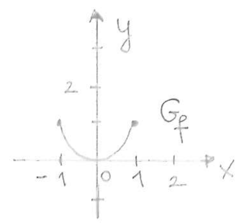
Università di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2015 – 2016 – Rovereto, 19 – 23 ottobre 2015 n. 5

1) i) $\underline{A = \mathbb{R} \setminus \{-1, 1\}}$ ($f(x) = \frac{1}{x^2 - 1}$ è ben definita tranne per gli $x \in \mathbb{R}$, per cui $x^2 - 1 = 0$).

ii) f è definita sull'intervallo $[0, 1]$, mentre g è definita solo sull'insieme costituito dai punti $x = 0$ e $x = 1$.
Quindi, per esempio, ha senso $f(\frac{1}{2})$, e vale $f(\frac{1}{2}) = \frac{1}{4} + 2 = \frac{9}{4}$, mentre non è definita $g(\frac{1}{2})$.

iii) f è ben definita se in $x = 2$ assume un solo valore; quindi deve coincidere $f(2) = -2 + 3 = 1$ e $f(2) = 2 + a$, ossia $1 = 2 + a$ e quindi $\underline{a = -1}$.

iv) Deve essere $f([-1, 1]) = B$, quindi $\underline{B = [0, 1]}$.



2) i) $\operatorname{dom} f = \underline{[-3, +\infty[}$.

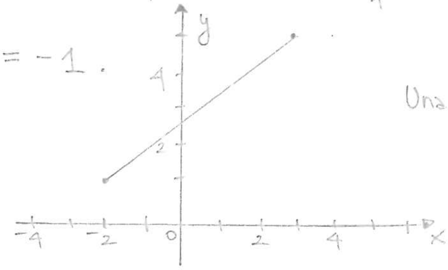
ii) $f(-3) = \underline{2}$; $f(-2) = \underline{0}$; $f(2) = \underline{-1}$.

iii) $(-2, 2) \notin \operatorname{graf} f$, poiché $f(-2) = 0 \neq 2$.
$(-2, 0) \in \operatorname{graf} f$ sì! poiché $-2 \in \operatorname{dom} f$ e $f(-2) = 0$.

iv) $f([-3, +\infty[) = \underline{[-1, 2]}$. È un sottoinsieme limitato di $\mathbb{R}$ ed è un intervallo.

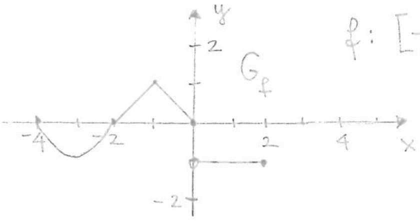
v) Possiamo prendere $x_1 = -2$, $x_2 = 0$, $x_0 = -1$.

3)



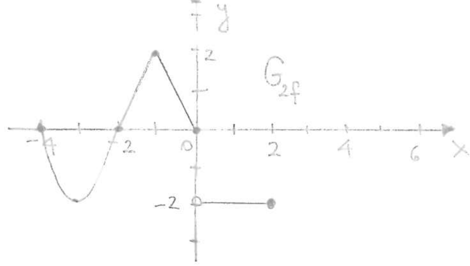
Una possibile $f : [-2, 3] \to [1, 5]$

$$\underline{f(x) = \frac{4}{5} x + \frac{13}{5}}$$

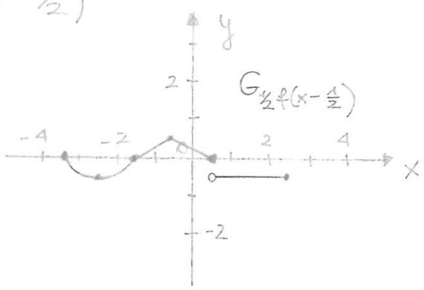
4) 

$f: [-4,2] \longrightarrow \mathbb{R}$

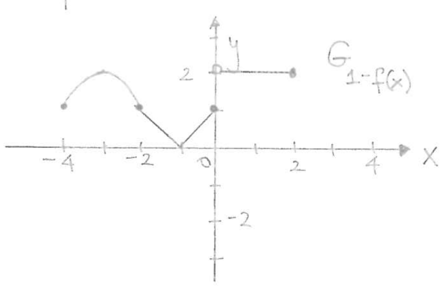
$x \longmapsto 2f(x)$
$\cap\!\!\!\!|$
$[-4,2]$



$x \longmapsto \frac{1}{2} f\left(x-\frac{1}{2}\right)$
$\cap\!\!\!\!|$
$\left[-4+\frac{1}{2}, 2+\frac{1}{2}\right]$
$\|$
$\left[-\frac{7}{2}, \frac{5}{2}\right]$

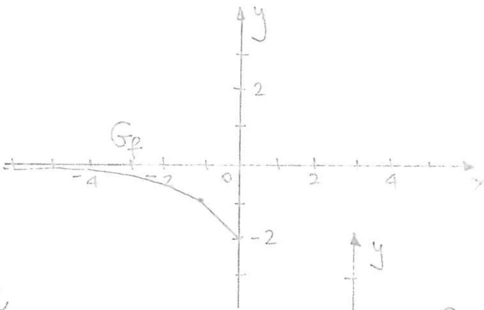


$x \longmapsto 1-f(x)$
$\cap\!\!\!\!|$
$[-4,2]$

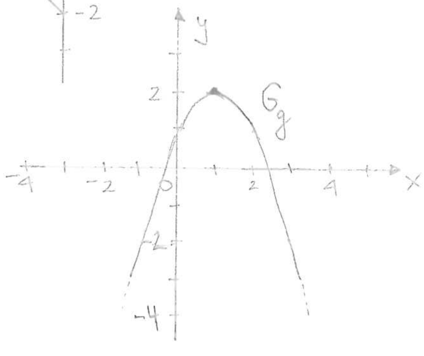


5) i) $f:]-\infty, 0] \to [-2,0[$, $g: \mathbb{R} \to \mathbb{R}$, $h: [-1,2] \to [-1,4]$

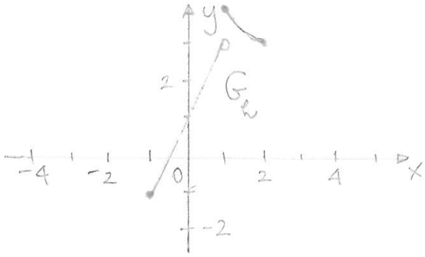
$$f(x) = \begin{cases} \frac{1}{x} & \text{se } x \leq -1 \\ -x-2 & \text{se } -1 < x \leq 0 \end{cases}$$



$$g(x) = -x^2+2x+1$$
$$= -(x^2-2x)+1 = -(x-1)^2+2$$



$$h(x) = \begin{cases} 2x+1 & \text{se } -1 \leq x < 1 \\ \frac{2}{x}+2 & \text{se } 1 \leq x \leq 2 \end{cases}$$

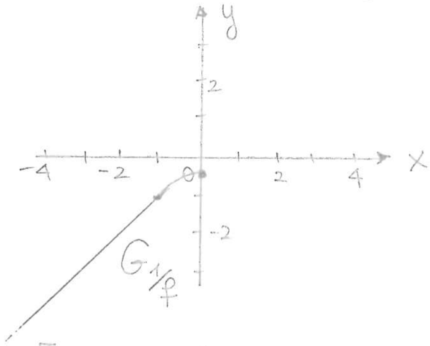


ii) $f(\{-1,0\}) = \underline{\{-2,-1\}}$; $f(-1) = \underline{-1}$.

iii) $\{x \in \text{dom}\, g : g(x) > 0\} = \{x \in \mathbb{R} : -x^2+2x+1 > 0\} = \underline{\left]1-\sqrt{2}, 1+\sqrt{2}\right[}$.

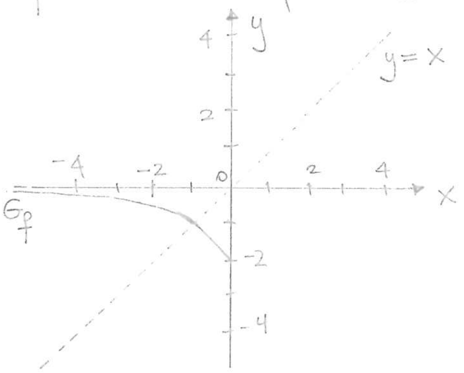
iv) $\dfrac{1}{f(x)}$ esiste $\forall x \in \operatorname{dom} f$: $f(x) \neq 0$. Poiché $f(x) \neq 0$ $\forall x \in]-\infty, 0]$, possiamo asserire che $\operatorname{dom} \dfrac{1}{f} =]-\infty, 0]$. Inoltre risulta

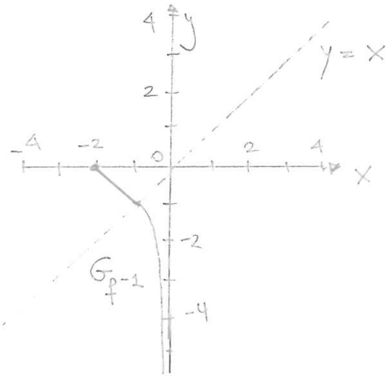
$$\frac{1}{f(x)} = \begin{cases} x & \text{se } x \leq -1 \\ \dfrac{1}{-x-2} & \text{se } -1 < x \leq 0 \end{cases} \iff \begin{cases} x & \text{se } x \leq -1 \\ -\dfrac{1}{x+2} & \text{se } -1 < x \leq 0 \end{cases}$$



□

v) $f:]-\infty, 0] \to [-2, 0[$ è una funzione iniettiva e suriettiva, quindi biiettiva. Dunque esiste la funzione inversa $f^{-1}: [-2, 0[\to]-\infty, 0]$





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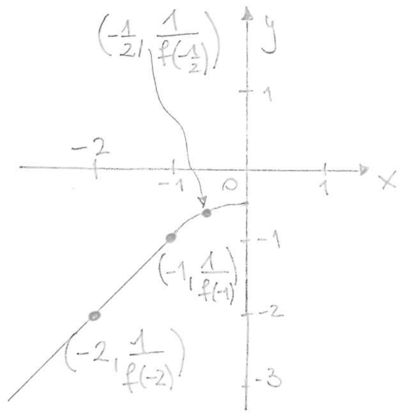
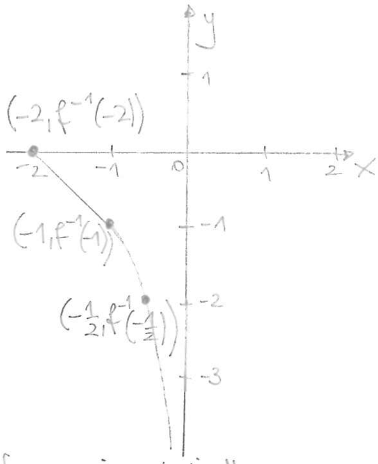
vi)





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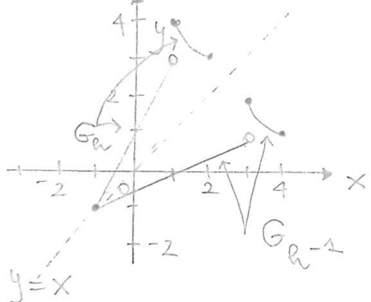
vii) $h: [-1, 2] \to [-1, 4]$ è una funzione biiettiva; quindi esiste la funzione inversa $h^{-1}: [-1, 4] \to [-1, 2]$



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