Università di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2015–2016 – Rovereto, 12–16 ottobre 2015 n. 4

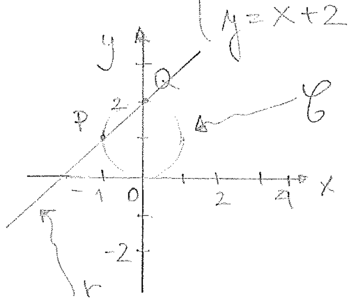
1) i) eq. di $\mathscr{C}$: $(x-0)^2+(y-1)^2=1$

r: $y=x+2$.

Per det. P e Q dobbiamo risolvere il sistema di eq. $\begin{cases} x^2+(y-1)^2=1 \\ y=x+2. \end{cases}$

Si ha $\begin{cases} x^2+(x+2-1)^2=1 \\ y=x+2 \end{cases} \iff \begin{cases} x^2+x^2+2x+1=1 \\ y=x+2 \end{cases}$

$\iff \begin{cases} 2x^2+2x=0 \\ y=x+2. \end{cases}$ Ne segue $\begin{cases} x_{1/2}\in\{-1,0\} \\ y=x+2, \end{cases}$ e si ha $P=(-1,1)$, $Q=(0,2)$



$$\overline{PQ}=\sqrt{(-1-0)^2+(1-2)^2}=\underline{\underline{\sqrt{2}}}.$$

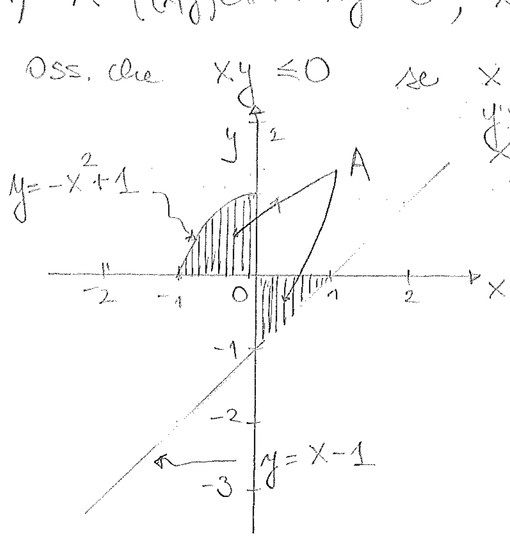
ii) $\begin{cases} x^2+(y-1)^2=k^2 \\ y=x+1+k \end{cases}$ (nell'es. i) si ha $k=1$).

$\iff \begin{cases} x^2+(x+\cancel{1}+k-\cancel{1})^2=k^2 \\ y=x+1+k \end{cases} \iff \begin{cases} 2x^2+2xk=0 \\ y=x+1+k \end{cases} \iff \begin{cases} x_{1/2}\in\{-k,0\} \\ y=x+1+k \end{cases}$

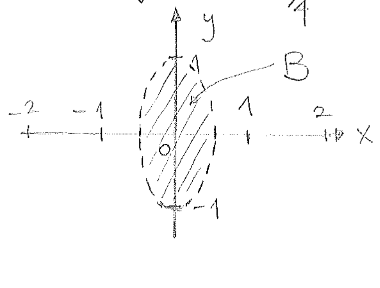
e si ha $P=(-k,1)$, $Q=(0,1+k)$; risolvendo il sistema di eq. dato si trovano i punti di intersezione della circonferenza di centro $(0,1)$ e raggio k, con la retta di eq. $y=x+1+k$ (al variare di $k>0$).

2) i) $A=\{(x,y)\in\mathbb{R}^2 : xy\le 0,\ x-1\le y\le -x^2+1\}$

Oss. che $xy\le 0$ se $x=0$ e $y\in\mathbb{R}$; $y=0$ e $x\in\mathbb{R}$; x ed y hanno segni discordi $\Big\}$: sono le coppie (x,y) che si trovano nel secondo e quarto quadrante



$$B=\{(x,y)\in\mathbb{R}^2 : 4x^2+y^2<1\}$$
$$=\left\{(x,y)\in\mathbb{R}^2 : \frac{x^2}{1/4}+y^2<1\right\}$$

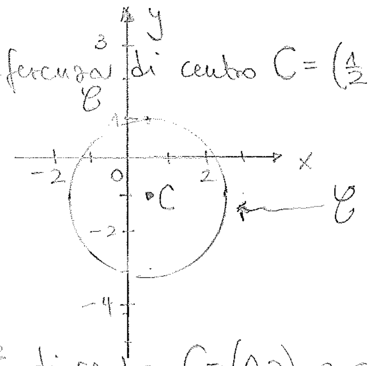


ii) $\begin{cases} y = -x^2+1 \\ y = x+k \end{cases} \iff \begin{cases} x+k = -x^2+1 \\ y = x+k \end{cases} \iff \begin{cases} x^2+x+k-1=0 \\ y = x+k \end{cases}$

Ora $x^2+x+k-1=0$ ha almeno una soluzione se e solo se $1-4(k-1) \geq 0$ $\iff 4k \leq 5 \iff k \leq 5/4$. Possiamo allora asserire che la retta $y=x+k$ interseca l'insieme A in almeno un pt. se e solo se $\underline{-1 \leq k \leq 5/4}$.

iii) $x = k$, con $k \leq -\frac{1}{2}$ o $k \geq \frac{1}{2}$ (possiamo anche scrivere per $k \in \mathbb{R}$, con $|k| \geq \frac{1}{2}$).

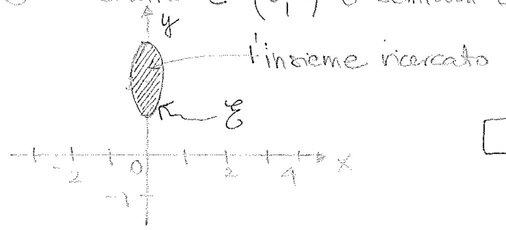
3) i) $(x-\frac{1}{2})^2 + (y+1)^2 = 4$: circonferenza $\mathcal{C}$ di centro $C=(\frac{1}{2},-1)$ e raggio 2.



ii) $9x^2 + (y-2)^2 \leq 1$

$\iff \dfrac{x^2}{1/9} + (y-2)^2 \leq 1$.

Ora $\dfrac{x^2}{1/9} + (y-2)^2 = 1$: ellisse $\mathcal{E}$ di centro $C=(0,2)$ e semiassi $a=\frac{1}{3}$, $b=1$

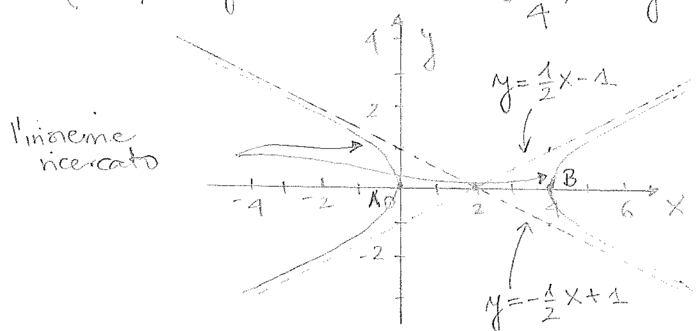


iii) $x^2 - 4y^2 - 4x = 0$

$\iff x^2 - 4x - 4y^2 = 0$

$\iff (x-2)^2 - 4 - 4y^2 = 0$

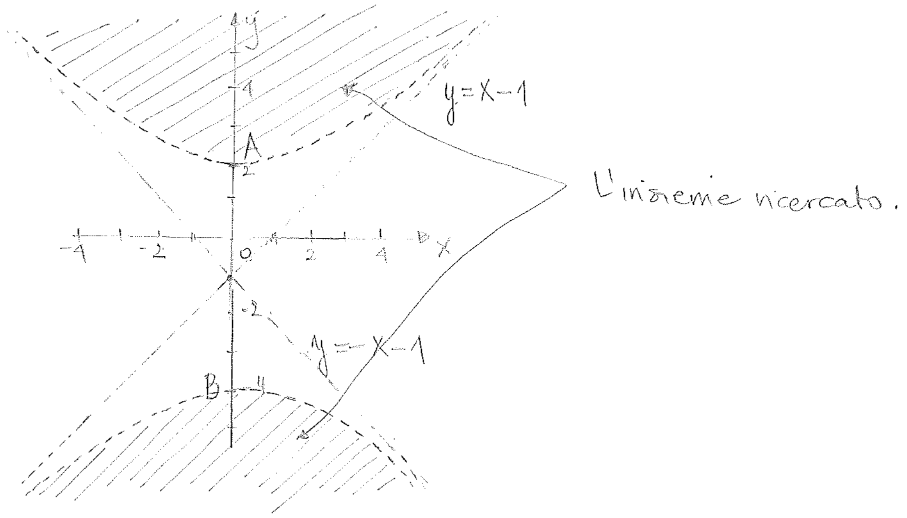
$\iff (x-2)^2 - 4y^2 = 4 \iff \dfrac{(x-2)^2}{4} - y^2 = 1$: iperbole di centro $C=(2,0)$ semiassi $a=2$, $b=1$ e vertici $A=(0,0)$ e $B=(4,0)$



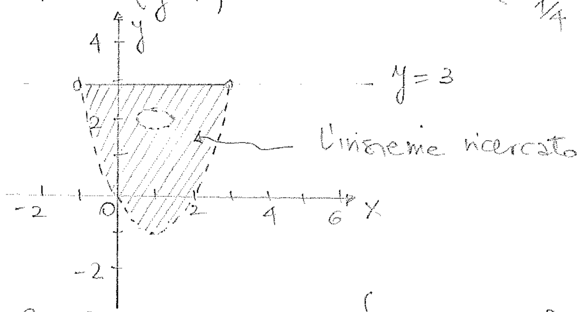
iv) $-x^2 + y^2 + 2y - 8 > 0$

$\iff -x^2 + (y+1)^2 - 1 - 8 > 0 \iff -x^2 + (y+1)^2 > 9 \iff -\dfrac{x^2}{9} + \dfrac{(y+1)^2}{9} > 1$

Ora $-\dfrac{x^2}{9} + \dfrac{(y+1)^2}{9} = 1$: iperbole di centro $C=(0,-1)$, semiassi $a=b=3$ e vertici $A=(0,2)$ e $B=(0,-4)$

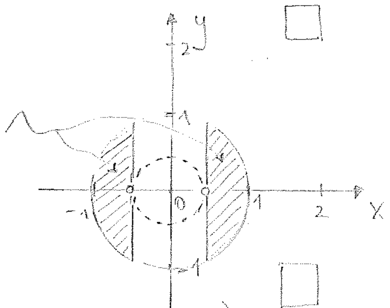


4) a) $\begin{cases} x^2-2x<y\le 3 \\ 4(x-1)^2+9(y-2)^2>1 \end{cases} \iff \begin{cases} (x-1)^2-1<y\le 3 \\ \dfrac{(x-1)^2}{1/4}+\dfrac{(y-2)^2}{1/9}>1 \end{cases}$



$x^2-2x=3 \iff x^2-2x-3=0$
$\iff (x-3)(x+1)=0$

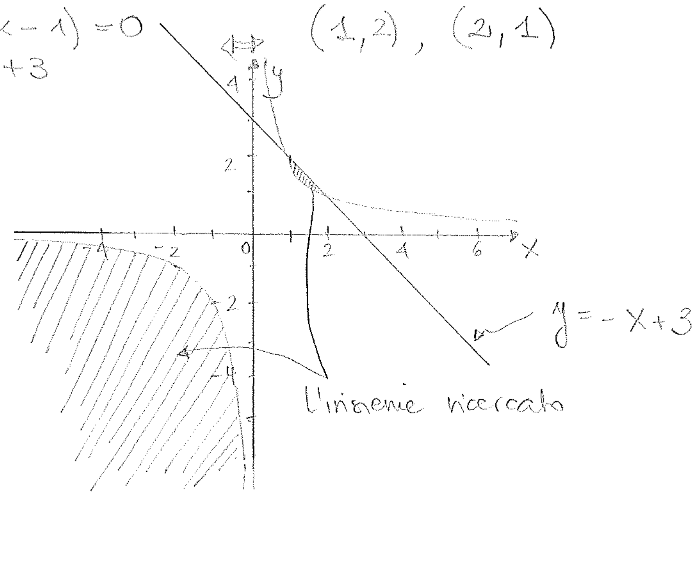
b) $\begin{cases} \frac{1}{4}<x^2+y^2\le 1 \\ x^2\ge \frac{1}{4} \end{cases} \iff \begin{cases} \frac{1}{4}<x^2+y^2\le 1 \\ x\le -\frac{1}{2} \text{ o } x\ge \frac{1}{2} \end{cases}$



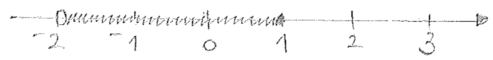
c) $\begin{cases} xy\ge 2 \\ y+x-3\le 0 \end{cases} \iff \begin{cases} \left(x>0 \text{ e } y\ge \frac{2}{x}\right) \text{ o } \left(x<0 \text{ e } y\le \frac{2}{x}\right) \\ y\le -x+3 \end{cases}$

$(xy=2 \text{ e } y=-x+3) \iff \begin{cases} x(-x+3)=2 \\ y=-x+3 \end{cases} \iff \begin{cases} x^2-3x+2=0 \\ y=-x+3 \end{cases}$

$\iff \begin{cases} (x-2)(x-1)=0 \\ y=-x+3 \end{cases} \iff (1,2),\ (2,1)$



5) i) $A =]-\infty, 1] \setminus]-\infty, -2] = \underline{]-2, 1]}$



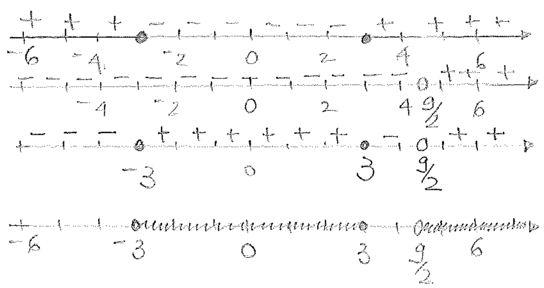
A

$B = \left\{ x \in \mathbb{R} : \dfrac{x^2-2x}{2x-9} \geq -1 \right\}$

$= \underline{\underline{[-3, 3] \cup]9/2, +\infty[}}$

$$\frac{x^2-2x}{2x-9} \geq -1 \iff \frac{x^2-2x}{2x-9} + 1 \geq 0$$

$$\iff \frac{x^2 \cancel{-2x} \cancel{+2x} - 9}{2x-9} \geq 0 \iff \frac{x^2-9}{2x-9} \geq 0$$



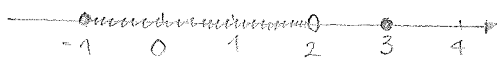
x^2-9

$2x-9$

$\dfrac{x^2-9}{2x-9}$

B

$C = [-1, 2[\cup \{3\}$



C

ii) A è limitato inferiormente e limitato superiormente;

$\underline{\nexists \min A}$; $\underline{\max A = 1}$.

B è limitato inferiormente, ma non è limitato superiormente;

$\underline{\min B = -3}$; $\underline{\nexists \max B}$.

C è limitato inferiormente e limitato superiormente;

$\underline{\min C = -1}$; $\underline{\max C = 3}$.