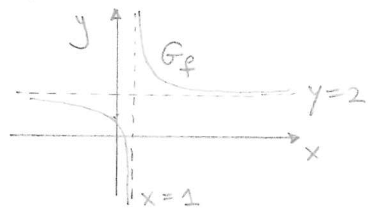
Università di Trento – Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
VERIFICA SETTIMANALE DI ANALISI MATEMATICA
a.a. 2015 – 2016 — Rovereto, 16 – 20 novembre 2015 n. 9

1) i) $\lim\limits_{x \to 1^+} f(x) = +\infty \quad : \forall M > 0, \exists \delta > 0 : \left(\forall x \in \,]1, 1+\delta[\;\Rightarrow f(x) > M \right);$

$\lim\limits_{x \to +\infty} f(x) = 2 \quad : \forall \varepsilon > 0, \exists K > 0 : \left(\forall x > K \Rightarrow 2 - \varepsilon < f(x) < 2 + \varepsilon \right).$

ii) Esempio: $f(x) = \dfrac{1}{x-1} + 2$



2) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} -1/x^2 & \text{se } x < -1 \\ 2 & \text{se } x = -1 \\ e^{-x} - e & \text{se } x > -1 \end{cases}$.

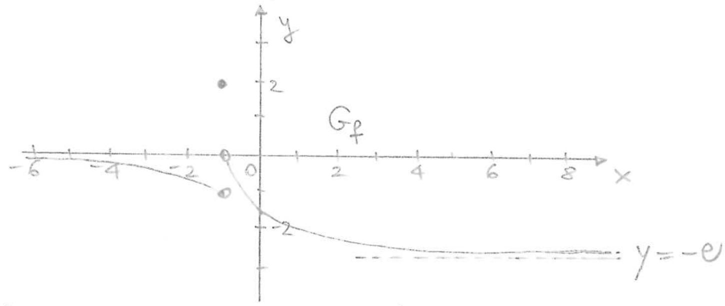
i) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \left(-\dfrac{1}{x^2} \right) = \underline{\underline{0}}$;

$\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^-} \left(-\dfrac{1}{x^2} \right) = \underline{\underline{-1}}$;

$\lim\limits_{x \to -1^+} f(x) = \lim\limits_{x \to -1^+} \left(e^{-x} - e \right) = e^1 - e = \underline{\underline{0}}$;

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \left(e^{-x} - e \right) = \underline{\underline{-e}}$.

ii)



iii) $\lim\limits_{x \to -1^-} f(x) = -1 \neq \lim\limits_{x \to -1^+} f(x) = 0 \neq f(-1) = 2$.

f non è continua in $x = -1$ (altrimenti dovrebbe essere $\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^+} f(x) = f(-1)$).

iv) f non ha asintoti verticali. Ha asintoti orizzontali però:

$\underline{y = 0 \text{ per } x \to -\infty}$, e $\underline{y = -e \text{ per } x \to +\infty}$.

3) a) $\lim\limits_{x \to 1^-} (x^4 - 2x) = 1^4 - 2 \cdot 1 = \underline{\underline{-1}}$;

$\lim\limits_{x \to 4^+} \left(-\sqrt{x} + \log_{1/2} x \right) = -\sqrt{4} + \log_{1/2} 4 = -2 - 2 = \underline{\underline{-4}}$;

$$\lim_{x\to 2^-} \frac{2x - 1/x}{2^x} = \frac{2\cdot 2 - 1/2}{2^2} = \frac{7/2}{4} = \frac{7}{8} ;$$

$$\lim_{x\to 0^-} \frac{2^x - 1}{x^3+2} = \frac{2^0 - 1}{0+2} = \frac{0}{2} = 0 .$$

□

b) $$\lim_{x\to 1^-} \frac{x \to 1}{x^2-1} = -\infty ;$$ infinitesimo negativo

$$\lim_{x\to 1^+} \frac{x \to 1}{x^2-1} = +\infty ;$$ infinitesimo positivo

$$\lim_{x\to 2^-} \frac{x^2 \to 4}{|x|-2} = -\infty ;$$ infinitesimo negativo

$$\lim_{x\to 2^+} \frac{x^2 \to 4}{|x|-2} = +\infty ;$$ infinitesimo positivo

$$\lim_{x\to 0^-} \frac{1}{x^3+x} = \lim_{x\to 0^-} \frac{1}{x(\underbrace{x^2+1}_{\to 1})} = -\infty .$$

□

c) $$\lim_{x\to 0^+} \frac{1}{x^3+x} = \lim_{x\to 0^+} \frac{1}{x(\underbrace{x^2+1}_{\to 1})} = +\infty ;$$

$$\lim_{x\to -\infty} \frac{e^x - 1}{e^{-x}} = 0 ;$$ ($e^x \to 0$, $e^{-x} \to +\infty$)

$k=1$ $$\lim_{x\to +\infty} \frac{5x - 2^{-x}}{1-x^3} = 0 \qquad \frac{\cancel{x}(5 - 2^{-x}/x)}{x^{\cancel{3}2}(1/x^3 - 1)} \xrightarrow[x\to+\infty]{} 0$$

$k=2$ $$\lim_{x\to +\infty} \frac{5x^2 - 2^{-x}}{1-x^3} = 0 \longrightarrow \frac{\cancel{x^2}(5 - 2^{-x}/x^2)}{x^3(1/x^3 - 1)} \xrightarrow[x\to+\infty]{} 0$$

$k=3$ $$\lim_{x\to +\infty} \frac{5x^3 - 2^{-x}}{1-x^3} = -5 \longrightarrow \frac{\cancel{x^3}(5 - 2^{-x}/x^3)}{\cancel{x^3}(1/x^3 - 1)} \xrightarrow[x\to+\infty]{} -5$$

$k=4$ $$\lim_{x\to +\infty} \frac{5x^4 - 2^{-x}}{1-x^3} = -\infty \qquad \frac{x^{\cancel{4}}(5 - 2^{-x}/x^4)}{\cancel{x^3}(1/x^3 - 1)} \xrightarrow[x\to+\infty]{} -\infty$$

■

4) i) $$\lim_{x\to -\infty} \frac{|x|}{x+2} = \lim_{x\to -\infty} \frac{-x}{x+2} = -1 ;$$

$$\lim_{x\to -\infty} \frac{2^x + x}{e^x + x} = 1 ;$$ ($2^x \to 0$, $e^x \to 0$)

$$\lim_{x\to +\infty} \frac{2^x + x}{e^x + x} = 0 ; \qquad \frac{2^x(1 + x/2^x)}{e^x(1 + x/e^x)} \xrightarrow[x\to+\infty]{} 0$$

$$\lim_{x\to0^+}\frac{e^x-1}{3x} = \lim_{x\to0^+}\frac{1}{3}\underbrace{\left(\frac{e^x-1}{x}\right)}_{\to 1} = \frac{1}{3}.$$

limite notevole! □

ii) $$\lim_{x\to-\infty}\frac{\overbrace{e^x}^{\to 0}-|x|}{\log|x|} = \lim_{x\to-\infty}\frac{x}{\log(-x)} = -\infty\ ;$$

$$\lim_{x\to0^-}\frac{x\,2^x}{e^x-1} = \lim_{x\to0^-}\underbrace{\left(\frac{x}{e^x-1}\right)}_{\to 1}\overbrace{2^x}^{\to 1} = 1\ ;$$

Nota: Se $\lim_{x\to0} f(x)=1$ anche $\lim_{x\to0}\frac{1}{f(x)}=1$.

$$\lim_{x\to+\infty}\frac{\log(1+x^2)}{x} = 0\ ;$$ (l'infinito al denominatore è più "potente" dell'infinito al numeratore).

$$\lim_{x\to-\infty}\frac{\overbrace{\log(1+x^2)}^{\to+\infty}}{2^x} = +\infty.$$
2^x infinitesimo positivo □

iii) $$\lim_{x\to0^-}\frac{e^{2x}-1}{4x} = \lim_{x\to0^-}\overbrace{\left(\frac{e^{2x}-1}{2x}\right)}^{\to 1}\cdot\frac{1}{2} = \frac{1}{2}\ ;$$

$\left(\frac{e^{\square}-1}{\square}\to 1 \text{ purché } \square \text{ tende a } 0!\right)$

$$\lim_{x\to0^+}\frac{\log(1+3x^2)}{x} = \lim_{x\to0^+}\overbrace{\frac{\log(1+3x^2)}{3x^2}}^{\to 1}\cdot\overbrace{3x}^{\to 0} = 0\ ;$$

$\left(\frac{\log(1+\square)}{\square}\to 1 \text{ purché } \square \text{ tende a } 0!\right)$

ricorda!

$$\lim_{x\to0^-}\frac{1+\overbrace{\log x^2}^{\to-\infty}}{x^2} = -\infty.$$
x^2 infinitesimo positivo □

iv) $$\lim_{x\to1^-}\frac{\log(1+(x^2-1))}{x-1} = \lim_{x\to1^-}\overbrace{\frac{\log(1+(x^2-1))}{x^2-1}}^{\to 1}\cdot\overbrace{(x+1)}^{\to 2} = 2\ ;$$

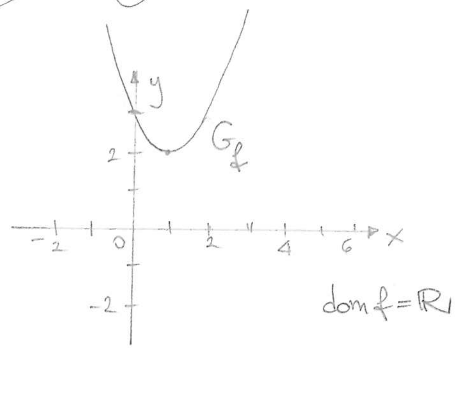
$$\lim_{x\to0}\frac{\log(1+x^2)}{2x^2} = \lim_{x\to0}\underbrace{\frac{\log(1+x^2)}{x^2}}_{\to 1}\cdot\frac{1}{2} = \frac{1}{2}\ ;$$

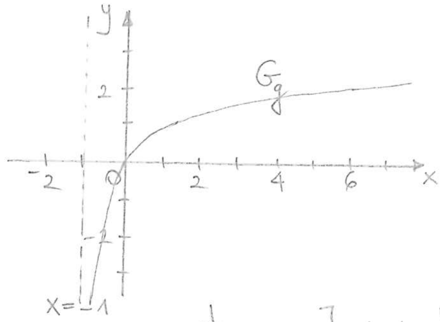
$$\lim_{x\to0^-}\frac{\log(1+x^2)}{x^3} = \lim_{x\to0^-}\overbrace{\frac{\log(1+x^2)}{x^2}}^{\to 1}\cdot\overbrace{\frac{1}{x}}^{\to-\infty} = -\infty$$ ■

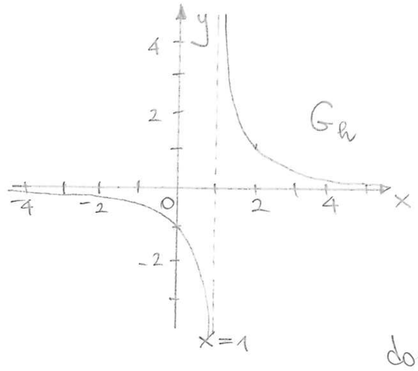
5) i) a) $f(x) = x^2-2x+3 = (x-1)^2+2$

b) $g(x) = \log(x+1)$

c) $h(x) = \frac{1}{(x-1)^3}$



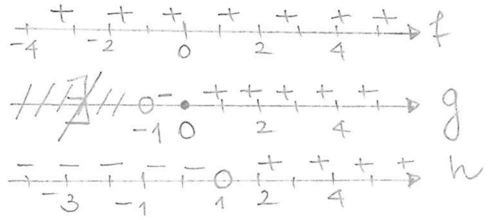




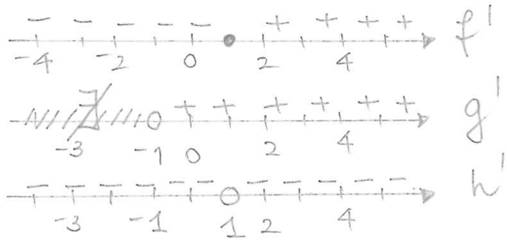
$\operatorname{dom} g =]-1, +\infty[$

$\operatorname{dom} h = \mathbb{R} \setminus \{1\}$

ii)



iii)



6) i) a) $f(x) = \frac{1}{x^2}$ $\boxed{(-1,1)}$

$f'(x) = (x^{-2})' = -2x^{-3} = -\frac{2}{x^3}$

$f'(-1) = +2$

retta tg: $y = f(-1) + f'(-1)(x+1)$

$= 1 + 2(x+1) \quad \Rightarrow \quad \underline{y = 2x+3}$;

in $\boxed{(1,1)}$

retta tg: $y = f(1) + f'(1)(x-1)$

$= 1 - 2(x-1) \quad \Rightarrow \quad \underline{y = -2x+3}$.

b) $g(x) = \sqrt{x}$ $\boxed{(9,3)}$

$g'(x) = (x^{\frac{1}{2}})' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}}$

$g'(9) = \frac{1}{2 \cdot 3} = \frac{1}{6}$

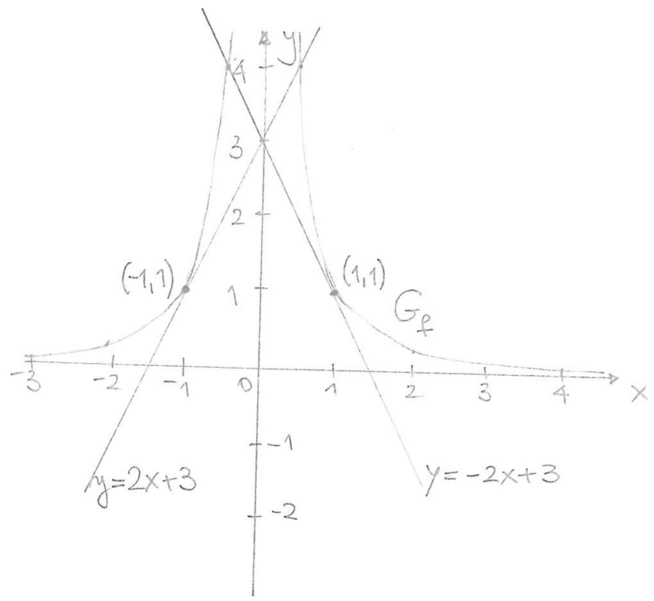
retta tg: $y = g(9) + g'(9)(x-9)$

$= 3 + \frac{1}{6}(x-9) \quad \Rightarrow \quad \underline{y = \frac{1}{6}x + \frac{3}{2}}$.

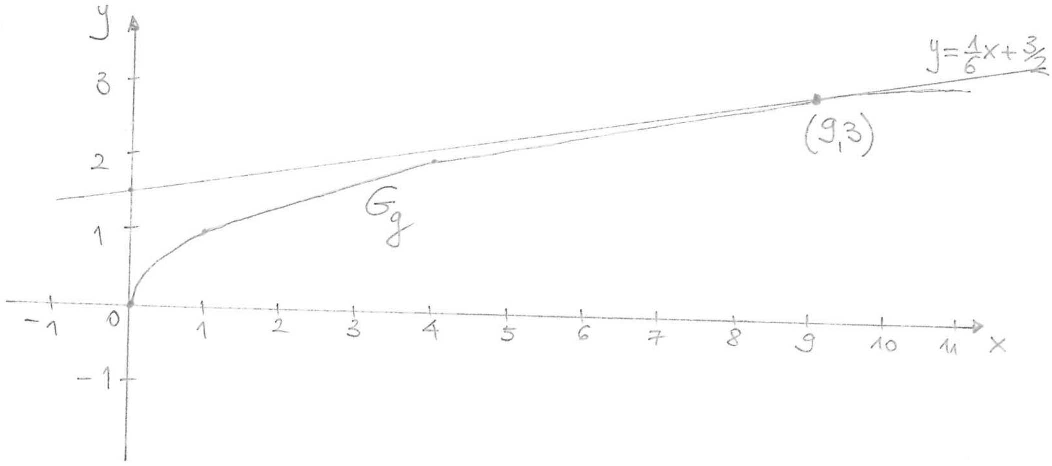
c) $h(x) = e^x$ $\boxed{(1, e)}$

$\underline{y = ex}$

ii) a)



b)



c)