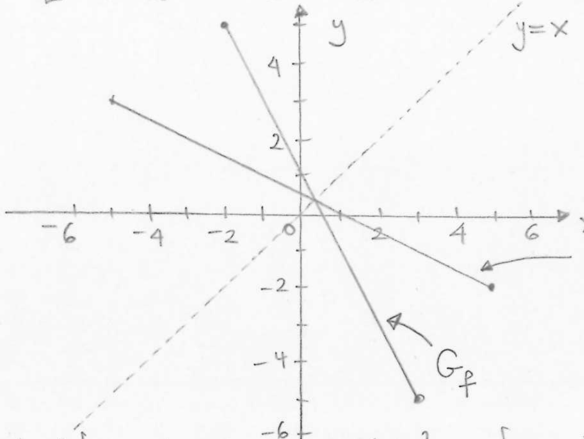
Università di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2015 - 2016 — Rovereto, 26 - 30 ottobre 2015 n. 6

1) $f : [-2,3] \to [-5,5] \quad f(x) = -2x+1$.

i) Proviamo che f è iniettiva: siano $x_1, x_2 \in [-2,3]$, $x_1 \neq x_2$. Allora $-2x_1+1 \neq -2x_2+1$, ossia $f(x_1) \neq f(x_2)$.

Proviamo che f è suriettiva: sia $y \in [-5,5]$. Dobbiamo trovare $x \in [-2,3] : f(x) = y$. Ora $-2x+1 = y \iff -2x = y-1 \iff x = \frac{y-1}{-2}$. Infine, se $y \in [-5,5]$, allora $y-1 \in [-6,4]$ e $\frac{y-1}{-2} \in [-2,3]$. Quindi, in conclusione, preso $y \in [-5,5]$ qualsiasi, esiste $x \in [-2,3]$ $\left(x = \frac{y-1}{-2}\right)$ tale che $f(x) = y$. $\square$

ii) $f^{-1} : [-5,5] \to [-2,3]$ è data da $f^{-1}(x) = \frac{x-1}{-2} = -\frac{1}{2}x + \frac{1}{2}$.



$\square$

iii) $\operatorname{dom} \frac{1}{f} = \{x \in \operatorname{dom} f : f(x) \neq 0\} = \{x \in [-2,3] : -2x+1 \neq 0\} = \underline{\underline{[-2,3] \setminus \{\frac{1}{2}\}}}$.

2) i) $A = \{x \in \operatorname{dom} f : f(x) > 0\} = \underline{\underline{]1,3[\cup]5,+\infty[\setminus \{6\}}}$.

A <u>non</u> è un insieme <u>limitato</u>. A non ha minimo e non ha massimo. $\square$

ii) $\max\limits_{[1,+\infty[} f = \underline{\underline{1}}$ pt. di massimo $\underline{\underline{x \in \{2\} \cup [7,+\infty[}}$.

$\min\limits_{[1,+\infty[} f = \underline{\underline{-1}}$ pt. di minimo $\underline{\underline{x = 4}}$. $\square$

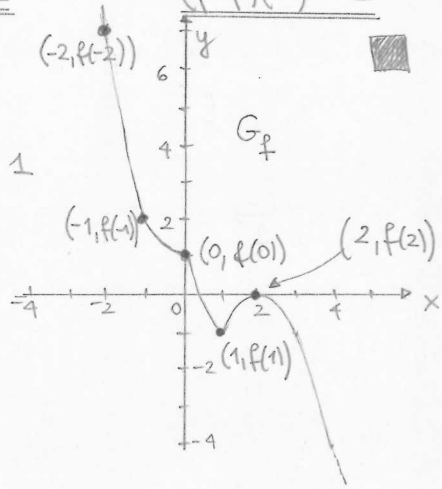
iii) $(f \circ f)(1) \nexists$ poichè $f(1) = 0$, ma $f(0)$ non è definita.

$(f \circ f)(2)$ esiste, poichè $f(2) = 1$ e $f(1) = 0$. Quindi $(f \circ f)(2) = f(f(2)) = f(1) = \underline{\underline{0}}$. $\square$

iv) $(f\circ f)(x)$ è ben definito per $x\in[1,+\infty[\ :\ f(x)\in[1,+\infty[$.

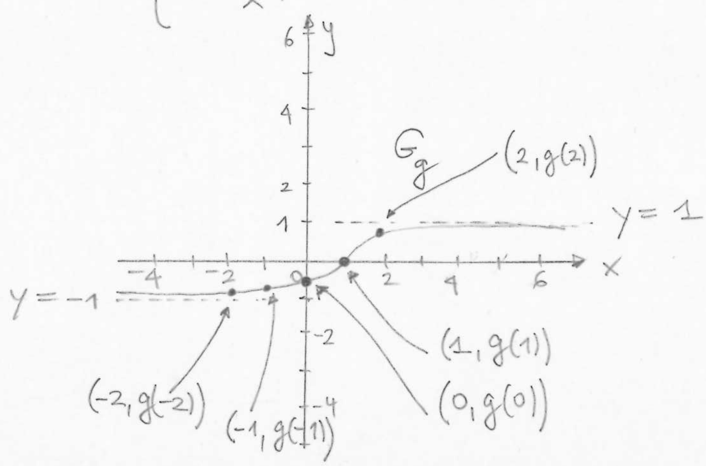
Ora $f(x)\in[1,+\infty[$ con $x\in[1,+\infty[$ se e solo se $x=2$ oppure $x\in[7,+\infty[$.

Quindi possiamo porre $\underline{B=\{2\}\cup[7,+\infty[}$. Inoltre $\underline{(f\circ f)(x)=1}$ $\underline{\text{per ogni } x\in B}$.



3) i) $f:\mathbb{R}\to\mathbb{R}$ $\quad f(x)=\begin{cases}-x^3+1 & \text{se } x<0\\ -2\sqrt{x}+1 & \text{se } 0\le x<1\\ -(x-2)^2 & \text{se } x\ge 1\end{cases}$

$g:\mathbb{R}\to\mathbb{R}$ $\quad g(x)=\begin{cases}-\dfrac{1}{x-2}-1 & \text{se } x<1\\ 1-\dfrac{1}{x^4} & \text{se } x\ge 1\end{cases}$



ii) $f(\mathbb{R})=\underline{\underline{\mathbb{R}}}$; $\quad g(\mathbb{R})=\underline{\underline{]-1,1[}}$.

<u>f non è una funzione limitata, mentre g lo è.</u> Nessuna delle due ammette massimo o minimo.

iii) Se $k<-1$, $f(x)=k$ ha una sola soluzione;

$k=-1$, " ha 2 soluzioni;

$-1<k<0$, " ha 3 soluzioni;

$k=0$, " ha 2 soluzioni;

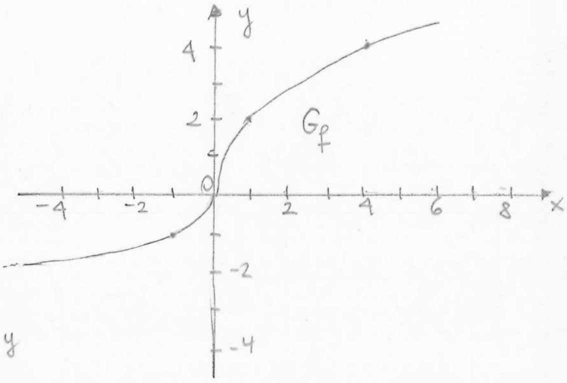
$0<k$, " ha una sola soluzione.

iv) $(f+g)(0)=f(0)+g(0)=1-\frac{1}{2}=\underline{\underline{\tfrac{1}{2}}}$; $\quad (fg)(1)=f(1)\,g(1)=(-1)0=\underline{\underline{0}}$;

$\left(\dfrac{f}{g}\right)(1)=\dfrac{f(1)}{g(1)}=\cancel{\dfrac{-1}{0}}\ \nexists$; $\quad \left(\dfrac{g}{f}\right)(1)=\dfrac{g(1)}{f(1)}=\dfrac{0}{-1}=\underline{\underline{0}}$;

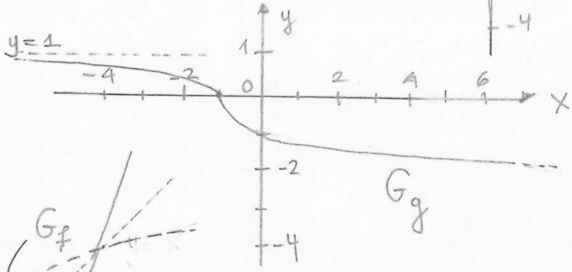
$(f\circ g)(1)=f(g(1))=f(0)=\underline{\underline{1}}$; $\quad (g\circ f)(-1)=g(f(-1))=g(2)=1-\dfrac{1}{16}=\underline{\underline{\dfrac{15}{16}}}$.

4) i) $f: \mathbb{R} \to \mathbb{R}$ $\quad f(x) = \begin{cases} \sqrt[3]{x} & \text{se } x<0 \\ 2\sqrt{x} & \text{se } x \geq 0 \end{cases}$



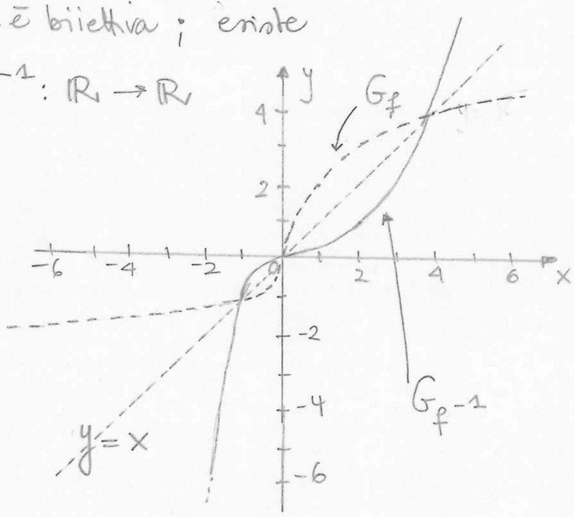
$g: \mathbb{R} \to]-\infty, 1[$

$g(x) = \begin{cases} \frac{1}{x}+1 & \text{se } x<-1 \\ -\sqrt[4]{x+1} & \text{se } x \geq -1 \end{cases}$



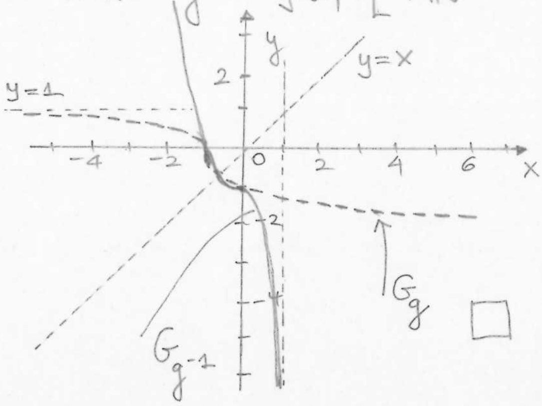
ii) f è biiettiva; esiste

$f^{-1}: \mathbb{R} \to \mathbb{R}$

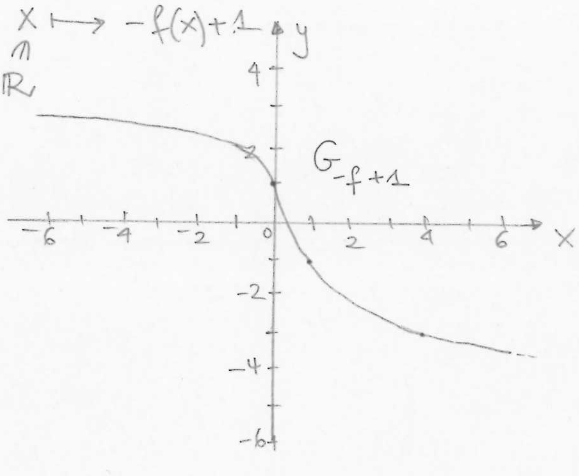


$g: \mathbb{R} \to]-\infty, 1[$ è biiettiva;

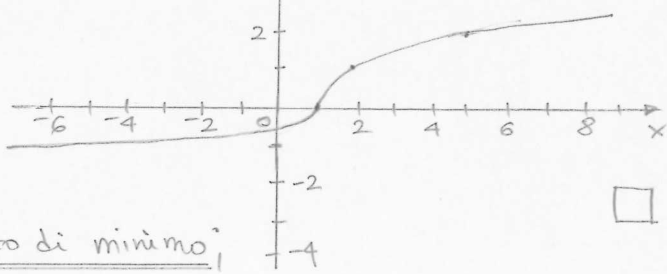
esiste $g^{-1}:]-\infty, 1[\to \mathbb{R}$



iii) $\underset{\mathbb{R}}{x} \longmapsto -f(x)+1$



$\underset{\mathbb{R}}{x} \longmapsto \frac{1}{2} f(x-1)$



iv) $\min\limits_{[-1,15]} g = g(15) = \underline{\underline{-2}}$

$\underline{\underline{x=15 \text{ punto di minimo}}}$;

$\max\limits_{[-1,15]} g = g(-1) = \underline{\underline{0}}$

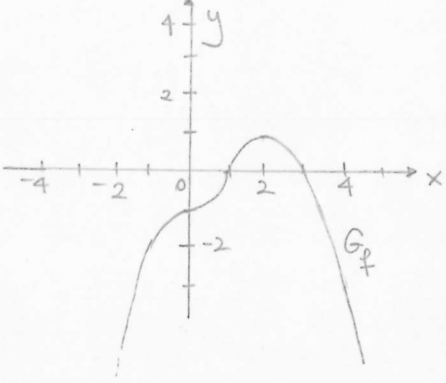
$\underline{\underline{x=-1 \text{ punto di massimo}}}$.

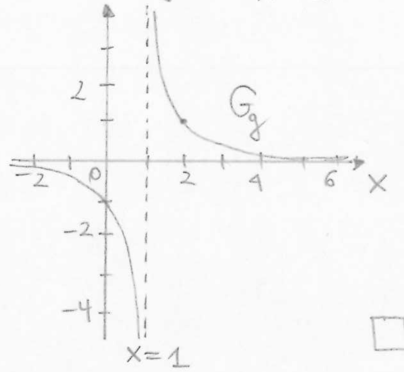
v) $\min\limits_{[-2,0]} g^{-1} = \underline{\underline{-1}}$ $\quad \underline{\underline{x=0 \text{ punto di minimo}}}$;

$\max\limits_{[-2,0]} g^{-1} = \underline{\underline{15}}$ $\quad \underline{\underline{x=-2 \text{ punto di massimo}}}$.

5) i) $f:\mathbb{R}\to\mathbb{R}$ $\quad f(x)=\begin{cases} x^3-1 & \text{se } x<1 \\ -(x-2)^2+1 & \text{se } x\geq 1\end{cases}$ $\qquad g:\mathbb{R}\setminus\{1\}\to\mathbb{R}$

$g(x)=\dfrac{1}{x-1}$





ii) $g\circ f$ è definita $\forall x\in\mathbb{R}:\ f(x)\neq 1$

quindi $\underline{\underline{\mathrm{dom}(g\circ f)=\mathbb{R}\setminus\{2\}}}$

$f\circ g$ è definita $\forall x\in \underline{\underline{\mathbb{R}\setminus\{1\}}}$.

$$(g\circ f)(x)=\begin{cases}\dfrac{1}{(x^3-1)-1} & \text{se } x<1\\[2ex] \dfrac{1}{[-(x-2)^2+1]-1} & \text{se } x\geq 1,\ x\neq 2\end{cases} = \begin{cases}\dfrac{1}{x^3-2} & \text{se } x<1\\[2ex] \dfrac{1}{-(x-2)^3} & \text{se } x\geq 1,\ x\neq 2.\end{cases}$$

$$(f\circ g)(x)=\begin{cases}\left(\dfrac{1}{x-1}\right)^3-1 & \text{se } x<1 \text{ o } x>2\\[2ex] -\left(\dfrac{1}{x-1}-2\right)^2+1 & \text{se } 1<x\leq 2.\end{cases}$$