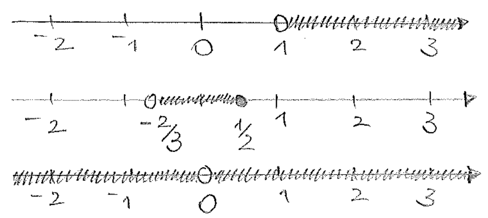
Università di Trento – Dip. di Psicologia e Scienze Cognitive
CDL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2015 – 2016 – Rovereto, 28 settembre – 2 ottobre 2015 n. 2

1) $A =]1,+\infty[$, $B = \{x \in \mathbb{R} : -1 < 3x+1 \leq x+2\} =$

$= \{x \in \mathbb{R} : -2 < 3x \leq x+1\} = \left\{x \in \mathbb{R} : \begin{matrix} 3x > -2 \text{ e} \\ 3x \leq x+1 \end{matrix}\right\}$

$= \{x \in \mathbb{R} : -\frac{2}{3} < x \leq \frac{1}{2}\} = \underline{]-\frac{2}{3}, \frac{1}{2}]}$

$C = \{x \in \mathbb{R} : \frac{1}{x^2} > 0\} = \underline{\mathbb{R} \setminus \{0\}} = \underline{]-\infty, 0[\cup]0,+\infty[}$.

a) 

A
B
C

b) $A \cap C = \underline{A =]1,+\infty[}$; $B \cup C = \underline{\mathbb{R}}$;

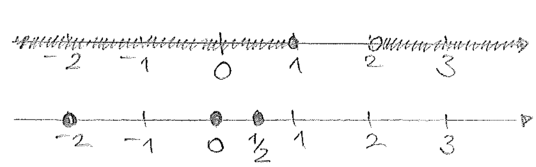
$A \setminus B = \underline{A}$; $\mathbb{R} \setminus C = \underline{\{0\}}$.

A e C non sono disgiunti, poiché $A \cap C \neq \emptyset$.

2) $A = \mathbb{R} \setminus]1,2] = \underline{]-\infty, 1] \cup]2,+\infty[}$;

$B = \{x \in \mathbb{R} : (2x-1)(x^2+2x) = 0\} = \{x \in \mathbb{R} : 2x-1 = 0 \text{ o } \underbrace{x^2+2x}_{x(x+2)} = 0\}$

$= \{x \in \mathbb{R} : 2x = 1 \text{ o } x = 0 \text{ o } x+2 = 0\} = \underline{\{-2, 0, \frac{1}{2}\}}$

a) 

A
B

b) $1 \in A$; (V) $\{0\} \subset B$; (V) $\{0\} \in \mathcal{P}(B)$; (V)

$A \cap B = B$; (V) (poiché $B \subset A$)

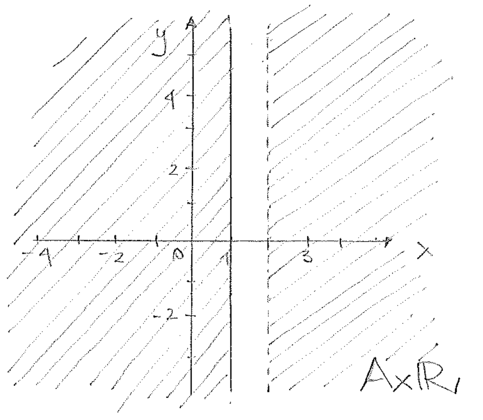
$\mathbb{R} \setminus B \subset A$; (F) (poiché $]1,2] \subset \mathbb{R} \setminus B$, ma $]1,2] \not\subset A$)

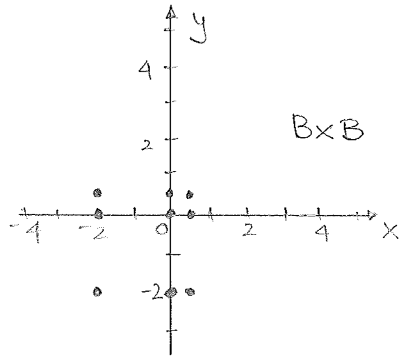
$B \setminus A = \emptyset$; (V) (poiché $B \subset A$)

$(0,0) \in A \times B$; (V) $[0,0] \in B \times A$ (F) (scrittura scorretta!)

$\{\{0\}, \{-2,0\}\} \subset \mathcal{P}(B)$ (V) (poiché $\{0\} \in \mathcal{P}(B)$, $\{-2,0\} \in \mathcal{P}(B)$)

c)



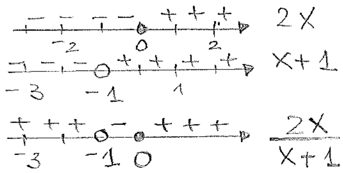


3) i) $A = \left\{ x \in \mathbb{R} : \dfrac{x^2+3x}{x+1} > x \right\}$

$= \,]-\infty, -1[\, \cup \,]0, +\infty[$

$\dfrac{x^2+3x}{x+1} - x > 0 \iff \dfrac{\cancel{x^2}+3x-\cancel{x^2}-x}{x+1} > 0$

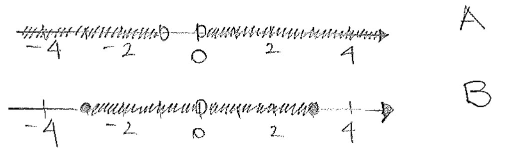
$\iff \dfrac{2x}{x+1} > 0$



$B = \left\{ x \in \mathbb{R} : \dfrac{x^2-9}{x^2} \le 0 \right\} = [-3,3] \setminus \{0\}$

Nota: $x^2 \geq 0 \;\forall x \in \mathbb{R}$ e $x^2 = 0 \iff x = 0$.

ii)

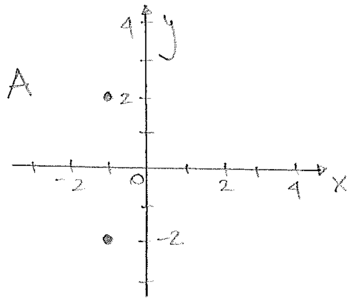


iii) $A \cup B = \mathbb{R} \setminus \{0\}$; $A \setminus B = \,]-\infty, -3[\, \cup \,]3, +\infty[$.

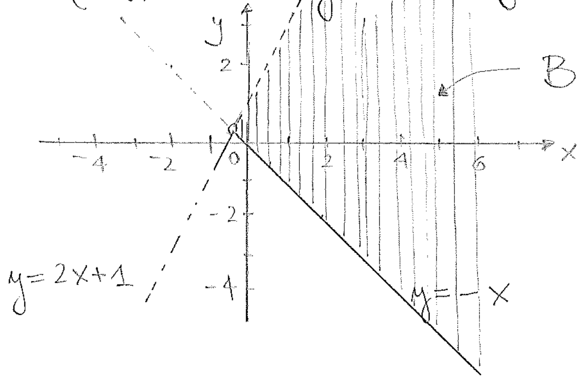
B non è un intervallo (l'intervallo ha un "buco").

4) i) $A = \{(x,y) \in \mathbb{R}^2 : x^2 = 1,\ y^2 = -4x\} = \{(x,y) \in \mathbb{R}^2 : x = \pm 1,\ y^2 = -4x\}$

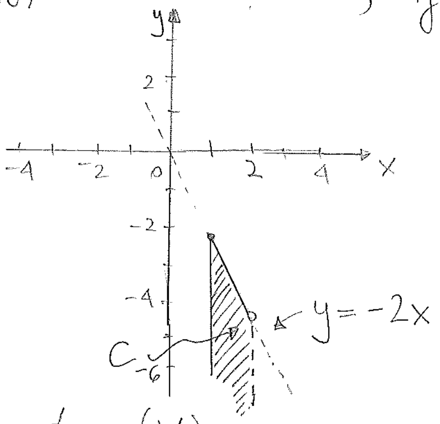
$= \{(x,y) \in \mathbb{R}^2 : x = -1,\ y^2 = 4\} = \{(-1,-2), (-1,2)\}$



$B = \{(x,y) \in \mathbb{R}^2 : -x \le y < 2x+1\}$

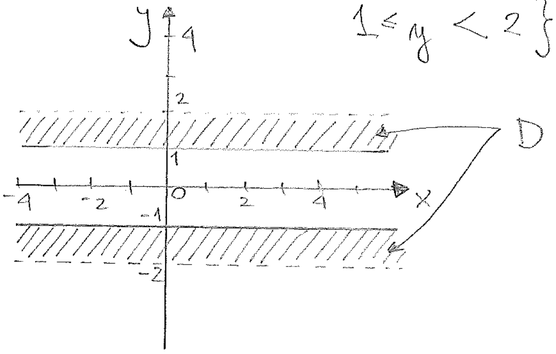


$C = \{(x,y) \in \mathbb{R}^2 : 1 \le x < 2 ,\ y \le -2x\}$



$D = \{(x,y) \in \mathbb{R}^2 : 1 \le y^2 < 4\} =$

$= \{(x,y) \in \mathbb{R}^2 : -2 < y \le -1$ o $1 \le y < 2\}$



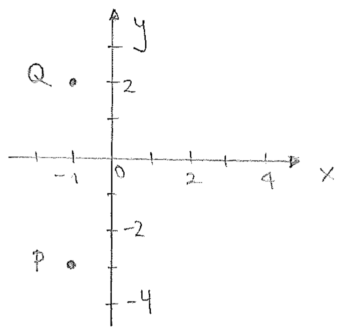
ii) $A \cap B = \emptyset$; (V)

$A \subset \mathbb{R}^2 \setminus B$; (V)

$(1,-2) \in C$; (V)

$\mathbb{R} \times \{-1\} \subset D$; (V).

5) i)



r: $\underline{\underline{x = -1}}$.

ii) r': $y = -3 + 2(x+1) \Rightarrow$

$\underline{\underline{y = 2x - 1}}$. Essa non è ortogonale alla retta r.

iii) $-2x + y + 2 = 0 \iff y = 2x - 2$ eq. della retta r''. Essa è parallela alla retta r', poiché hanno entrambe pendenza $m = 2$.

iv) R = pt. di intersezione di r'' con l'asse delle ascisse

$= (1,0)$

r''' = retta passante per $Q = (-1,2)$ e $R = (1,0)$ ha eq.

$$\frac{y-2}{0-2} = \frac{x+1}{1+1}$$

$\iff y - 2 = -(x+1) \iff \underline{\underline{y = -x + 1}}$

v)