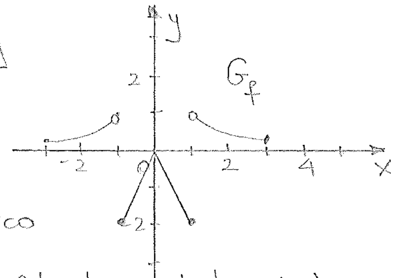
Università di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2015–2016 – Rovereto, 2–6 novembre 2015 n. 7

1) i) $f: [-3,3] \to \mathbb{R}$, $f(x) = \begin{cases} -2|x| & \text{se } x \in [-1,1] \\ \frac{1}{|x|} & \text{altrimenti} \end{cases}$



ii) Oss. innanzitutto che $[-3,3]$ è un insieme simmetrico (risp. all'origine). Inoltre $\forall x \in [-1,1]$ $f(-x) = -2|-x| = -2|x| = f(x)$

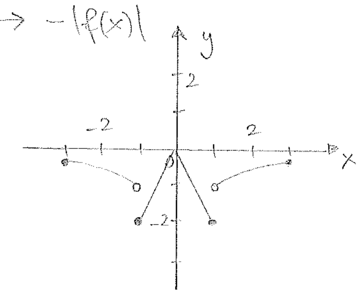
$\forall x \in [-3,3] \setminus [-1,1]$ $f(-x) = \frac{1}{|-x|} = \frac{1}{|x|} = f(x)$ □

iii) f ristretta a $[-3,-1[$ è crescente;

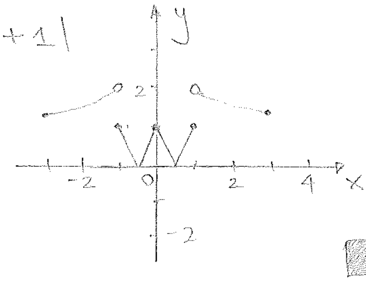
f ristretta a $[-1,0]$ è crescente;

f ristretta a $[0,1]$ è decrescente e f ristretta a $[1,3]$ è decrescente. □

iv) $x \longmapsto -|f(x)|$
$\in [-3,3]$

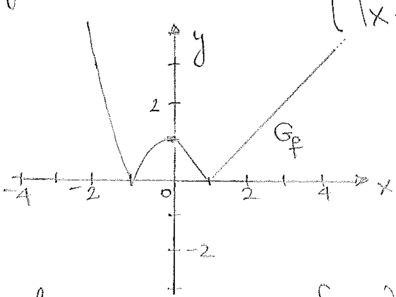


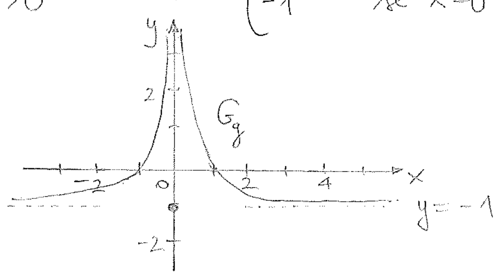
$x \longmapsto |f(x)+1|$
$\in [-3,3]$



■

2) i) $f, g: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} |-x^2+1| & \text{se } x \le 0 \\ |x-1| & \text{se } x > 0 \end{cases}$ $g(x) = \begin{cases} \frac{1}{x^2} - 1 & \text{se } x \ne 0 \\ -1 & \text{se } x = 0 \end{cases}$.





□

ii) $\max_{[-1,2]} f = \underline{1}$; $\underline{x \in \{0,2\}}$ pt. di massimo;

$\min_{[-1,2]} f = \underline{0}$; $\underline{x \in \{-1,1\}}$ pt. di minimo. □

iii) $(f \circ g)(0) = f(g(0)) = f(-1) = \underline{0}$;

$(g \circ f)(0) = g(f(0)) = g(1) = \underline{0}$. ■

3) • $|2x-1| < 2 \iff -2 < 2x-1 < 2 \iff -1 < 2x < 3 \iff$
$-\frac{1}{2} < x < \frac{3}{2}$ $\quad S = \left]-\frac{1}{2}, \frac{3}{2}\right[$.

- $|x^2-1|>0 \iff x^2-1\neq 0 \iff x^2\neq 1 \Rightarrow \underline{S=\mathbb{R}\setminus\{-1,1\}}$;

- $|-3x|-|x|=4 \iff 3|x|-|x|=4 \iff 2|x|=4 \iff |x|=2 \Rightarrow \underline{S=\{-2,2\}}$;

- $|x^3(1-x^2+x^3)|>-1 \qquad \underline{S=\mathbb{R}}$;

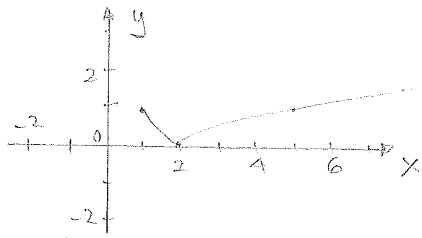
- $|x^2+2x|\leq 0 \iff |x^2+2x|=0 \iff x^2+2x=0 \Rightarrow \underline{S=\{-2,0\}}$;

- $||x|+1|\geq 3 \iff \cancel{|x|+1\leq -3}$ o $|x|+1\geq 3 \iff |x|\geq 2$

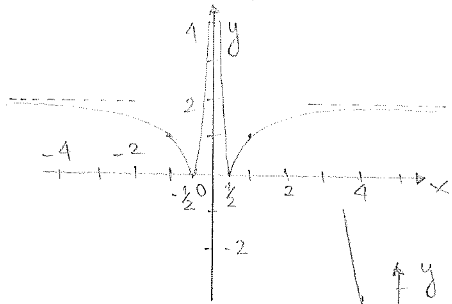
  impossibile

  $\iff x\leq -2$ o $x\geq 2 \qquad \underline{S=\,]-\infty,-2]\cup[2,+\infty[}$.

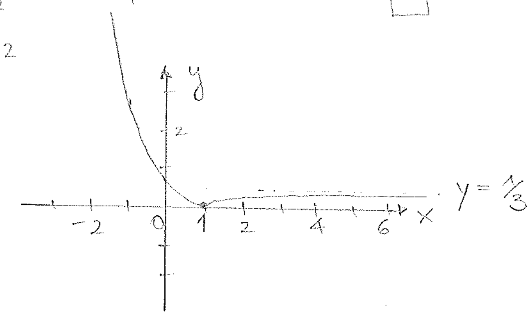
4) $\underset{[1,+\infty[}{x} \longmapsto |\sqrt{x-1}-1|$



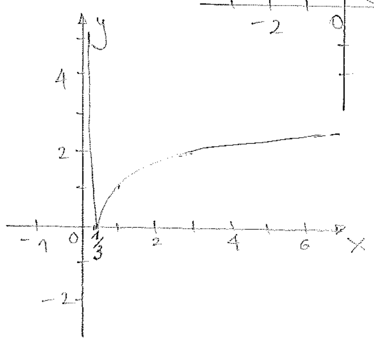
$\underset{\mathbb{R}\setminus\{0\}}{x} \longmapsto \left|2-\frac{1}{|x|}\right|$



$\underset{\mathbb{R}}{x} \longmapsto |3^{-x}-\frac{1}{3}|$



$\underset{]0,+\infty[}{x} \longmapsto |\log_{1/3} x-1|$



5) i) • $x^2+2x|x|-9\geq 0 \iff \begin{cases} x\geq 0\\ x^2+2x^2-9\geq 0\end{cases}$ o $\begin{cases} x<0\\ x^2-2x^2-9\geq 0\end{cases}$

$\iff \begin{cases} x\geq 0\\ x^2\geq 3\end{cases}$ o $\cancel{\begin{cases} x<0\\ -x^2\geq 9\end{cases}} \iff \begin{cases} x\geq 0\\ x\leq -\sqrt{3} \text{ o } x\geq\sqrt{3}\end{cases} \qquad \underline{S=[\sqrt{3},+\infty[}$.

- $|x-1|+x^2=0$ impossibile, poiché $|x-1|\geq 0$ e $=0 \iff x=1$
  poiché $x^2\geq 0$ e $=0 \iff x=0$;

  $\underline{S=\emptyset}$.

- $x^2+|x-2| \le 2x \iff \begin{cases} x \ge 2 \\ x^2+x-2-2x \le 0 \end{cases} \quad \text{o} \quad \begin{cases} x<2 \\ x^2-x+2-2x \le 0 \end{cases}$

$$\iff \begin{cases} x \ge 2 \\ x^2-x-2 \le 0 \end{cases} \quad \text{o} \quad \begin{cases} x<2 \\ x^2-3x+2 \le 0 \end{cases}$$

$$\iff \begin{cases} x \ge 2 \\ (x-2)(x+1) \le 0 \end{cases} \quad \text{o} \quad \begin{cases} x<2 \\ (x-2)(x-1) \le 0 \end{cases}$$

$$\iff \begin{cases} x \ge 2 \\ -1 \le x \le 2 \end{cases} \quad \text{o} \quad \begin{cases} x<2 \\ 1 \le x \le 2 \end{cases} \quad \Rightarrow \quad \underline{\underline{S=[1,2]}}.$$

$\square$

ii) - $2^{2x} \ge 32 \iff 2^{2x} \ge 2^5 \underset{2^x\uparrow}{\iff} 2x \ge 5 \iff x \ge \frac{5}{2} \qquad \underline{\underline{S=\left[\frac{5}{2},+\infty\right[}}.$

- $3^{x|x-1|} \le 1 \iff 3^{x|x-1|} \le 3^0 \underset{3^x\uparrow}{\iff} x|x-1| \le 0 \iff x \le 0 \ \text{o} \ x=1$

$$\underline{\underline{S=\left]-\infty,0\right] \cup \{1\}}}.$$

- $\left(\frac{1}{2}\right)^{x^2+|x|} > \frac{1}{4} \iff \left(\frac{1}{2}\right)^{x^2+|x|} > \left(\frac{1}{2}\right)^2 \underset{\left(\frac{1}{2}\right)^x\downarrow}{\iff} x^2+|x| < 2$

$$\iff \begin{cases} x \ge 0 \\ x^2+x-2 < 0 \end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ x^2-x-2 < 0 \end{cases}$$

$$\iff \begin{cases} x \ge 0 \\ (x+2)(x-1) < 0 \end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ (x-2)(x+1) < 0 \end{cases}$$

$$\iff \begin{cases} x \ge 0 \\ -2<x<1 \end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ -1<x<2 \end{cases} \quad \Rightarrow \quad \underline{\underline{S=\left]-1,1\right[}}.$$

$\square$

iii) - $\log_2(4x+1) \le 0 \iff \log_2(4x+1) \le \log_2 1$

$$\iff \begin{cases} 4x+1>0 & \text{(altrimenti non è definito il membro a sinistra della diseq)} \\ 4x+\cancel{1} \le \cancel{1} & \text{(perché } \log_2 x \text{ è strett. crescente)} \end{cases}$$

$$\iff \begin{cases} x > -\frac{1}{4} \\ x \le 0 \end{cases} \quad \Rightarrow \quad \underline{\underline{S=\left]-\tfrac{1}{4},0\right]}}.$$

- $\log(x^2-1) < 2 \iff \log(x^2-1) < \log e^2$

$$\iff \begin{cases} x^2-1>0 \\ x^2-1<e^2 \end{cases} \quad \text{(come sopra!)}$$

$$\iff \begin{cases} x^2>1 \\ x^2<e^2+1 \end{cases} \iff \begin{cases} x<-1 \ \text{o} \ x>1 \\ -\sqrt{e^2+1}<x<\sqrt{e^2+1} \end{cases}$$

$$\underline{\underline{S=\left]-\sqrt{e^2+1},-1\right[\cup \left]1,\sqrt{e^2+1}\right[}}.$$

• $x^2 \log_{1/3} 3 + |x| \log_{1/4} 1 = \log_4 1/16 \iff -x^2 = -2$,

poiché $\log_{1/3} 3 = -1$ poiché $\left(\frac{1}{3}\right)^{-1} = 3$;

$\log_{1/4} 1 = 0$ poiché $\left(\frac{1}{4}\right)^{0} = 1$;

$\log_4 \frac{1}{16} = -2$ poiché $4^{-2} = \frac{1}{16}$.

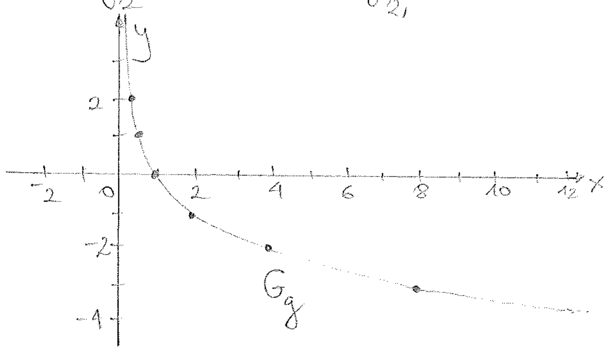
Dunque $x^2 = 2$, ossia $\underline{\underline{S = \{-\sqrt{2}, \sqrt{2}\}}}$.

6) i) $g :]0,+\infty[\to \mathbb{R}$, $g(x) = \log_{1/2} x$

$\log_{1/2} \frac{1}{4} = 2$ poiché $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\log_{1/2} \frac{1}{2} = 1$ poiché $\left(\frac{1}{2}\right)^1 = \frac{1}{2}$, e così via

$\log_{1/2} 1 = 0$, $\log_{1/2} 2 = -1$, $\log_{1/2} 4 = -2$, $\log_{1/2} 8 = -3$



ii) $x \longmapsto \log_{1/2}(x+1)$

$]-1,+\infty[$

$x \longmapsto -|\log_{1/2}|x||$

$\mathbb{R}\setminus\{0\}$