset{\nearrow 1}{\frac{\log(1-\sin^2 x)}{-\sin^2 x}} \cdot \overset{\nearrow -1}{\frac{(-\sin^2 x)}{x^2}}\right] \overset{\uparrow -2}{\frac{x^4}{\cos x^2 - 1}} \cdot \left(\frac{1}{x^2}\right)$$

infinitesimo positivo

$[1^\infty]$ $\quad \lim\limits_{x\to+\infty}\left(1+\dfrac{x-5}{25-x^2}\right)^x = \lim\limits_{x\to+\infty}\left(1-\dfrac{\cancel{x-5}}{(\cancel{x-5})(x+5)}\right)^x = \underline{\underline{e^{-1}}}$;

infatti $\quad \left(1-\dfrac{1}{x+5}\right)^x = e^{x\left[\overbrace{\left[\log\left(1-\frac{1}{x+5}\right)\right]\Big/-\frac{1}{x+5}}^{\to 1}\right]\left(-\frac{1}{x+5}\right)} \xrightarrow[x\to+\infty]{} e^{-1}$

$\left[\dfrac{0}{0}\right]$ $\quad \lim\limits_{x\to 0}\dfrac{e^x-e^{-x}}{\tan x} = \underline{\underline{2}}$; $\quad$ infatti $\quad \dfrac{e^x-e^{-x}}{\tan x} = \dfrac{e^x-1+1-e^{-x}}{\tan x} =$

$$= \underbrace{\frac{e^x-1}{x}}_{\to 1}\cdot\underbrace{\frac{x}{\tan x}}_{\to 1} + \underbrace{\frac{e^x-1}{e^x\cdot x}}_{\to 1}\cdot\underbrace{\frac{x}{\tan x}}_{\to 1} \xrightarrow[x\to 0]{} 2$$

$[\infty^0]$ $\quad \lim\limits_{x\to 0}\left(1+\dfrac{1}{2x}\right)^{\tan(\log(1+3x))} = \underline{\underline{1}}$;

infatti $\quad \left(1+\dfrac{1}{2x}\right)^{\tan(\log(1+3x))} =$

$$= e^{\tan(\log(1+3x))\,\log\left(1+\frac{1}{2x}\right)} =$$

$$= e^{\underbrace{\frac{\tan(\log(1+3x))}{\log(1+3x)}}_{\to 1}\,\underbrace{\frac{\log(1+3x)}{3x}}_{\to 1}\,\cdot\,\overbrace{3x\log\left(1+\frac{1}{2x}\right)}^{\to 0}} \xrightarrow[x\to 0]{} e^0 = 1$$

$\left[\dfrac{0}{0}\right]$ $\quad \lim\limits_{x\to 1}\dfrac{e^{-1/2}\cos(x-1)-e^{\frac{x^2-2x}{2}}}{(x-1)(x+1)} = 0$; $\quad$ infatti

$$\frac{e^{-1/2}\left[\cos(x-1)-e^{\frac{x^2-2x+1}{2}}\right]}{(x-1)(x+1)} = \frac{e^{-1/2}\left[\cos(x-1)-e^{\frac{(x-1)^2}{2}}\right]}{(x-1)(x+1)}$$

$$= e^{-1/2}\left[\underbrace{\frac{\cos(x-1)-1}{(x-1)^2}}_{\to 1/2}\cdot\underbrace{\frac{(x-1)}{x+1}}_{\to 0} - \underbrace{\frac{e^{\frac{(x-1)^2}{2}}-1}{(x-1)^2}}_{\to 1/2}\cdot\underbrace{\frac{x-1}{x+1}}_{\to 0}\right]$$

$\boxed{5}$ (i) $\quad \lim\limits_{x\to-\infty}\left[\dfrac{x^2+2x-\alpha}{x-1}-\alpha(x-1)\right] = \begin{cases} -\infty & \text{se } \alpha<1 \\ 4 & \text{se } \alpha=1 \\ +\infty & \text{se } \alpha>1 \end{cases}$;

infatti

$$\frac{x^2+2x-\alpha-\alpha(x-1)^2}{x-1} = \frac{x^2+2x-\alpha-\alpha x^2+2\alpha x-\alpha}{x-1}$$

$$= \frac{(1-\alpha)x^2+(2+2\alpha)x-2\alpha}{x-1}.$$

□

(ii) $$\lim_{x\to 0^+} \frac{\log(1+x^\alpha)}{3x^5} = \begin{cases} +\infty & \text{se } \alpha<5 \\ 1/3 & \text{se } \alpha=5 \\ 0 & \text{se } \alpha>5\,; \end{cases}$$

infatti $\frac{\log(1+x^\alpha)}{3x^5}$

$\to +\infty$ se $\alpha<0$ poiché $\log(1+x^\alpha) \xrightarrow[x\to0^+]{} +\infty$ e $x^5 \xrightarrow[x\to 0^+]{} 0$

$\to +\infty$ se $\alpha=0$ poiché $\frac{\log 2}{3x^5} \xrightarrow[\text{se } x\to 0^+]{} +\infty$

$$\underbrace{\frac{\log(1+x^\alpha)}{x^\alpha}}_{\to 1} \cdot \frac{x^\alpha}{3x^5} \begin{cases} \to +\infty & \text{se } 0<\alpha<5 \\ \to \frac{1}{3} & \text{se } \alpha=5 \\ \to 0 & \text{se } \alpha>5. \end{cases}$$

$\square$

(iii) $$\lim_{x\to+\infty} \left(\frac{e^x-x}{e^{\alpha x}}\right)^x = \begin{cases} +\infty & \text{se } \alpha<1 \\ 1 & \text{se } \alpha=1 \\ 0 & \text{se } \alpha>1\,; \end{cases}$$

infatti $\left(\frac{e^x-x}{e^{\alpha x}}\right)^x = e^{x\log\left(\frac{e^x-x}{e^{\alpha x}}\right)}$

ora
- $x\log\left(\frac{e^x-x}{e^{\alpha x}}\right) \xrightarrow[x\to+\infty]{} +\infty$ se $\alpha<1$
- $x\log\left(\frac{e^x-x}{e^x}\right) = x\log\left(1-\frac{x}{e^x}\right) =$ se $\alpha=1$

  $= \underbrace{x\left(-\frac{x}{e^x}\right)}_{\to 0} \underbrace{\frac{\log\left(1-\frac{x}{e^x}\right)}{-\frac{x}{e^x}}}_{\to 1} \xrightarrow[x\to+\infty]{} 0$
- $x\log\Big(\underbrace{\frac{e^x-x}{e^{\alpha x}}}_{\text{infinitesimo positivo}}\Big) \xrightarrow[x\to+\infty]{} -\infty$ se $\alpha>1$.

$\blacksquare$

$\boxed{6}$ (i) $$f(x) = \begin{cases} x^2+3x+\alpha & \text{se } x<-1 \\ -\frac{x}{2^x} & \text{se } x\geq -1. \end{cases}$$

f è continua in ogni pt. $x\in\mathbb{R}\setminus\{-1\}$ $\forall \alpha\in\mathbb{R}$. Basta ora richiedere che $\lim_{x\to-1^-} f(x) = \lim_{x\to-1^+} f(x) = f(-1)$ per avere la continuità di f anche in $x=-1$.

Ora
$$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-}(x^2+3x+\alpha) = -2+\alpha$$
$$\lim_{x\to-1^+} f(x) = \lim_{x\to-1^+}\left(-\frac{x}{2^x}\right) = 2 = f(-1).$$

Quindi deve essere $-2+\alpha=2$, ossia $\underline{\underline{\alpha=4}}$. $\square$

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(ii)
$$f(x)=\begin{cases}\alpha(x+1) & \text{se } x\le 0\\ \dfrac{e^{2x}-1}{\sin 4x\cos x} & \text{se } x>0.\end{cases}$$

f è continua in ogni pt. $x\in\mathbb{R}\setminus\{0\}$ $\forall \alpha\in\mathbb{R}$. Basta ora imporre che $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$ per avere la continuità di f anche in $x=0$.

Ora
$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\alpha(x+1)=\alpha=f(0)$$
$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{e^{2x}-1}{2x}\cdot\frac{2x}{\sin 4x\cos x}=\lim_{x\to 0^+}\underbrace{\frac{e^{2x}-1}{2x}}_{\downarrow 1}\cdot\underbrace{\frac{4x}{\sin 4x}}_{\downarrow 1}\cdot\underbrace{\frac{1}{2\cos x}}_{\downarrow 1/2}$$
$$=1/2.$$

Quindi dobbiamo imporre $\underline{\alpha=1/2}$, per avere la continuità di f anche in $x=0$. $\square$

(iii)
$$f(x)=\begin{cases}\cos x-2\beta & \text{se } x<-\pi/2\\ \alpha\sin x+\beta & \text{se } -\pi/2\le x\le 0\\ \frac{1}{\alpha}\arctan 1/x & \text{se } x>0.\end{cases}$$

Analogamente agli esercizi precedenti, basta richiedere
$$\lim_{x\to -\frac{\pi}{2}^-} f(x)=\lim_{x\to -\frac{\pi}{2}^-}(\cos x-2\beta)=\lim_{x\to -\frac{\pi}{2}^+} f(x)=\lim_{x\to -\frac{\pi}{2}^+}(\alpha\sin x+\beta)$$
$$=f\left(-\tfrac{\pi}{2}\right),$$

ossia
$$-2\beta=-\alpha+\beta$$

e
$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}(\alpha\sin x+\beta)=f(0)=\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\left(\frac{1}{\alpha}\arctan\frac{1}{x}\right)$$

ossia
$$\beta=\frac{1}{\alpha}\frac{\pi}{2}.$$

Ora
$$\begin{cases}-2\beta=-\alpha+\beta\\ \beta=\dfrac{\pi}{2\alpha}\end{cases}\iff\begin{cases}\alpha=+3\beta\\ 3\beta^2=\pi/2\end{cases}\qquad \beta^2=\frac{\pi}{6}\ ;$$

Ne segue $(\alpha,\beta)=\left(3\sqrt{\frac{\pi}{6}},\sqrt{\frac{\pi}{6}}\right)$ e $(\alpha,\beta)=\left(-3\sqrt{\frac{\pi}{6}},-\sqrt{\frac{\pi}{6}}\right)$. ■