Università di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2015 – 2016 – Rovereto, 23 – 27 novembre 2015 n. 10

1) a) $f(x) = \frac{3}{x-2}$ nel pt. $(1,-3)$: $f'(x) = -\frac{3}{(x-2)^2}$ $f'(1) = -3$

retta tg : $y = -3 - 3(x-1)$ $\Rightarrow$ $\underline{y = -3x}$.

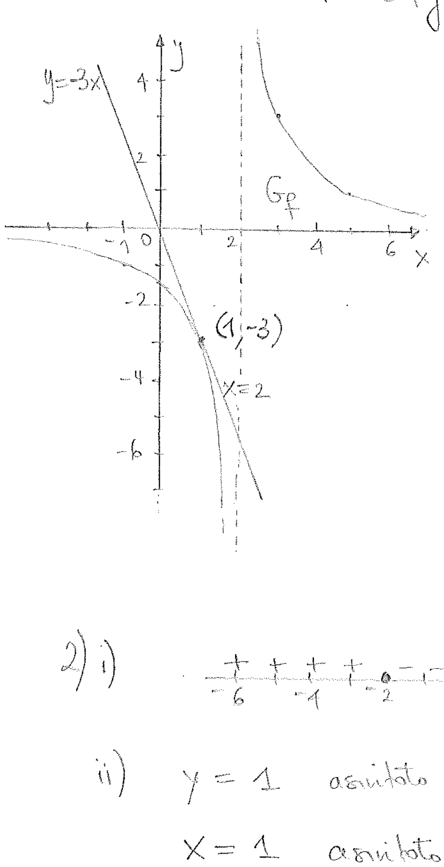
b) $g(x) = \sqrt[3]{x}$ nel pt. $(8,2)$: $g'(x) = (x^{1/3})' = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

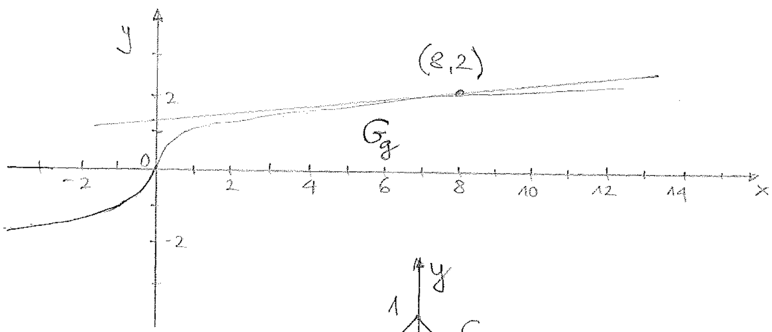
$g'(8) = \frac{1}{3 \cdot 4} = \frac{1}{12}$

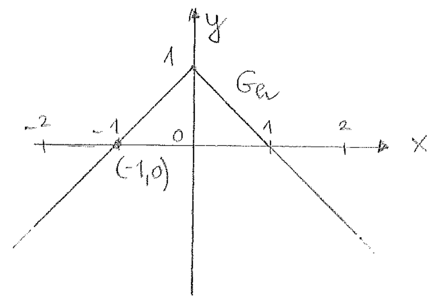
retta tg : $y = 2 + \frac{1}{12}(x-8)$ $\Rightarrow$ $\underline{y = \frac{1}{12}x + \frac{4}{3}}$.

c) $h(x) = 1 - |x|$ nel pt. $(-1,0)$: $h'(x) = (1+x)' = 1$ se $x < 0$ $h'(-1) = 1$

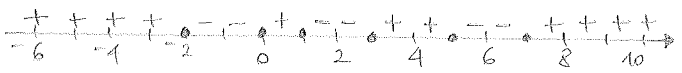
retta tg : $y = 0 + 1(x+1)$ $\Rightarrow$ $\underline{y = x+1}$







2) i)

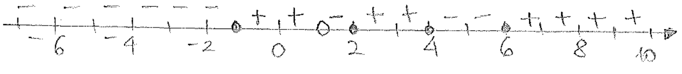


f

ii) $y = 1$ asintoto orizzontale per f, per $x \to -\infty$;

$x = 1$ asintoto verticale (da sinistra) per f.

iii)



f' $\mathrm{dom} f' = \mathbb{R} \setminus \{1\}$.

iv) $\underline{x \in \{1,3\} \text{ punto di massimo}}$ di f su $[1,3]$. Non sono punti in cui si annulla la derivata prima.

$\underline{x = 2 \text{ punto di minimo}}$ di f su $[1,3]$. Abbiamo $f'(2) = 0$.

3) i) a) $\left(4x^5 - x^{-2}\right)' = 4 \cdot 5x^4 - (-2)x^{-3} = \underline{20x^4 + 2x^{-3}}$;

$$\left(\frac{x}{3+x^2}\right)' = \frac{1\cdot(3+x^2) - x\cdot 2x}{(3+x^2)^2} = \frac{3-x^2}{(3+x^2)^2} \quad ;$$

$$\left(\frac{\sqrt{x}}{e^x-2}\right)' = \frac{(\sqrt{x})'\,(e^x-2) - \sqrt{x}\cdot e^x}{(e^x-2)^2} = \frac{\frac{1}{2\sqrt{x}}\,(e^x-2) - \sqrt{x}\,e^x}{(e^x-2)^2} \quad ;$$

$$\left(\sqrt[3]{x} + \log_2 x\right)' = \frac{1}{3}x^{-2/3} + \frac{1}{x\log 2}\,.$$

$\square$

b) $\left[\left(x + \frac{1}{x^{-2}}\right)\left(x^{-1} + 2^x\right)\right]' = \left[(x+x^2)\left(x^{-1}+2^x\right)\right]' = (1+2x)\left(x^{-1}+2^x\right) + (x+x^2)\left(-\frac{1}{x^2} + 2^x\log 2\right)$;

$$\left[(\log x + x^3)\left(e^{-x} + \frac{1}{x^2-1}\right)\right]' = \left(\frac{1}{x} + 3x^2\right)\left(e^{-x} + \frac{1}{x^2-1}\right) + (\log x + x^3)\left(-e^{-x} - \frac{2x}{(x^2-1)^2}\right).$$

$\square$

ii) $\left[(2x^3+e^x)^4\right]' = 4\left(2x^3+e^x\right)^3\cdot\left(6x^2+e^x\right)$;

$$\left(e^{3x^2+x}\right)' = e^{3x^2+x}\,(6x+1)\ ;$$

$$\left[\log\left(1+4x+x^2\right)\right]' = \frac{4+2x}{1+4x+x^2} \quad ;$$

$$\left[x e^{-x}\log(1-x^3)\right]' = e^{-x}\log(1-x^3) - xe^{-x}\log(1-x^3) - \frac{3x^3e^{-x}}{1-x^3}\,.$$

$\blacksquare$

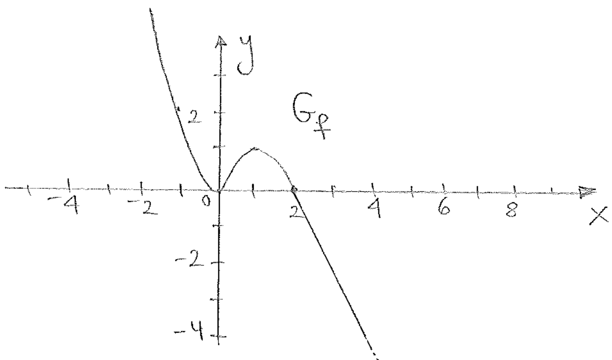
4) $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} 2x^2 & \text{se } x \le 0 \\ -x^2+2x & \text{se } 0 < x \le 2 \\ -2x+4 & \text{se } x > 2\,. \end{cases}$$

i) $\lim\limits_{x\to 0^-} 2x^2 = 0 = f(0) = \lim\limits_{x\to 0^+}(-x^2+2x) \Rightarrow f$ è continua in $x=0$.

$\lim\limits_{x\to 2^-}(-x^2+2x) = 0 = f(2) = \lim\limits_{x\to 2^+}(-2x+4) \Rightarrow f$ è continua in $x=2$.

$$y = -x^2+2x = -(x^2-2x) = -(x-1)^2+1$$



$\square$

ii) $\lim\limits_{h\to 0^-}\frac{f(h)-f(0)}{h} = \lim\limits_{h\to 0^-}\frac{2h^{\cancel{2}} - 0}{\cancel{h}} = \underline{0}$;

$\lim\limits_{h\to 0^+}\frac{f(h)-f(0)}{h} = \lim\limits_{h\to 0^+}\frac{-h^{\cancel{2}}+2\cancel{h} - 0}{\cancel{h}} = \underline{2}$;

$\Big\}$ $\underline{f \text{ non è derivabile in } x=0}$

$\square$

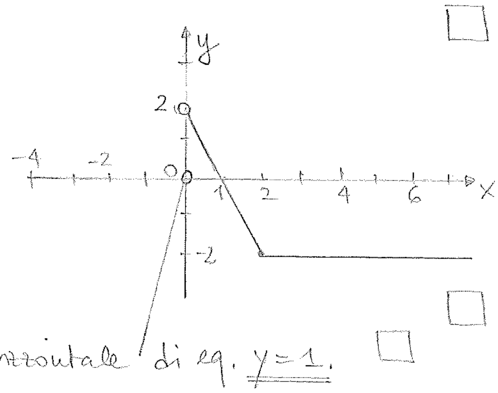
iii) $\lim\limits_{h\to 0^-} \frac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0^-} \frac{-(2+h)^2+2(2+h)-0}{h} = \lim\limits_{h\to 0^-} \frac{\cancel{-4}-4h-h^2\cancel{+4}+2h}{h}$

$= \lim\limits_{h\to 0^-} \frac{-h^2-2\cancel{h}}{\cancel{h}} = \underline{-2}$;

$\lim\limits_{h\to 0^+} \frac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0^+} \frac{-2(2+h)+4-0}{h} = \lim\limits_{h\to 0^+} \frac{\cancel{-4}-2h\cancel{+4}}{h} = \underline{-2}$;

$\Rightarrow$ <u>f è derivabile in $x=2$.</u>

iv) $f'(x) = \begin{cases} 4x & \text{se } x<0 \\ -2x+2 & \text{se } 0<x\le 2 \\ -2 & \text{se } x>2 . \end{cases}$

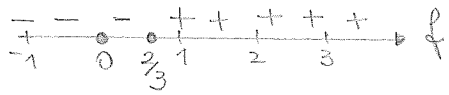


dom $f' = \mathbb{R}\setminus\{0\}$.

v) in $(1,1)$ la retta tg al grafico è orizzontale di eq. $\underline{y=1}$.

5) $\boxed{f(x) = 3x^3-2x^2}$ • dom $f = \mathbb{R}$

• $f(x) = x^2(3x-2)$



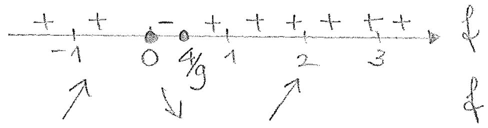
• $\lim\limits_{x\to-\infty} f(x) = \underline{\underline{-\infty}}$; $\lim\limits_{x\to+\infty} f(x) = \underline{\underline{+\infty}}$;

• f è continua essendo somma di funzioni continue su $\mathbb{R}$;

• dom $f' = \mathbb{R}$ $f'(x) = 9x^2-4x = x(9x-4)$

$f'(x)=0 \iff x=0,\ x=4/9$ pt. critici di f

$x=0$ pt. di massimo locale stretto



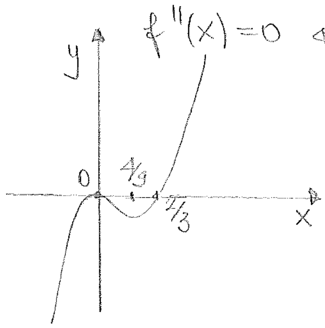
$f(0)=0$

$x=4/9$ pt. di min. loc. stretto $f\left(\frac{4}{9}\right) = \cancel{3}\cdot\frac{64}{81\cdot\cancel{9}3} - 2\cdot\frac{16}{81} = \frac{64-6\cdot16}{81\cdot3}$

$= \frac{64-96}{81\cdot3} = -\frac{32}{81\cdot3} \sim -0.1$

• dom $f'' = \mathbb{R}$ $f''(x) = 18x-4$

$f''(x)=0 \iff x=\frac{2}{9}$ pt. di flesso



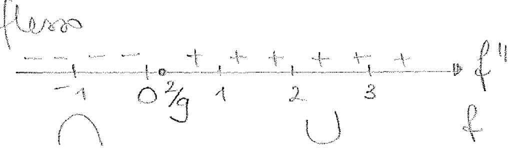
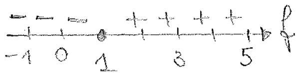


grafico qualitativo di f

- $\boxed{f(x) = \dfrac{x-1}{x^2-2x+2}}$ • dom $f = \mathbb{R}$ $x^2-2x+2 = 0 \iff x_{1/2} = \dfrac{2 \pm \sqrt{4-8}}{2}$ impossibile

•  $x^2-2x+2 > 0 \quad \forall x \in \mathbb{R}$

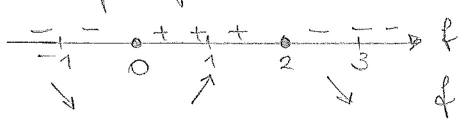
- $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to+\infty} f(x) = 0$ $y = 0$ asintoto orizzontale per f, per $x \to -\infty$
per $x \to +\infty$.

- f è continua in $\mathbb{R}$, essendo rapporto di funzioni continue con denominatore che non si annulla mai.

- dom $f' = \mathbb{R}$ $f'(x) = \dfrac{(x^2-2x+2)-(x-1)(2x-2)}{(x^2-2x+2)^2} = \dfrac{x^2-2x+\cancel{2}-2x^2+4x-\cancel{2}}{(x^2-2x+2)^2}$

$$= \frac{-x^2+2x}{(x^2-2x+2)^2}.$$

$f'(x) = 0 \iff x = 0,\ x = 2$ pt. critici per f



$x=0$ pt. di min. loc. stretto per f
$f(0) = -\frac{1}{2}$

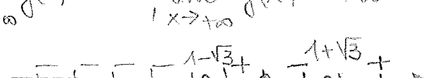
$x=2$ pt. di max. loc. stretto per f
$f(2) = \frac{1}{2}$.

- dom $f'' = \mathbb{R}$ $f''(x) = \dfrac{(-2x+2)(x^2-2x+2)^{\cancel{2}} - (-x^2+2x)\,2\,\cancel{(x^2-2x+2)}(2x-2)}{(x^2-2x+2)^{\cancel{4}\,3}}$

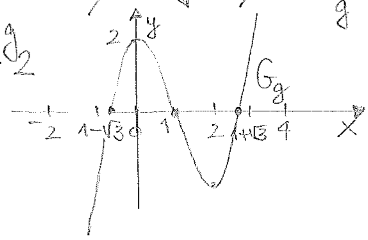
$$= \frac{-2x^3+4x^2-\cancel{4x}+2x^2-\cancel{4x}+4x^3-12x^2+\cancel{8x}+4}{(x^2-2x+2)^3}$$

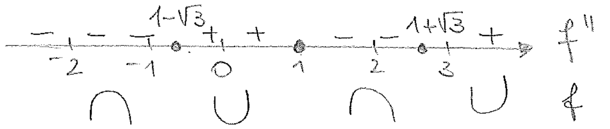
$$= \frac{2x^3-6x^2+4}{(x^2-2x+2)^3} = \frac{2(x^3-3x^2+2)}{(x^2-2x+2)^3}$$

Studiamo $g(x) = x^3-3x^2+2 = (x-1)(x^2-2x-2)$: abbiamo $\lim\limits_{x\to-\infty} g(x) = -\infty$, $\lim\limits_{x\to+\infty} g(x) = +\infty$



$g'(x) = 3x^2-6x = 3x(x-2)$ $x=0$ pt. di max.loc. stretto per g
$g(0) = 2$,
$x=2$ pt. di min. loc. stretto per g
$g(2) = 8-3\cdot4+2 = -2$





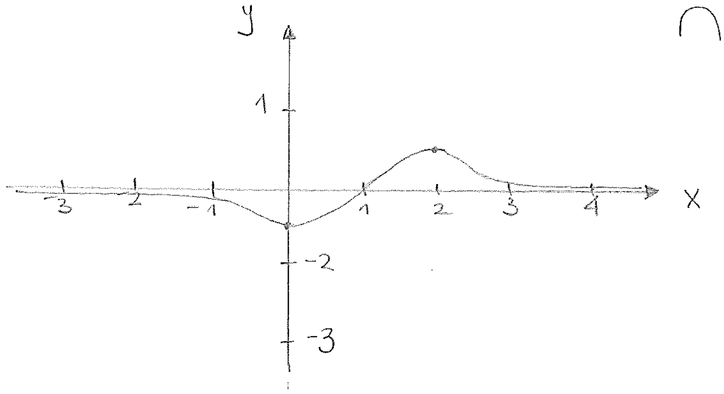
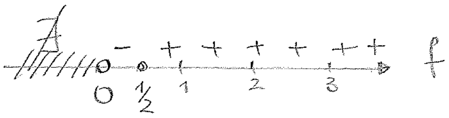


grafico qualitativo di f

$\square$

- $\boxed{f(x) = \frac{\log(2x)}{x}}$

- $\mathrm{dom} f =]0, +\infty[$
- segno di f



- $\lim_{x \to 0^+} f(x) = -\infty$ $x = 0$ asintoto verticale (da destra) di f

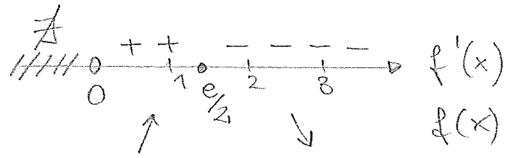
  $\lim_{x \to +\infty} f(x) = 0$ $y = 0$ asintoto orizzontale di f, per $x \to +\infty$.

- f continua perchè rapporto di funzioni continue su $]0, +\infty[$.

- $\mathrm{dom} f' =]0, +\infty[$ $f'(x) = \frac{\frac{1}{2\cancel{x}} \cdot 2 \cdot \cancel{x} - \log(2x)}{x^2} = \frac{1 - \log(2x)}{x^2}$

  $f'(x) = 0 \iff \log(2x) = 1 \iff 2x = e \iff x = \frac{e}{2}$ pt. critico di f

  $x = \frac{e}{2}$ pt. di max. loc. stretto per f



$$f\left(\frac{e}{2}\right) = \frac{\log\left(\cancel{2} \cdot \frac{e}{\cancel{2}}\right)}{\frac{e}{2}} = \frac{2}{e} \sim 0.7$$

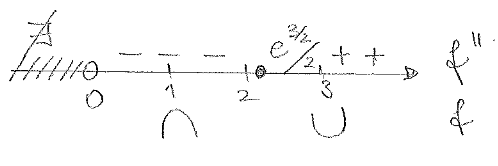
- $\mathrm{dom} f'' =]0, +\infty[$ $f''(x) = \frac{-\frac{1}{\cancel{2x}} \cdot \cancel{2} \cdot x^{\cancel{2}} - (1 - \log(2x)) \cdot 2\cancel{x}}{x^{\cancel{4}3}}$

$$= \frac{-3 + 2\log(2x)}{x^3}$$

$-3 + 2\log(2x) = 0$

$\iff \log(2x) = \frac{3}{2}$

$\iff 2x = e^{3/2}$ $x = \frac{e^{3/2}}{2} \sim 2.2$



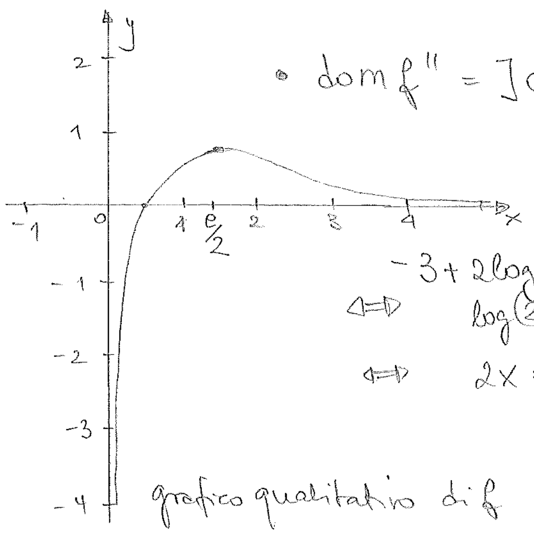


grafico qualitativo di f

$\square$

- $\boxed{f(x) = \dfrac{e^{1-x}}{x-1}}$
  - dom $f = \mathbb{R} \setminus \{1\}$
  - segno di f $- - - - \, \circ \, + + +$ (-2, 0, 2) f

- $\lim\limits_{x \to -\infty} f(x) = -\infty$, $\lim\limits_{x \to 1^-} f(x) = -\infty$ $x = 1$ asintoto verticale (da sinistra) per f

  $\lim\limits_{x \to 1+} f(x) = +\infty$ $x = 1$ asintoto verticale (da destra) per f

  $\lim\limits_{x \to +\infty} f(x) = 0$

  $y = 0$ asintoto orizzontale per f, per $x \to +\infty$.

- f continua in dom f, poiché composizione e rapporto di funz. continue.
- dom $f' = \mathbb{R} \setminus \{1\}$

$$f'(x) = \frac{-e^{1-x}(x-1) - e^{1-x}}{(x-1)^2} = \frac{e^{1-x}(-x)}{(x-1)^2}.$$

$f'(x) = 0 \iff x = 0$ pt. critico per f ; $f(0) = -e$.

$x = 0$ pt. di max. loc. stretto per f $+ + + + + \, \circ \, - - - -$ f' (-4, -2, 0, 2, 4); f: $\nearrow$ $\searrow$ $\searrow$

- dom $f'' = \mathbb{R} \setminus \{1\}$

$$f''(x) = \frac{\left[-e^{1-x}(-x) - e^{1-x}\right](x-1)^{\cancel{2}} + xe^{1-x} \cdot 2\cancel{(x-1)}}{(x-1)^{\cancel{4}\,3}}$$

$$= \frac{e^{1-x}\left[(x-1)^2 + 2x\right]}{(x-1)^3} = \frac{e^{1-x}(x^2+1)}{(x-1)^3}$$

$- - - - - - \, \circ \, + + + +$ f'' (-4, -2, 0, 2, 4); f: $\cap$ $\cup$

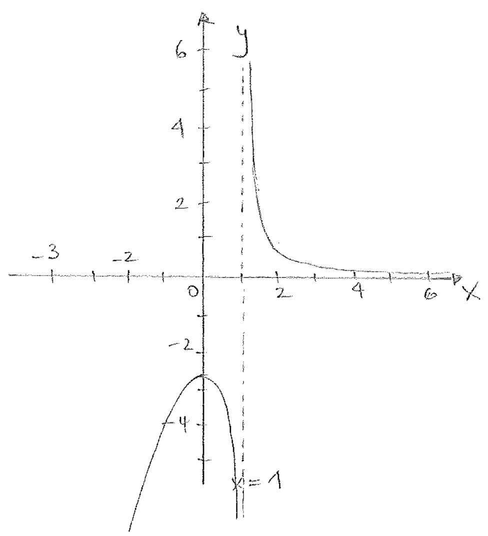


grafico qualitativo di f

$\square$

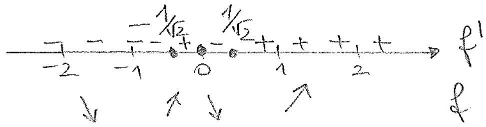
- $\boxed{f(x) = (x^2 - 1)x^2}$
  - dom $f = \mathbb{R}$

  f è pari!
  - segno di f $+ \, \bullet \, - \, \bullet \, - \, \bullet \, + +$ f (-2, -1, 0, 1, 2)

- $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to +\infty} f(x) = +\infty$
- f è continua in $\mathbb{R}$, perché prodotto di funzioni continue.

• $\operatorname{dom} f' = \mathbb{R}$, $\quad f'(x) = 4x^3 - 2x = 2x(2x^2 - 1)$

$f'(x) = 0 \iff x = 0\,,\ x = \pm\frac{1}{\sqrt{2}}$ pt. critici di f

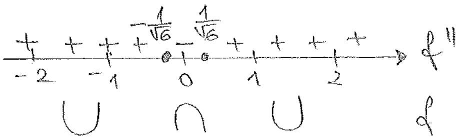


$x = -\frac{1}{\sqrt{2}}\,,\ x = \frac{1}{\sqrt{2}}$

pt. di min. loc. stretti per f; $\quad f\left(-\frac{1}{\sqrt{2}}\right) = f\left(\frac{1}{\sqrt{2}}\right) = \left(\frac{1}{2} - 1\right)\left(\frac{1}{2}\right) = -\frac{1}{4}$

$x = 0$ pt. di max. loc. stretto per f, $\quad f(0) = 0$

• $\operatorname{dom} f'' = \mathbb{R}$, $\quad f''(x) = 12x^2 - 2 = 2(6x^2 - 1)$



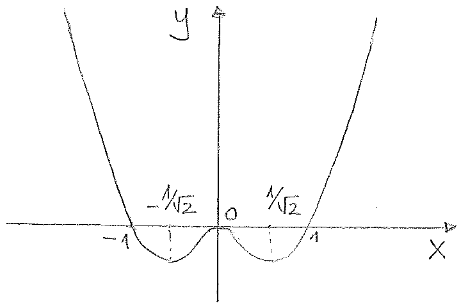


grafico qualitativo di f

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