Università di Trento - Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2015-2016 - Rovereto, 30 novembre - 4 dicembre 2015 n. 11

1) i) $\displaystyle\sum_{n=0}^{10} (-1)^n \frac{a^{2n+1}}{2n+3}$; $\displaystyle\sum_{n=1}^{10} \frac{n}{2^{nx}}$;

$\displaystyle\sum_{n=1}^{9} (-1)^n \frac{b^n n^2}{(n+1)}$; $\displaystyle\sum_{n=1}^{7} \int_{1/n+1}^{1/n} x\,dx$. $\square$

ii) a) $f_n(x) = x^2 + n$ $n \in \{1,2,3,\dots\}$, $x \in \mathbb{R}$ $m_n = \min\limits_{x\in[-1,1]} f_n(x) = f_n(0) = n$

$$\sum_{n=1}^{7} m_n = \sum_{n=1}^{7} n = 1+2+3+\dots+7 = \underline{\underline{28}} .$$

b) $\displaystyle\sum_{j=1}^{5} f^{(j)}\left(\frac{1}{j}\right)$ $f(x) = \log x$ $f^{(1)}(x) = \frac{1}{x}$ $f^{(2)}(x) = -\frac{1}{x^2}$

$f^{(3)}(x) = \frac{2}{x^3}$ $f^{(4)}(x) = -\frac{6}{x^4}$ $f^{(5)}(x) = \frac{24}{x^5}$

$$= f^{(1)}(1) + f^{(2)}\left(\tfrac{1}{2}\right) + f^{(3)}\left(\tfrac{1}{3}\right) + \dots + f^{(5)}\left(\tfrac{1}{5}\right) =$$

$$= 1 - 4 + 2\cdot 27 - 6\cdot 16^2 + 24\cdot 5^5 = -3 + 54 - 1536 + 24\cdot 3125$$

$$= 51 - 1536 + 75.000 = \underline{\underline{73.515}} .$$

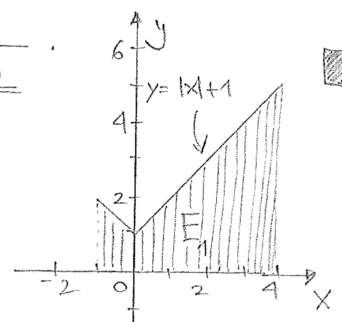
c) $\displaystyle\sum_{m=1}^{4} (-1)^{m+1} \frac{1-m}{m^2} = -\frac{(-1)}{4} + \frac{(-2)}{9} - \frac{(-3)}{16} = \frac{1}{4} - \frac{2}{9} + \frac{3}{16} = \frac{36-32+27}{16\cdot 9}$

$$= \underline{\underline{\frac{31}{144}}} ;$$

$$\sum_{k=3}^{30} \left(\frac{3}{k^2} - \frac{3}{(k+1)^2}\right) = 3\left[\left(\frac{1}{9} - \cancel{\frac{1}{16}}\right) + \left(\cancel{\frac{1}{16}} - \frac{1}{25}\right) + \dots + \left(\cancel{\frac{1}{29^2}} - \cancel{\frac{1}{30^2}}\right) + \left(\cancel{\frac{1}{30^2}} - \frac{1}{31^2}\right)\right]$$

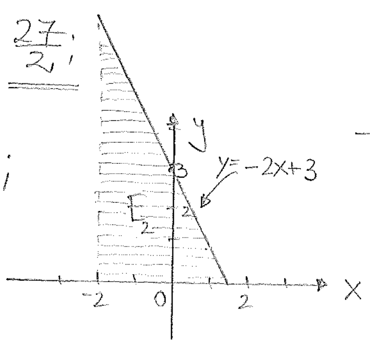
$$= 3\left[\frac{1}{9} - \frac{1}{31^2}\right] = \frac{\cancel{3}(31^2-9)}{\cancel{9}_3\cdot 31^2} = \underline{\underline{\frac{952}{2883}}} .$$ $\blacksquare$

2) $\displaystyle\int_{-1}^{4} (|x|+1)\,dx = \text{area}(E_1) = \underline{\underline{\frac{27}{2}}} ;$

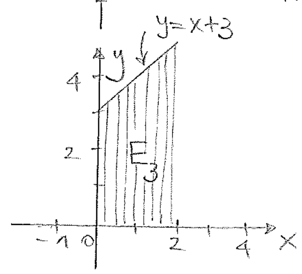


$\displaystyle\int_{-2}^{3/2} (-2x+3)\,dx = \frac{7}{2}\cdot\frac{7}{2} = \underline{\underline{\frac{49}{4}}} ;$

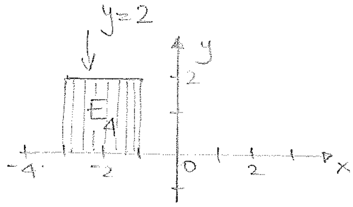
area(E_2)



$\displaystyle\int_{0}^{2} (x+3)\,dx = \text{area}(E_3) = \underline{8} ;$



$$\int_{-3}^{-1} 2\,dx = \text{area}(E_1) = \underline{4}$$



3) i) $$\int_0^1 (x^2+3x)\,dx = \left[\frac{x^3}{3}+3\frac{x^2}{2}\right]_0^1 = \frac{1}{3}+\frac{3}{2} = \frac{2+9}{6} = \underline{\frac{11}{6}};$$

$$\int_1^3 (1-x^{-2})\,dx = \left[x+x^{-1}\right]_1^3 = \left(3+\frac{1}{3}\right)-(1+1) = 1+\frac{1}{3} = \underline{\frac{4}{3}};$$

$$\int_{-2}^{-1}\left(e^x+\frac{3}{x}\right)dx = \left[e^x+3\log|x|\right]_{-2}^{-1} = \left(e^{-1}+3\cancel{\log 1}\right)-\left(e^{-2}+3\log 2\right)$$

$$= \underline{\frac{1}{e}-\frac{1}{e^2}-\log 8}.$$

ii) $$\int_0^1 \left(\sqrt[4]{x}-2\sqrt{x}\right)dx = \int_0^1 \left(x^{1/4}-2x^{1/2}\right)dx = \left[\frac{x^{1/4+1}}{1/4+1}-\frac{2x^{1/2+1}}{1/2+1}\right]_0^1 =$$

$$= \left[\frac{4}{5}x^{5/4}-\frac{4}{3}x^{3/2}\right]_0^1 = \frac{4}{5}-\frac{4}{3} = \frac{12-20}{15} = \underline{-\frac{8}{15}};$$

$$\int_1^2 \frac{4x^2-3x^3}{x}\,dx = \int_1^2 (4x-3x^2)\,dx = \left[2x^2-x^3\right]_1^2 = (\cancel{8}-\cancel{8})-(2-1) = \underline{-1};$$

$$\int_0^1 \frac{x^2-4}{x+2}\,dx = \int_0^1 \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}\,dx = \left[\frac{x^2}{2}-2x\right]_0^1 = \frac{1}{2}-2 = \underline{-\frac{3}{2}}.$$

iii) $$\int_{-1}^0 \left(e^{-x}-xe^{x^2}\right)dx = -\int_{-1}^0 -e^{-x}\,dx - \frac{1}{2}\int_{-1}^0 2xe^{x^2}\,dx = \left[-e^{-x}-\frac{1}{2}e^{x^2}\right]_{-1}^0$$

$$= \left(-1-\frac{1}{2}\right)-\left(-e-\frac{1}{2}e\right) = \underline{-\frac{3}{2}+\frac{3}{2}e}$$

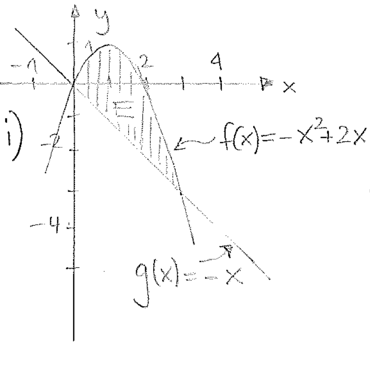
$$\int_1^2 \frac{3}{2x+1}\,dx = \frac{3}{2}\int_1^2 \frac{2}{2x+1}\,dx = \frac{3}{2}\left[\log|2x+1|\right]_1^2 = \frac{3}{2}(\log 5-\log 3)$$

$$= \underline{\frac{3}{2}\log\frac{5}{3}}.$$

$$\int_0^1 x(3x^2-1)^3\,dx = \frac{1}{6}\int_0^1 6x(3x^2-1)^3\,dx = \frac{1}{6}\left[\frac{(3x^2-1)^4}{4}\right]_0^1 =$$

$$= \frac{1}{6}\cdot 4-\frac{1}{6}\cdot\frac{1}{4} = \frac{1}{6}\cdot\frac{15}{4} = \frac{\cancel{15}^{\,5}}{\cancel{24}_{\,8}}.$$

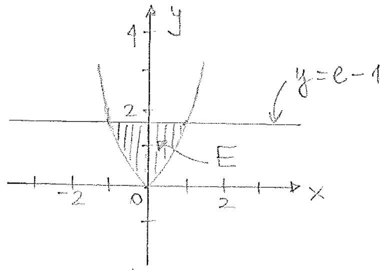
4) i)



$$\text{area}(E) = \int_0^3 (-x^2+2x+x)\,dx = \int_0^3 (-x^2+3x)\,dx =$$

$$= \left[-\frac{x^3}{3}+\frac{3x^2}{2}\right]_0^3 = -9+\frac{27}{2} = \frac{-18+27}{2} = \underline{\frac{9}{2}}.$$ $\square$

ii)

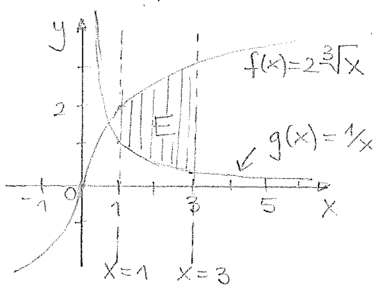


$$\text{area}(E) = 2\int_0^1 (e-\cancel{1}-e^x+\cancel{1})\,dx =$$

$$= 2\left[ex-e^x\right]_0^1 = 2(\cancel{e-e}) - 2(-1)$$

$$= \underline{2}.$$ $\square$

iii)



$$\text{area}(E) = \int_1^3 \left(2\sqrt[3]{x}-\frac{1}{x}\right)dx = \left[2\,\frac{x^{4/3}}{4/3} - \log|x|\right]_1^3$$

$$= \left(\frac{3}{2}\cdot 3^{4/3} - \log 3\right) - \left(\frac{3}{2} - \cancel{\log 1}\right)$$

$$= \underline{\frac{9}{2}\sqrt[3]{3} - \frac{3}{2} - \log 3}.$$ $\blacksquare$

5) i) $F(x) = \frac{x^2}{4}(2\log x - 1)$ è derivabile in $]0,+\infty[$ e

$$F'(x) = \frac{\cancel{2}x}{\cancel{4}_2}(2\log x - 1) + \frac{x^{\cancel{2}}}{\cancel{4}_2}\cdot\frac{\cancel{2}}{\cancel{x}} = x\log x - \cancel{\frac{x}{2}} + \cancel{\frac{x}{2}} = f(x) \text{ in }]0,+\infty[$$ $\square$

ii) $$\int_1^3 f(x)\,dx = \left[F(x)\right]_1^3 = \left[\frac{x^2}{4}(2\log x - 1)\right]_1^3 = \frac{9}{4}(2\log 3 - 1) - \frac{1}{4}(2\cancel{\log 1} - 1)$$

$$= \frac{9}{2}\log 3 - \frac{9}{4} + \frac{1}{4} = \underline{\frac{9}{2}\log 3 - 2}.$$ $\square$

iii) Osserviamo che $f(x) \geq 0$ in $[1,4]$, mentre $g(x) < 0$ in $[1,4]$, quindi $f(x) \geq g(x)$ in $[1,4]$. Quindi

$$\text{area}(E) = \int_1^4 [f(x)-g(x)]\,dx = \int_1^4 f(x)\,dx + \int_1^4 x^2\,dx = \left[F(x)\right]_1^4 + \left[\frac{x^3}{3}\right]_1^4 =$$

$$= F(4) - F(1) + \left(\frac{64}{3}-\frac{1}{3}\right) = 4(2\log 4 - 1) - \frac{1}{4}(2\cancel{\log 1} - 1)$$

$$+ \frac{63}{3} = 8\log 4 - 4 + \frac{1}{4} + \frac{63}{3} = 8\log 4 + \frac{-48+3+252}{12}$$

$$= \underline{8\log 4 + \frac{\cancel{207}^{\,69}}{\cancel{12}_{\,4}}}.$$

6) i) $\boxed{h(x) = (2x + x^2)e^x}$

- $\mathrm{dom}\, h = \mathbb{R}$
- segno di h

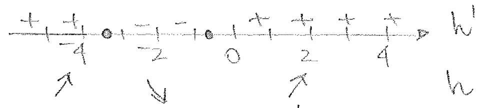
- $\lim_{x \to -\infty} h(x) = \lim_{x \to -\infty} \frac{2x + x^2}{e^{-x}} = 0$ $\quad$ $y = 0$ asintoto orizzontale per h, per $x \to -\infty$.

$\lim_{x \to +\infty} h(x) = +\infty$

- h è continua essendo somma e prodotto di funzioni continue.
- $\mathrm{dom}\, h' = \mathbb{R}$ $\quad$ $h'(x) = (2 + 2x)e^x + (2x + x^2)e^x = e^x(x^2 + 4x + 2)$

$h'(x) = 0 \iff x^2 + 4x + 2 = 0 \iff x_{1/2} = \frac{-4 \pm \sqrt{16 - 8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} =$

$x_{1/2} = -2 \pm \sqrt{2}$ pr. critici per h $\quad = -2 \pm \sqrt{2}$



$x = -2 - \sqrt{2}$ pr. di max. loc. stretto per h

$x = -2 + \sqrt{2}$ pr. di min. loc. stretto per h

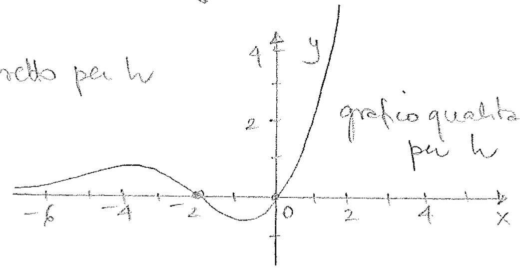
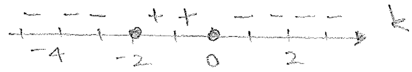


grafico qualitativo per h

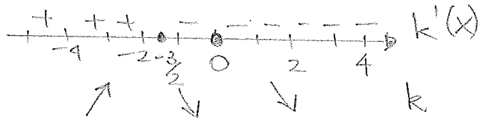
$\boxed{k(x) = -x^3(x + 2)}$

- $\mathrm{dom}\, k = \mathbb{R}$
- segno di k



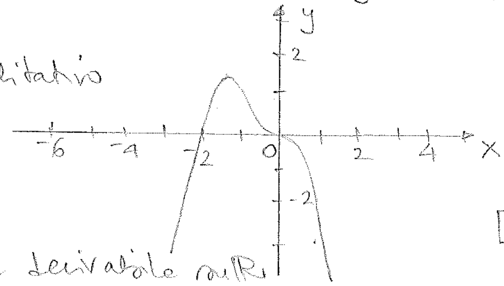
- $\lim_{x \to -\infty} k(x) = -\infty$ $\quad$ $\lim_{x \to +\infty} k(x) = -\infty$
- k(x) è continua essendo prodotto di funzioni continue
- $\mathrm{dom}\, k' = \mathbb{R}$ $\quad$ $k'(x) = -4x^3 - 6x^2 = 2x^2(-2x - 3)$

$k'(x) = 0 \iff x = 0,\ x = -3/2$ pr. critici per k



$x = -3/2$ pr. di max. loc. stretto per k $\quad$ $k\left(-\frac{3}{2}\right) = +\frac{27}{8}\left(-\frac{3}{2} + 2\right) = \frac{27}{16}$

grafico qualitativo di k



ii) $H(x) = x^2 e^x + 1$ è una funzione derivabile su $\mathbb{R}$ $\square$

$H'(x) = 2xe^x + x^2 e^x = (2x + x^2)e^x = h(x) \quad \forall x \in \mathbb{R}$ $\square$

iii) $$\text{area}(E) = \int_{-2}^{0} \left(k(x) - h(x)\right) dx = \int_{-2}^{0} k(x)\,dx - \int_{-2}^{0} h(x)\,dx$$

$$= \int_{-2}^{0} \left(-x^4 - 2x^3\right) dx - \int_{-2}^{0} h(x)\,dx =$$

$$= \left[-\frac{x^5}{5} - \frac{x^4}{2}\right]_{-2}^{0} - \left[H(x)\right]_{-2}^{0} =$$

$$= -\left(\frac{32}{5} - 8\right) - \left(H(0) - H(-2)\right) = \frac{8}{5} - \left[\cancel{1} - \frac{4}{e^2} - \cancel{1}\right]$$

$$= \underline{\underline{\frac{8}{5} + \frac{4}{e^2}}} \; .$$

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