

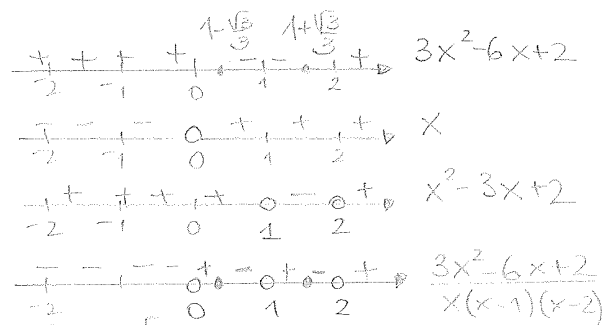
Risolvere le seguenti disequazioni

$$1) \frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-2} \geq 0$$

$$\Leftrightarrow \frac{(x-1)(x-2) + x(x-2) + x(x-1)}{x(x-1)(x-2)} \geq 0$$

$$\Leftrightarrow \frac{x^2 - 3x + 2 + x^2 - 2x + x^2 - x}{x(x-1)(x-2)} \geq 0$$

$$\Leftrightarrow \frac{3x^2 - 6x + 2}{x(x-1)(x-2)} \geq 0$$

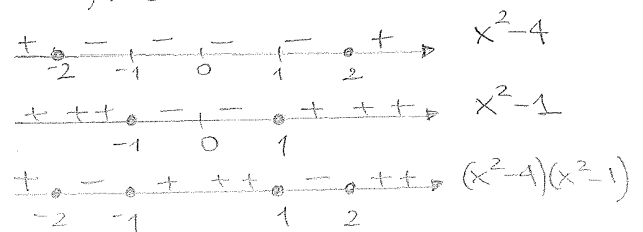


$$S =]0, 1 - \frac{\sqrt{3}}{3}] \cup [1 + \frac{\sqrt{3}}{3}, 2] \cup]2, +\infty[$$

$$2) \frac{10}{x^2+1} > 6 - x^2 \Leftrightarrow 10 - 6(x^2+1) + x^2(x^2+1) > 0 \quad \text{NOTA: } x^2+1 > 0 \forall x \in \mathbb{R}$$

$$\Leftrightarrow x^4 - 5x^2 + 4 > 0$$

$$\Leftrightarrow (x^2 - 4)(x^2 - 1) > 0$$



$$S =]-\infty, -2[\cup]-1, 1[\cup]2, +\infty[$$

$$3) x^2 + 2|x| - 3 < 0 \Leftrightarrow \begin{cases} x \geq 0 \\ x^2 + 2x - 3 < 0 \end{cases} \quad \circ \quad \begin{cases} x < 0 \\ x^2 - 2x - 3 < 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq 0 \\ (x+3)(x-1) < 0 \end{cases} \quad \circ \quad \begin{cases} x < 0 \\ (x-3)(x+1) < 0 \end{cases}$$

$$\downarrow S_1 = [0, 1[$$

$$\downarrow S_2 =]-4, 0[$$

$$\Rightarrow S = S_1 \cup S_2 =]-4, 1[$$

$$4) \frac{4|x|}{x^2-2|x|-3} < -1 \Leftrightarrow \begin{cases} x \geq 0 \\ \frac{4x+x^2-2x-3}{x^2-2x-3} < 0 \end{cases} \vee \begin{cases} x < 0 \\ \frac{-4x+x^2+2x-3}{x^2+2x-3} < 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq 0 \\ \frac{x^2+2x-3}{x^2-2x-3} < 0 \end{cases} \vee \begin{cases} x < 0 \\ \frac{x^2-2x-3}{x^2+2x-3} < 0 \end{cases}$$

$$= \frac{x^2+2x-3}{(x+3)(x-1)} \quad \begin{array}{c} + \quad - \quad - \quad + \quad + \\ -3 \quad -1 \quad 0 \quad 1 \quad 2 \end{array}$$

$$= \frac{x^2-2x-3}{(x-3)(x+1)} \quad \begin{array}{c} + \quad + \quad - \quad - \quad + \\ -3 \quad -1 \quad 1 \quad 3 \end{array}$$

$$\frac{x^2+2x-3}{x^2-2x-3} \quad \begin{array}{c} + \quad - \quad - \quad + \quad + \\ -3 \quad -1 \quad 0 \quad 1 \quad 3 \end{array} \quad \begin{array}{c} + \quad - \quad - \quad + \quad + \\ -3 \quad -1 \quad 0 \quad 1 \quad 3 \end{array} \quad \begin{array}{c} x^2-2x-3 \\ x^2+2x-3 \end{array}$$

$$S_1 =]1, 3[$$

$$S_2 =]-3, -1[$$

$$S =]-3, -1[\cup]1, 3[.$$

$$5) \left| \frac{x-1}{x-3} \right| > 1 \Leftrightarrow \frac{x-1}{x-3} < -1 \vee \frac{x-1}{x-3} > 1$$

$$\Leftrightarrow \frac{x-1+x-3}{x-3} < 0 \vee \frac{x-1-x+3}{x-3} > 0$$

$$\Leftrightarrow \frac{2x-4}{x-3} < 0 \vee \frac{2}{x-3} > 0$$

$$\frac{2x-4}{x-3} \quad \begin{array}{c} - \quad - \quad - \quad + \quad + \\ 0 \quad 2 \quad 3 \quad 4 \end{array}$$

$$\frac{x-3}{x-3} \quad \begin{array}{c} - \quad - \quad - \quad + \quad + \\ 0 \quad 3 \end{array}$$

$$\frac{2x-4}{x-3} \quad \begin{array}{c} + \quad + \quad + \quad - \quad - \\ 0 \quad 2 \quad 3 \end{array}$$

$$S_1 =]2, 3[$$

$$\updownarrow S_2 =]3, +\infty[$$

$$S = S_1 \cup S_2 =]2, +\infty[\setminus \{3\}.$$

$$6) \left| \frac{2x-1}{5-x} \right| < 2 \Leftrightarrow -2 < \frac{2x-1}{5-x} < 2 \Rightarrow \begin{cases} \frac{2x-1}{5-x} + 2 > 0 \\ \frac{2x-1}{5-x} - 2 < 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \frac{2x-1+10-2x}{5-x} > 0 \\ \frac{2x-1-10+2x}{5-x} < 0 \end{cases} \Leftrightarrow \begin{cases} \frac{9}{5-x} > 0 \\ \frac{4x-11}{5-x} < 0 \end{cases}$$

$$S =]-\infty, 5[\cap \left(\left] -\infty, \frac{11}{4} \right[\cup \right] 5, +\infty \left[\right) \\ = \underline{\underline{]-\infty, \frac{11}{4}[}}$$

$$\begin{aligned} 7) |x^2 - 6x| < x + x^2 &\Leftrightarrow \begin{cases} x^2 - 6x \geq 0 \\ -x^2 - 6x < x + x^2 \end{cases} \quad \text{ou} \quad \begin{cases} x^2 - 6x < 0 \\ -x^2 + 6x < x + x^2 \end{cases} \\ &\Leftrightarrow \begin{cases} x^2 - 6x \geq 0 \\ 7x > 0 \end{cases} \quad \text{ou} \quad \begin{cases} x^2 - 6x < 0 \\ 2x^2 - 5x > 0 \end{cases} \\ &\Leftrightarrow \begin{aligned} &\downarrow \\ S_1 &= [6, +\infty[\end{aligned} \quad \begin{aligned} &\downarrow \\ S_2 &=]\frac{5}{2}, 6[\end{aligned} \\ &\Rightarrow \underline{\underline{S = S_1 \cup S_2 =]\frac{5}{2}, +\infty[}}. \end{aligned}$$

$$\begin{aligned} 8) |x^2 - 4| \geq 4 + 2x &\Leftrightarrow \begin{cases} x^2 - 4 \geq 0 \\ x^2 - 4 \geq 4 + 2x \end{cases} \quad \text{ou} \quad \begin{cases} x^2 - 4 < 0 \\ -x^2 + 4 \geq 4 + 2x \end{cases} \\ &\Leftrightarrow \begin{cases} x^2 - 4 \geq 0 \\ x^2 - 2x - 8 \geq 0 \end{cases} \quad \text{ou} \quad \begin{cases} x^2 - 4 < 0 \\ x^2 + 2x \leq 0 \end{cases} \\ &\quad \downarrow \quad \downarrow \\ (x-4)(x+2) \quad S_1 &=]-\infty, -2] \cup [4, +\infty[\quad S_2 =]-2, 0] \\ &\Rightarrow \underline{\underline{S = S_1 \cup S_2 =]-\infty, 0] \cup [4, +\infty[}}. \end{aligned}$$

9) $|5x+1| < |x+1|$ $x = -1$ non solution !

$\forall x \neq -1$ on a $|5x+1| < |x+1| \Leftrightarrow \left| \frac{5x+1}{x+1} \right| < 1$

$$\Leftrightarrow -1 < \frac{5x+1}{x+1} < 1$$

$$\Leftrightarrow \begin{cases} \frac{5x+1}{x+1} + 1 > 0 \\ \frac{5x+1}{x+1} - 1 < 0 \end{cases} \Leftrightarrow \begin{cases} \frac{5x+1+x+1}{x+1} > 0 \\ \frac{5x+1-x-1}{x+1} < 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \frac{6x+2}{x+1} > 0 \\ -\frac{4x}{x+1} < 0 \end{cases} \Leftrightarrow \begin{cases} x \in]-\infty, -1[\cup]-\frac{1}{3}, +\infty[= S_1 \\ x \in]-4, 0[= S_2 \end{cases}$$

$$\underline{\underline{S = S_1 \cap S_2 =]-\frac{1}{3}, 0[}}.$$

$$10) |2-(x+1)| > 1-|x| \Leftrightarrow |1-x| > 1-|x|$$

$$\Leftrightarrow \begin{cases} 1-x \geq 0 \\ x \geq 0 \\ 1-x > 1-x \end{cases} \circ \begin{cases} 1-x \geq 0 \\ x < 0 \\ 1-x > 1+x \end{cases} \circ \begin{cases} 1-x < 0 \\ x \geq 0 \\ x-1 > 1-x \end{cases} \circ \begin{cases} 1-x < 0 \\ x < 0 \\ x-1 > 1+x \end{cases}$$

$$\Leftrightarrow \emptyset \circ \{x < 0\} \circ \{x > 1\} \circ \emptyset$$

$$\underline{\underline{S =]-\infty, 0[\cup]1, +\infty[.}}$$

$$11) |x| + 2|x-1| > 1-x$$

$$\Leftrightarrow \begin{cases} x \geq 0 \\ x-1 \geq 0 \\ x+2x-2 > 1-x \end{cases} \circ \begin{cases} x \geq 0 \\ x-1 < 0 \\ x-2x+2 > 1-x \end{cases} \circ \begin{cases} x < 0 \\ x-1 \geq 0 \\ -x+2x-2 > 1-x \end{cases} \circ \begin{cases} x < 0 \\ x-1 < 0 \\ x-2x+2 > 1-x \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq 1 \\ x \geq \frac{3}{4} \end{cases} \circ \begin{cases} 0 \leq x < 1 \\ \mathbb{R}_1 \end{cases} \circ \emptyset \circ \begin{cases} x < 0 \\ x < \frac{1}{2} \end{cases}$$

$$\Rightarrow \underline{\underline{S = \mathbb{R}_1.}}$$

$$12) |2x-1| + |x-2| > 1 \Leftrightarrow \begin{cases} 2x-1 < -1 \\ 2x-1+|x-2| < -1 \end{cases} \circ \begin{cases} 2x-1 > 1 \\ 2x-1+|x-2| > 1 \end{cases}$$

$$\Leftrightarrow |x-2| < -2x \circ |x-2| > 2-2x$$

$$\Leftrightarrow \begin{cases} x-2 \geq 0 \\ x-2 < -2x \end{cases} \circ \begin{cases} x-2 < 0 \\ -x+2 < -2x \end{cases} \circ \begin{cases} x-2 \geq 0 \\ x-2 > 2-2x \end{cases} \circ \begin{cases} x-2 < 0 \\ -x+2 > 2-2x \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq 2 \\ x < \frac{2}{3} \end{cases} \circ \begin{cases} x < 2 \\ x < -2 \end{cases} \circ \begin{cases} x \geq 2 \\ x > \frac{4}{3} \end{cases} \circ \begin{cases} x < 2 \\ x > 0 \end{cases}$$

$$\Rightarrow \underline{\underline{S =]-\infty, -2[\cup]0, +\infty[.}}$$

$$13) \frac{|x^2-2x|+x^2}{2+|x|} \leq 1 \Leftrightarrow |x^2-2x|+x^2 \leq 2+|x| \Leftrightarrow$$

$$\begin{cases} x^2-2x \geq 0 \\ x \geq 0 \\ x^2-2x+x^2 \leq 2+x \end{cases} \circ \begin{cases} x^2-2x \geq 0 \\ x < 0 \\ x^2-2x+x^2 \leq 2-x \end{cases} \circ \begin{cases} x^2-2x < 0 \\ x \geq 0 \\ x^2+2x+x^2 \leq 2+x \end{cases} \circ \begin{cases} x^2-2x < 0 \\ x < 0 \\ -x^2+2x+x^2 \leq 2-x \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq 2 \\ 2x^2-3x-2 \leq 0 \end{cases} \circ \begin{cases} x < 0 \\ 2x^2-x-2 \leq 0 \end{cases} \circ \begin{cases} x \in [0, 2[\\ x \leq 2 \end{cases} \circ \emptyset$$

$$\downarrow 2 \quad \frac{1-\sqrt{17}}{4} \leq x < 0 \quad [0, 2[\quad \underline{\underline{S = \left[\frac{1-\sqrt{17}}{4}, 2 \right].}}$$

$$14) \sqrt{2x-x^2} > x \Leftrightarrow \begin{cases} 2x-x^2 \geq 0 \\ x < 0 \end{cases} \quad \text{ou} \quad \begin{cases} 2x-x^2 \geq 0 \\ x \geq 0 \\ 2x-x^2 > x^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2-2x \leq 0 \\ x < 0 \end{cases} \quad \text{ou} \quad \begin{cases} x^2-2x \leq 0 \\ x \geq 0 \\ 2x^2-2x < 0 \end{cases}$$

$$S =]0, 1[.$$

$$15) 1+2x > \sqrt{4x^2-5x+1} \Leftrightarrow \begin{cases} 4x^2-5x+1 \geq 0, \quad 1+2x \geq 0 \\ (1+2x)^2 > 4x^2-5x+1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \leq \frac{1}{4} \text{ ou } x \geq 1, \quad x \geq -\frac{1}{2} \\ 1+4x+4x^2 > 4x^2-5x+1 \end{cases}$$

$$S =]0, \frac{1}{4}] \cup [1, +\infty[.$$

$$16) \sqrt{4x^2+3x-1} + 3-2x > 0 \Leftrightarrow \sqrt{4x^2+3x-1} > 2x-3$$

$$\Leftrightarrow \begin{cases} 4x^2+3x-1 \geq 0 \\ 2x-3 < 0 \end{cases} \quad \text{ou} \quad \begin{cases} 4x^2+3x-1 \geq 0 \\ 2x-3 \geq 0 \\ 4x^2+3x-1 > (2x-3)^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \leq -1 \text{ ou } x \geq \frac{1}{4} \\ x < \frac{3}{2} \end{cases} \quad \text{ou} \quad \begin{cases} x \leq -1 \text{ ou } x \geq \frac{1}{4} \\ x \geq \frac{3}{2} \\ 4x^2+3x-1 > 4x^2-12x+9 \end{cases}$$

$$S_1 =]-\infty, -1] \cup [\frac{1}{4}, \frac{3}{2}[$$

$$S_2 = [\frac{3}{2}, +\infty[.$$

$$\Rightarrow S = S_1 \cup S_2 =]-\infty, -1] \cup [\frac{1}{4}, +\infty[.$$

$$17) 2(x - \sqrt{x^2+2x+5}) > 3 \Leftrightarrow x - \sqrt{x^2+2x+5} > \frac{3}{2}$$

$$\Leftrightarrow \sqrt{x^2+2x+5} < x - \frac{3}{2} \Leftrightarrow \begin{cases} x - \frac{3}{2} \geq 0 \\ x^2+2x+5 < x^2-3x+\frac{9}{4} \end{cases}$$

$$x^2+2x+5 \geq 0 \quad \forall x \in \mathbb{R}$$

$$5x < \frac{9}{4} - 5 \Rightarrow 5x < -\frac{11}{4} \Rightarrow x < -\frac{11}{20}$$

$$\Leftrightarrow \begin{cases} x \geq \frac{3}{2} \\ x < -\frac{11}{20} \end{cases} \Rightarrow S = \emptyset.$$

$$\begin{aligned}
 18) \quad \sqrt[3]{x^3+2} &> x-1 \Leftrightarrow x^3+2 > (x-1)^3 \\
 &\Leftrightarrow \cancel{x^3}+2 > \cancel{x^3}-3x^2+3x-1 \\
 &\Leftrightarrow 3x^2-3x+3 > 0 \\
 &\Leftrightarrow x^2-x+1 > 0 \quad \forall x \in \mathbb{R}
 \end{aligned}$$

$$\underline{S = \mathbb{R}}$$

$$\begin{aligned}
 19) \quad \sqrt[3]{2x-7} - \sqrt[3]{x-3} &> 0 \Leftrightarrow \sqrt[3]{2x-7} > \sqrt[3]{x-3} \\
 &\Leftrightarrow 2x-7 > x-3 \\
 &\Leftrightarrow x > 4
 \end{aligned}$$

$$\underline{S =]4, +\infty[}$$

$$\begin{aligned}
 20) \quad \frac{3^{x+1}}{27^{2x}} < \frac{1}{3^{x^2+5}} &\Leftrightarrow 3^{x+1} \cdot 3^{-3(2x)} < 3^{-x^2-5} \\
 &\Leftrightarrow 3^{x+1-6x} < 3^{-x^2-5} \\
 &\Leftrightarrow x+1-6x < -x^2-5 \\
 &\Leftrightarrow x^2-5x+6 < 0 \\
 &\quad (x-3)(x-2) < 0
 \end{aligned}$$

$$\underline{S =]2, 3[}$$

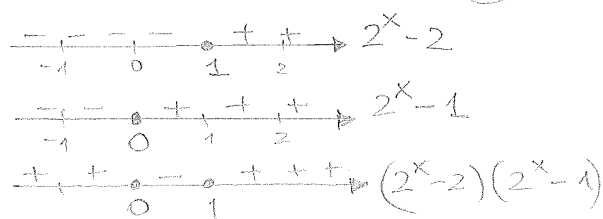
$$\begin{aligned}
 21) \quad \sqrt{9^{x+1}} &\geq 25 \cdot 5^{2x} \Leftrightarrow \sqrt{(3^{x+1})^2} \geq 5^2 \cdot 5^{2x} \\
 &\Leftrightarrow 3^{x+1} \geq 5^{2+2x} = 25^{x+1} \\
 &\Leftrightarrow \left(\frac{3}{25}\right)^{x+1} \geq 1 \Leftrightarrow x+1 \leq 0
 \end{aligned}$$

$$\underline{S =]-\infty, -1]}$$

$$\begin{aligned}
 22) \quad \frac{2^x \cdot 15}{2^3(2^3+1)} < 5 \left(\frac{1}{3}\right)^{4-x} &\Leftrightarrow \frac{2^x \cdot \cancel{15}}{8 \cdot \cancel{8} \cdot \cancel{5}} < \left(\frac{1}{3}\right)^{4-x} \\
 &\Leftrightarrow 2^{x-3} < 3 \cdot 3^{-4+x} \\
 &\Leftrightarrow 2^{x-3} < 3^{x-3} \\
 &\Leftrightarrow \left(\frac{2}{3}\right)^{x-3} < 1 \quad x-3 > 0
 \end{aligned}$$

$$\underline{S =]3, +\infty[}$$

$$\begin{aligned}
 23) \quad 4^x - 3 \cdot 2^x + 2 &> 0 \quad (2^x)^2 - 3 \cdot 2^x + 2 > 0 \\
 &\Rightarrow (2^x - 2)(2^x - 1) > 0
 \end{aligned}$$



$$\underline{S =]-\infty, 0[\cup]1, +\infty[}$$

$$24) \quad 8\left(\frac{1}{2}\right)^x - 6\left(\frac{1}{2}\right)^x + 1 > 0 \quad \text{Poniamo } \left(\frac{1}{2}\right)^x = t$$

$$8t^2 - 6t + 1 > 0$$

$$\text{ora } 8t^2 - 6t + 1 = 0 \Leftrightarrow t_{1/2} = \frac{6 \pm \sqrt{36 - 32}}{16} = \frac{6 \pm 2}{16} \begin{cases} \frac{1}{2} \\ \frac{1}{4} \end{cases}$$

$$\text{Quindi } 8t^2 - 6t + 1 > 0 \Leftrightarrow t < \frac{1}{4} \text{ o } t > \frac{1}{2}$$

$$\text{Quindi due aree } \left(\frac{1}{2}\right)^x < \frac{1}{4} \text{ o } \left(\frac{1}{2}\right)^x > \frac{1}{2}$$

$$\left(\frac{1}{2}\right)^x \overset{\updownarrow}{<} \left(\frac{1}{2}\right)^2 \text{ o } \left(\frac{1}{2}\right)^x > \left(\frac{1}{2}\right)^1$$

$$\Rightarrow \underline{S =]-\infty, 1[\cup]2, +\infty[}$$

$$25) \quad \log(2x^2 - 5x + 3) < 0 \Leftrightarrow \begin{cases} 2x^2 - 5x + 3 > 0 \\ 2x^2 - 5x + 3 < 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x < 1 \text{ o } x > \frac{3}{2} \\ \frac{1}{2} < x < 2 \end{cases}$$

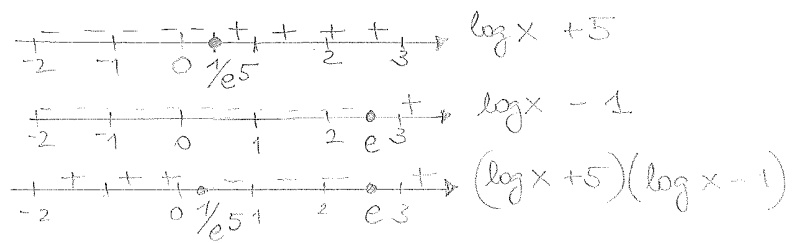
$$\Rightarrow \underline{S =]\frac{1}{2}, 1[\cup]\frac{3}{2}, 2[}$$

$$26) \quad \log(x^2 - 4x + 3) - \log(x - 2) \geq \log(x + 1)$$

$$\Leftrightarrow \begin{cases} x^2 - 4x + 3 > 0 \\ x - 2 > 0 \\ x + 1 > 0 \\ x^2 - 4x + 3 \geq (x - 2)(x + 1) \end{cases}$$

$$\Leftrightarrow \begin{cases} x < 1 \text{ o } x > 3 \\ x > 2 \\ x > -1 \\ \cancel{x^2 - 4x + 3} \geq \cancel{x^2} - x - 2 \end{cases} \Leftrightarrow \begin{cases} x > 3 \\ 3x \leq 5 \end{cases} \quad \underline{S = \emptyset}$$

$$27) \quad \log^2 x + 4 \log x - 5 \geq 0 \quad \begin{cases} x > 0 \\ (\log x + 5)(\log x - 1) \geq 0 \end{cases}$$



$$S =]-\infty, \frac{1}{e^5}] \cup [e, +\infty[$$

