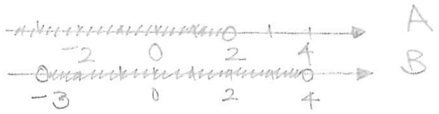
Università di Trento — Dip. di Psicologia e Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
PRIMA PROVA INTERMEDIA DI ANALISI MATEMATICA
a.a. 2015–2016 — Rovereto, 30 ottobre 2015

FILA (A)

a1) $A = \,]-\infty, 2[\,, \quad B = \,]-3, 4[$



$\underline{A \cup B = \,]-\infty, 4[\,}, \quad \underline{A \cap B = \,]-3, 2[}\,.$ ☐

a2) non $(\exists x \in \mathbb{R} : x < 1 \text{ o } x \geq 3) \iff$ "$\forall x \in \mathbb{R}, (x \geq 1) \text{ e } (x < 3)$". ☐

a3) $\dfrac{1}{x^2} = a - 2 \iff a - 2 > 0 \iff \underline{a \in \,]2, +\infty[}\,.$ ☐

a4) $(x-4)(x^3 - 5x^2 + 6x) = 0 \iff (x-4)\,x\,(x^2 - 5x + 6) = 0$

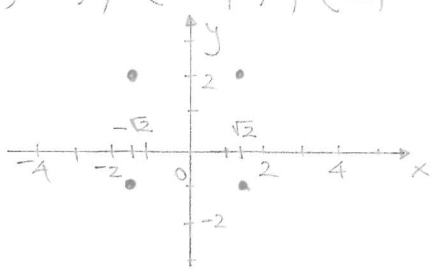
$\iff x(x-4)(x-3)(x-2) = 0$

$\underline{S = \{0, 2, 3, 4\}}\,.$

Si ha che $\underline{S \subseteq \mathbb{N}}$. ☐

a5) $E = \{x \in \mathbb{R} : x^2 = 2\} = \{-\sqrt{2}, \sqrt{2}\}; \quad F = \{-1, 2\}$

$E \times F = \{(-\sqrt{2}, -1), (-\sqrt{2}, 2), (\sqrt{2}, -1), (\sqrt{2}, 2)\}$



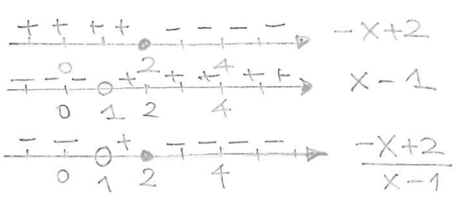
☐

a6) Punto di intersezione delle rette di eq. $x = -2$ e $y = 3x - 4$

è $\underline{S = (-2, -10)}$. ☐

a7) $\dfrac{x}{x-1} \leq 2 \iff \dfrac{x - 2x + 2}{x-1} \leq 0 \iff \dfrac{-x+2}{x-1} \leq 0$

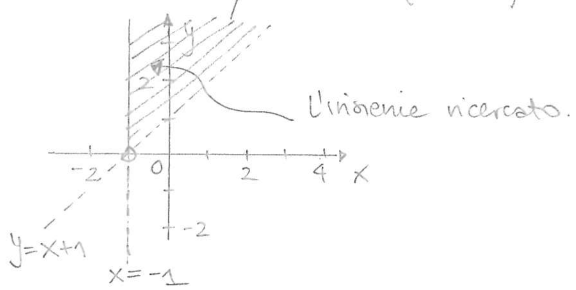
$\underline{S = \,]-\infty, 1[\, \cup \,[2, +\infty[}\,.$



☐

a8) $P=(-1,2) \quad m=2 \qquad y-2=2(x+1) \qquad \underline{\underline{y=2x+4}}.$

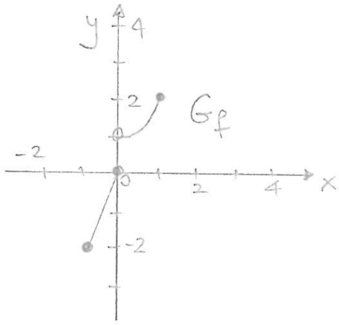
a9) $\begin{cases} y > x+1 \\ x \geq -1 \end{cases}$



a10) $f: [-1,1] \to \mathbb{R}$

$$f(x) = \begin{cases} 2x & \text{se } -1 \leq x \leq 0 \\ x^2+1 & \text{se } 0 < x \leq 1 \end{cases}$$

$$f([-1,1]) = \underline{\underline{[-2,0] \cup \,]1,2]}}.$$



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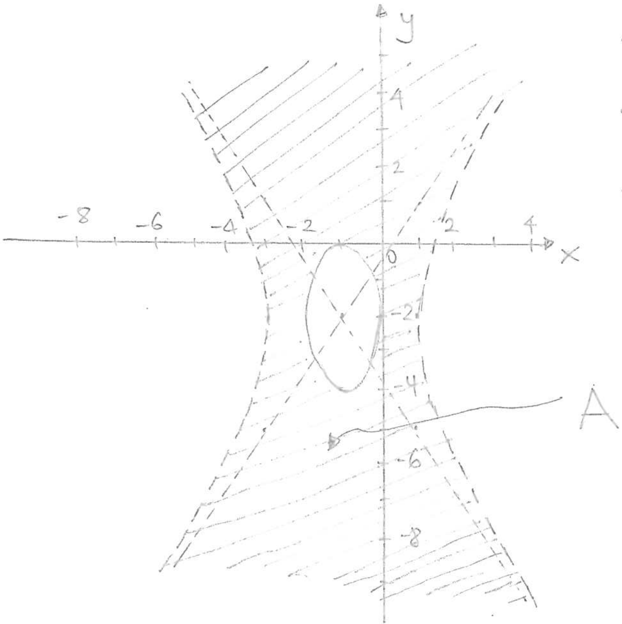
b1) i) $\text{non}\,[\forall n \in \mathbb{N}, \exists m \in \mathbb{N} : n^2-2m=0] \iff$ "$\exists n \in \mathbb{N} : \forall m \in \mathbb{N}, n^2-2m \neq 0$".

ii) È vera $\underline{\underline{\text{non}\,A}}$. Infatti, sia $n=1$; allora $n^2 \neq 2m \;\; \forall m \in \mathbb{N}$

b2) $4(x+1)^2+(y+2)^2 \geq 4 \iff (x+1)^2 + \dfrac{(y+2)^2}{4} \geq 1$.

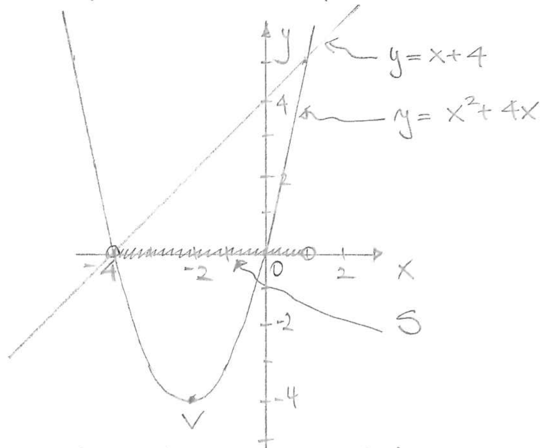
$$\begin{aligned} 9x^2-4y^2+18x-16y<43 &\iff 9(x^2+2x)-4(y^2+4y)<43 \\ &\iff 9(x+1)^2-9-4(y+2)^2+16<43 \\ &\iff 9(x+1)^2-4(y+2)^2<36 \\ &\iff \frac{(x+1)^2}{4}-\frac{(y+2)^2}{9}<1. \end{aligned}$$



b3) i) $x^2+4x < x+4 \iff x^2+3x-4<0$

$\iff (x+4)(x-1)<0 \Rightarrow \underline{S =]-4,1[}\,.$

$y = x^2+4x$
$= (x+2)^2-4 \quad V=(-2,-4)$



<u>Interpretazione geometrica della diseq $x^2+4x<x+4$:</u>

la soluzione corrisponde alle coordinate x dei pt. (x,y) appartenenti alla parabola $y=x^2+4x$, per i quali $y=x^2+4x$ è minore di $y=x+4$ che intuitiv. significa le x per cui la parabola "sta sotto" la retta $y=x+4$. □

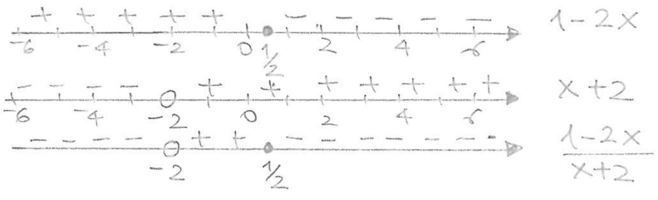
ii) $y+4 = -(x+2)$ (retta passante per $V=(-2,-4)$ con pendenza $m=-1$)

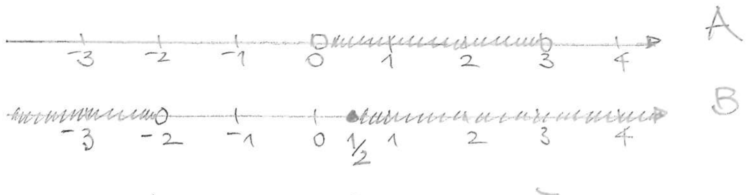
$\underline{y=-x-6}\,.$

b4) i) $A = \left\{ x\in\mathbb{R} : \dfrac{(x^2-3x)\,x^2}{x^2+1} < 0 \right\} = \left\{ x\in\mathbb{R} : x^2-3x<0 \right\}$
$= \underline{]0,3[}\,.$

$B = \left\{ x\in\mathbb{R} : \dfrac{1+x^2}{x+2} \le x \right\} = \left\{ x\in\mathbb{R} : \dfrac{1+\cancel{x^2}-\cancel{x^2}-2x}{x+2} \le 0 \right\} =$
$= \left\{ x\in\mathbb{R} : \dfrac{1-2x}{x+2} \le 0 \right\}$

$= \underline{]-\infty,-2[\cup [\tfrac{1}{2},+\infty[}\,.$



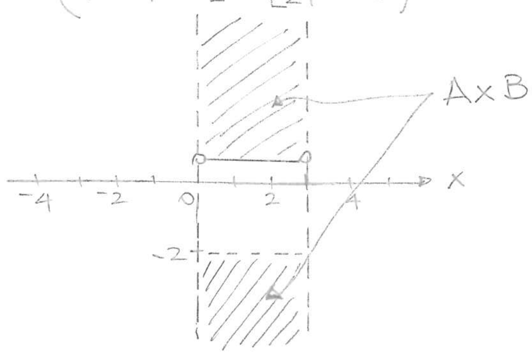


A è un intervallo, B non lo è. □

ii) A è un insieme limitato; B non è limitato (né superiormente, né inferiormente).

$\mathbb{R}\setminus B = [-2, \frac{1}{2}[$ $\qquad$ $\underline{\min(\mathbb{R}\setminus B) = -2}$ $\qquad$ $\underline{\nexists \max(\mathbb{R}\setminus B)}$.

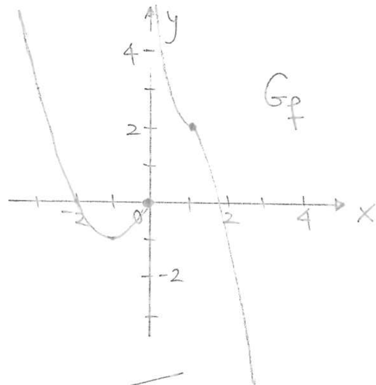
iii) $A \times B =]0,3[\times \left( ]-\infty,-2[\cup [\frac{1}{2},+\infty[\right)$



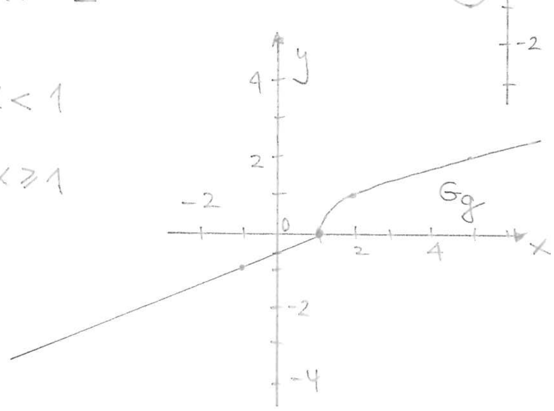
b5) i) $f, g : \mathbb{R} \to \mathbb{R}$

$(x+1)^2-1$

$$f(x) = \begin{cases} x^2+2x & \text{se } x \le 0 \\ \frac{1}{x^2}+1 & \text{se } 0 < x \le 1 \\ -x^2+3 & \text{se } x > 1 \end{cases}$$



$$g(x) = \begin{cases} \frac{x}{2}-\frac{1}{2} & \text{se } x < 1 \\ \sqrt{x-1} & \text{se } x \ge 1 \end{cases}$$



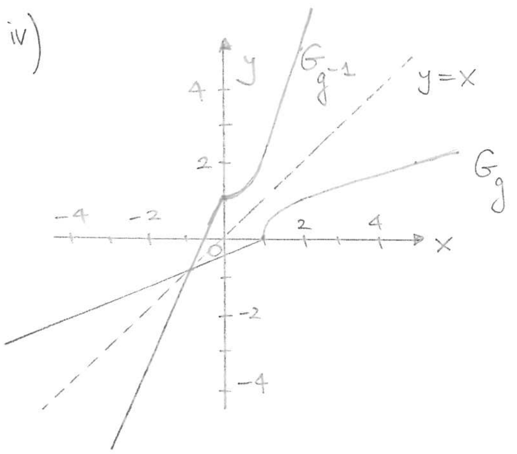
ii) f non è iniettiva; basta prendere $x_1 = -2 \neq x_2 = 0$ e si ha $f(x_1) = f(x_2)$.

iii) Se $k < -1$ $\exists$ una soluzione dell'eq. $f(x) = k$;

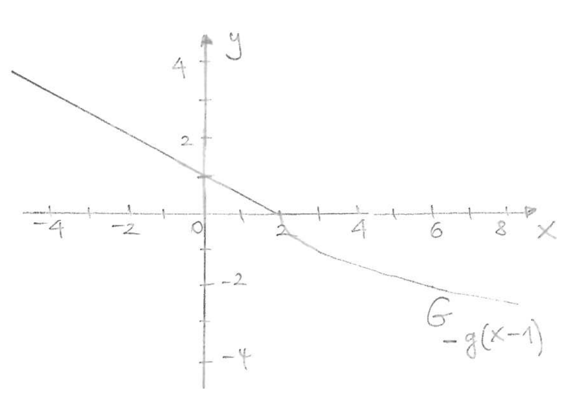
$k = -1$ o $k > 0$ $\exists$ due soluzioni dell'eq. $f(x) = k$;

$-1 < k \le 0$ $\exists$ tre soluzioni dell'eq. $f(x) = k$.

iv)

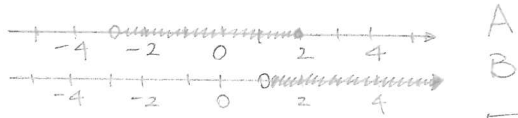


v)



FILA (B)

a1) $A =]-3,2]$, $B =]1,+\infty[$.



$A \cap B =]1,2]$; $A \setminus B =]-3,1]$.

a2) non $(\exists y \in \mathbb{Z} : y \leq -1 \text{ e } y^2 > 0) \iff$ "$\forall y \in \mathbb{Z}$, $y > -1$ o $y^2 \leq 0$."

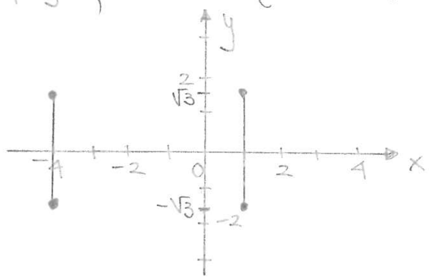
a3) $\frac{1}{x^2} = a+1 \iff a+1 > 0 \iff a \in]-1,+\infty[$.

a4) $(x-3)(x^3 - \frac{3}{2}x^2) = 0 \iff (x-3)x^2(x-\frac{3}{2}) = 0$

$S = \{0, \frac{3}{2}, 3\}$.

Si ha che $S \not\subseteq \mathbb{N}$.

a5) $A = \{-4,1\}$; $B = \{x \in \mathbb{R} : x^2 - 3 \leq 0\} = [-\sqrt{3}, \sqrt{3}]$.

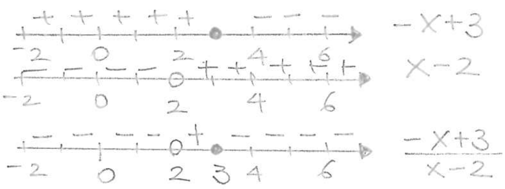


a6) Punto di intersezione delle rette di eq. $y=-2$ e $y=2x-4$

è $S = (1,-2)$.

a7) $\frac{x}{x-2} \geqslant 3 \iff \frac{x-3x+6}{x-2} \geqslant 0 \iff \frac{-2x+6}{x-2} \geq 0$

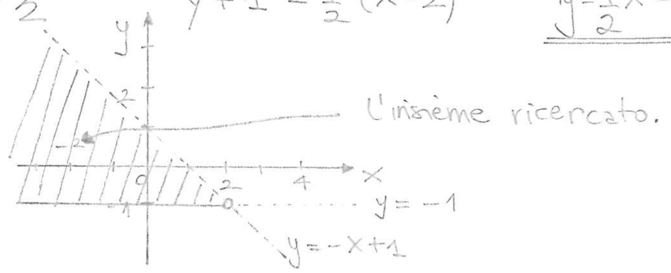
$\iff \frac{-x+3}{x-2} \geq 0$



$S =]2,3]$.

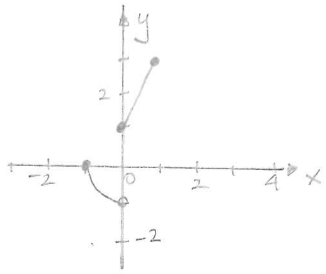
a8) $Q = (2,-1)$ $m = \frac{1}{2}$ $y+1 = \frac{1}{2}(x-2)$ $y = \frac{1}{2}x - 2$

a9) $\begin{cases} y < -x+1 \\ y \geqslant -1 \end{cases}$



a10) $f : [-1,1] \to \mathbb{R}$

$$f(x) = \begin{cases} x^2 - 1 & \text{se } -1 \le x < 0 \\ 2x+1 & \text{se } 0 \le x \le 1 . \end{cases}$$



$$f([-1,1]) = \underline{]-1,0] \cup [1,3]} .$$

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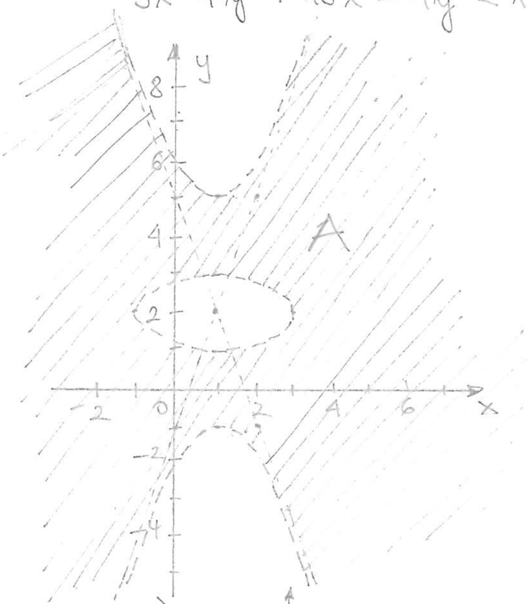
b1) i) non $\left[\exists n \in \mathbb{N} : \forall m \in \mathbb{N} , \ n^2 - 2m < 0\right] \iff$ "$\forall n \in \mathbb{N}, \exists m \in \mathbb{N} : n^2 - 2m \ge 0$.

ii) È vera $\underline{A}$. Infatti, sia $n = 1$, allora $1 \le 2m \ \forall m \in \mathbb{N}$.

b2) $(x-1)^2 + 4(y-2)^2 > 4 \iff \dfrac{(x-1)^2}{4} + (y-2)^2 > 1$.

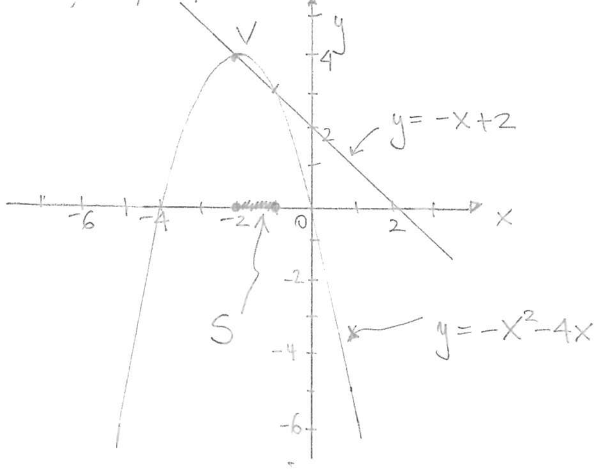
$-9x^2 + y^2 + 18x - 4y < 14 \iff -9(x^2 - 2x) + y^2 - 4y < 14$



$\iff -9(x-1)^2 + 9 + (y-2)^2 - 4 < 14$

$\iff -9(x-1)^2 + (y-2)^2 < 9$

$\iff -(x-1)^2 + \dfrac{(y-2)^2}{9} < 1$.



b3) i) $-x^2 - 4x \ge -x + 2$

$\iff x^2 + 3x + 2 \le 0$

$\iff (x+2)(x+1) \le 0$

$\implies \underline{S = [-2,-1]}$.

$y = -x^2 - 4x = -(x^2 + 4x)$

$= -(x+2)^2 + 4$

$V = (-2, 4)$

Interpretazione geometrica della diseq. $-x^2 - 4x \ge -x + 2$: la soluzione alle coordinate x dei pt. (x,y) appartenenti alla parabola $y=-x^2-4x$, per i quali $y=-x^2-4x$ è maggiore o uguale di $y=-x+2$ che intuitivamente significa le x per cui la parabola "sta sopra o interseca" la retta $y=-x+2$.

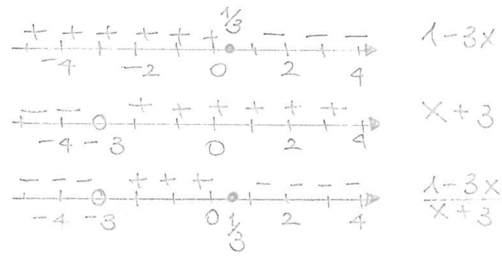
ii) $y-4=(x+2)$ (retta passante per $V=(-2,4)$ con pendenza $m=1$.

$\underline{y=x+6}$.

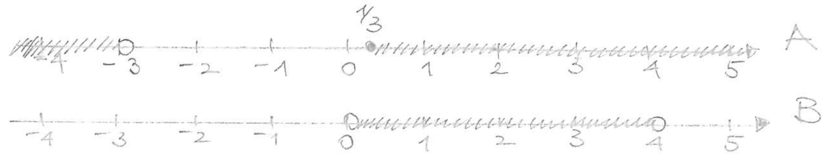
b4) i) $A=\left\{x\in\mathbb{R} : \frac{1+x^2}{x+3}\le x\right\} = \left\{x\in\mathbb{R} : \frac{1+\cancel{x^2}-\cancel{x^2}-3x}{x+3}\le 0\right\}$

$= \left\{x\in\mathbb{R} : \frac{1-3x}{x+3}\le 0\right\}$

$= \underline{\left]-\infty,-3\right[\cup \left[\tfrac{1}{3},+\infty\right[}$



$B=\left\{x\in\mathbb{R} : \frac{(x^2-4x)x^2}{x^2+1}<0\right\} = \left\{x\in\mathbb{R} : x^2-4x<0\right\} = \underline{\left]0,4\right[}$.

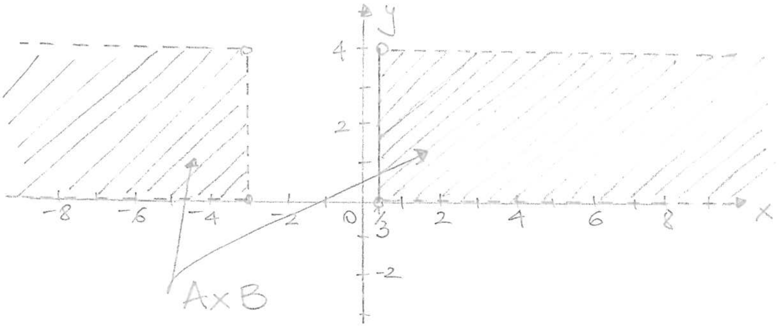


B è un intervallo, A non lo è.

ii) B è un insieme limitato, A non è limitato (né superiormente, né inferiormente).

$\mathbb{R}\setminus A = \left[-3,\tfrac{1}{3}\right[$ $\quad\underline{\min(\mathbb{R}\setminus A)=-3}$ $\quad\underline{\nexists \max(\mathbb{R}\setminus A)}$.

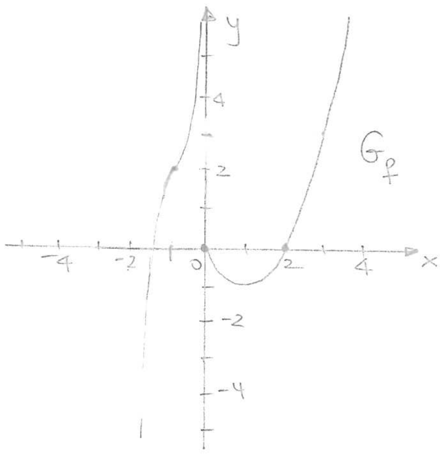
iii) $A\times B = \left(\left]-\infty,-3\right[\cup \left[\tfrac{1}{3},+\infty\right[\right) \times \left]0,4\right[$



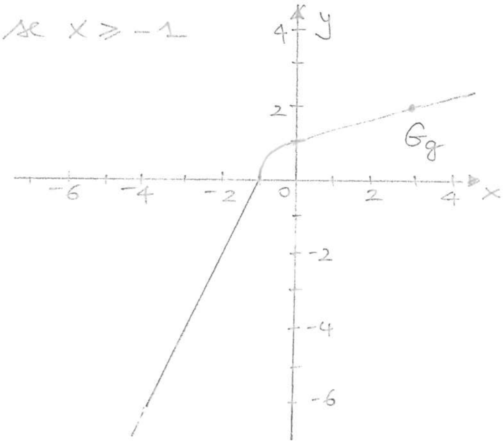
b5) i) $f, g : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} x^3+3 & \text{se } x<-1 \\ \dfrac{1}{x^2}+1 & \text{se } -1 \le x < 0 \\ x^2-2x & \text{se } x \ge 0 \end{cases}$$

$x^2-2x \to (x-1)^2-1$



$$g(x) = \begin{cases} 2x+2 & \text{se } x<-1 \\ \sqrt{x+1} & \text{se } x \ge -1 \end{cases}$$



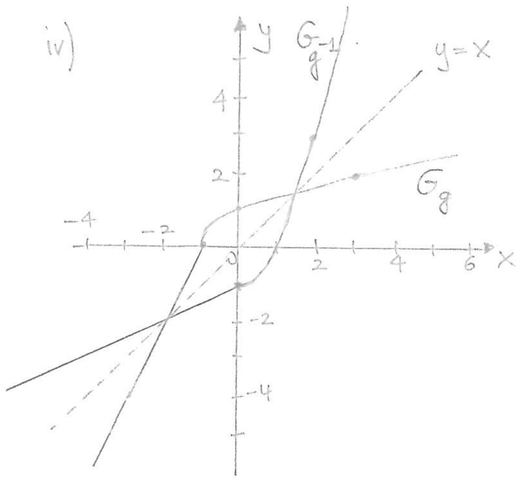
ii) f non è iniettiva: basta prendere $x_1 = 0 \neq x_2 = 2$, e si ha $f(x_1) = f(x_2)$.

iii) Se $k<-1$ $\exists$ una soluzione dell'eq. $f(x)=k$;

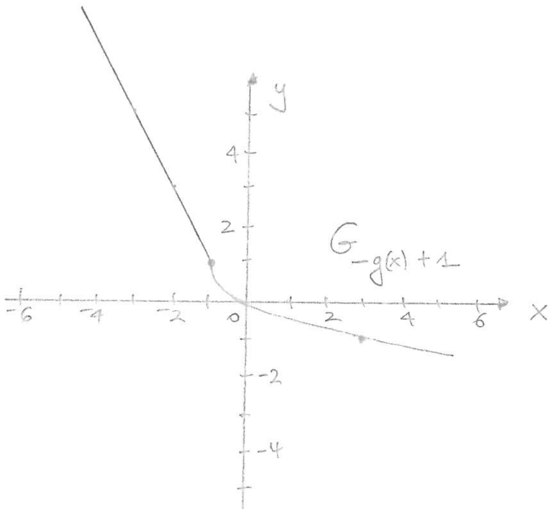
$k=-1$ o $k>0$ Sono due soluzioni dell'eq. $f(x)=k$;

$-1<k\le 0$ Sono tre soluzioni dell'eq. $f(x)=k$.

iv)

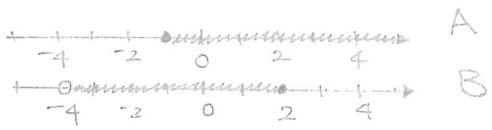


v)



FILA C

a1) $A = [-1, +\infty[\,, \quad B =]-4, 2]$



$A \cap B = [-1, 2]$; $B \setminus A =]-4, -1[$.

a2) non $(\forall x \in \mathbb{R},\ x^2 > 0 \text{ o } x^2 = 0) \iff$ "$\exists x \in \mathbb{R} : x^2 < 0 \text{ e } x^2 \neq 0$".

a3) $\frac{1}{x^2} = 1 - a \iff 1 - a > 0 \iff a \in]-\infty, 1[$.

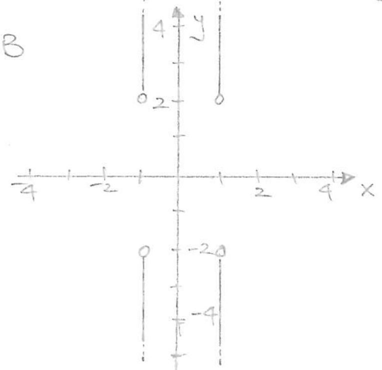
a4) $(x-1)(x^3 - \frac{2}{3}x^2) = 0 \iff (x-1)x^2(x - \frac{2}{3}) = 0$

$S = \{0, \frac{2}{3}, 1\}$.

Si ha che $S \not\subseteq \mathbb{N}$.

a5) $A = \{-1, 1\}$; $B = \{x \in \mathbb{R} : x^2 > 4\} =]-\infty, -2[\cup]2, +\infty[$.

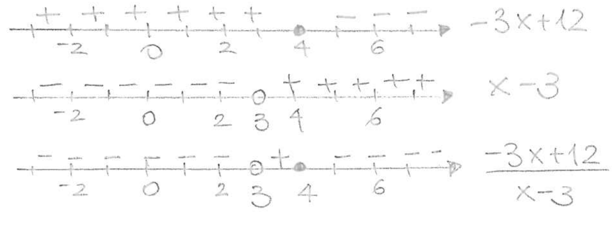
A×B



a6) Punto di intersezione delle rette di eq. $y = 2$ e $y = -2x - 4$ è $S = (-3, 2)$.

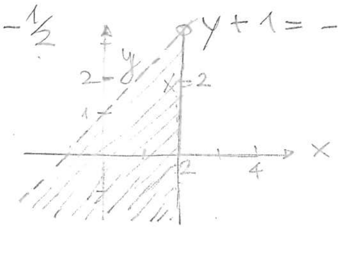
a7) $\frac{x}{x-3} < 4 \iff \frac{x - 4x + 12}{x-3} < 0 \iff \frac{-3x+12}{x-3} < 0$

$S =]-\infty, 3[\cup]4, +\infty[$.



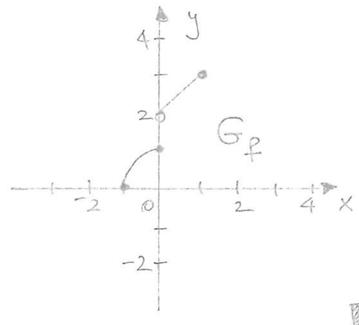
a8) $R = (2, -1)$ $m = -\frac{1}{2}$ $y + 1 = -\frac{1}{2}(x - 2)$ $y = -\frac{1}{2}x$

a9) $\begin{cases} y < x + 1 \\ x \leq 2 \end{cases}$



a10) $f : [-1, 1] \to \mathbb{R}$,

$$f(x) = \begin{cases} -x^2+1 & \text{se } -1 \le x \le 0 \\ x+2 & \text{se } 0 < x \le 1. \end{cases}$$



$f([-1,1]) = \underline{[0,1] \cup]2,3]}$. ■

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b1) i) non $\left[\forall n \in \mathbb{N}, \exists m \in \mathbb{N} : n^2 - \frac{m^2}{2} > 0\right]$

$\iff$ "$\exists n \in \mathbb{N} : \forall m \in \mathbb{N},\ n^2 - \frac{m^2}{2} \le 0$".

ii) È vera $\underline{A}$. Infatti, sia $n \in \mathbb{N}$ qualsiasi; basta prendere $m = n$

e si ha $n^2 - \frac{m^2}{2} = n^2 - \frac{n^2}{2} = \frac{n^2}{2} > 0$. ■

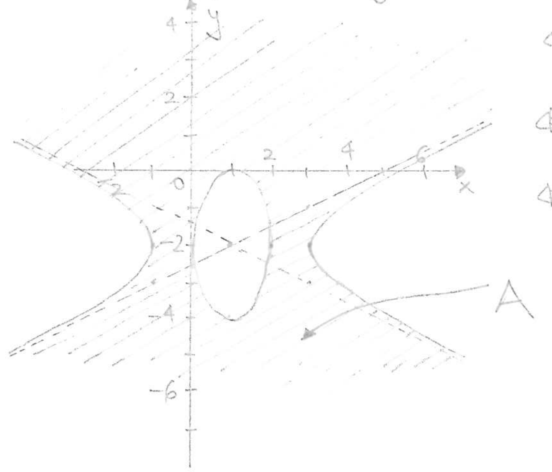
b2) $4(x-1)^2 + (y+2)^2 \ge 4 \iff (x-1)^2 + \frac{(y+2)^2}{4} \ge 1$.

$x^2 - 4y^2 - 2x - 16y \le 19 \iff x^2 - 2x - 4(y^2 + 4y) \le 19$

$\iff (x-1)^2 - 1 - 4(y+2)^2 + 16 \le 19$

$\iff (x-1)^2 - 4(y+2)^2 \le 4$

$\iff \frac{(x-1)^2}{4} - (y+2)^2 \le 1$.



■

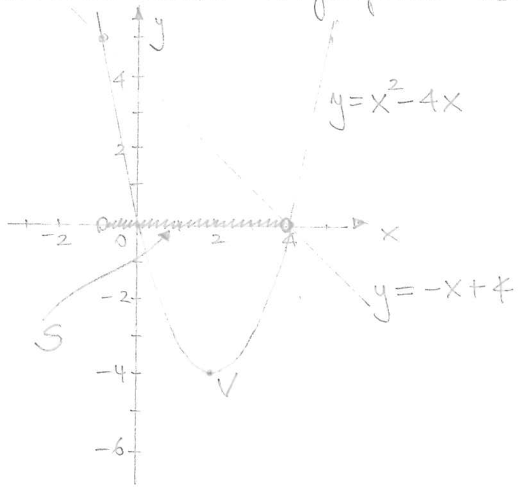
b3) i) $x^2 - 4x < -x + 4 \iff x^2 - 3x - 4 < 0$

$\iff (x-4)(x+1) < 0 \Rightarrow \underline{S =]-1, 4[}$.

$y = x^2 - 4x$

$= (x-2)^2 - 4 \qquad V = (2, -4)$

Interpretazione geom. della diseq. $x^2 - 4x < -x + 4$: la soluzione corrisponde alle coordinate x dei pt. (x, y) appartenenti alla parabola $y = x^2 - 4x$, per i quali $y = x^2 - 4x$ è minore di $y = -x + 4$ che intuitivamente significa le x per cui la parabola "sta sotto" la retta $y = -x+4$. □



ii) $y + 4 = x - 2$ (retta passante per $V = (2,-4)$ con pendenza $m = 1$)

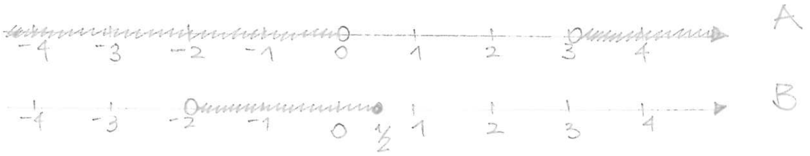
$\underline{\underline{y = x - 6}}$ ■

b4) i) $A = \left\{ x \in \mathbb{R} : \dfrac{(3x - x^2)\, x^2}{x^2+2} < 0 \right\} = \left\{ x \in \mathbb{R} : 3x - x^2 < 0 \right\}$

$= \underline{\underline{\left]-\infty, 0\right[\cup \left]3, +\infty\right[}}.$

$B = \left\{ x \in \mathbb{R} : \dfrac{1+x^2}{x+2} \geq x \right\} = \left\{ x \in \mathbb{R} : \dfrac{1 + \cancel{x^2} - \cancel{x^2} - 2x}{x+2} \geq 0 \right\}$

$= \left\{ x \in \mathbb{R} : \dfrac{1-2x}{x+2} \geq 0 \right\} = \underline{\underline{\left]-2, \tfrac{1}{2}\right]}}$ (vedi FILA A Es. b4)

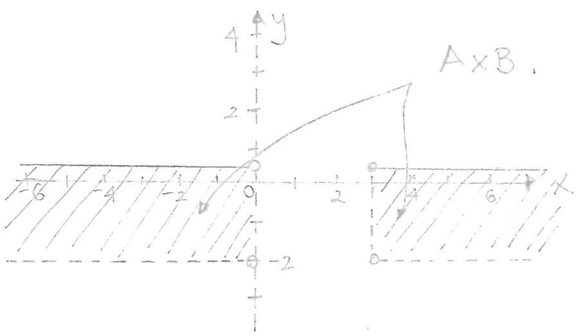


B è un intervallo, A non lo è. □

ii) B è un insieme limitato; A non è limitato (nè superiormente, nè inferiormente).

$\mathbb{R} \setminus A = [0, 3]$ $\underline{\underline{\min(\mathbb{R} \setminus A) = 0}}$ $\underline{\underline{\max(\mathbb{R} \setminus A) = 3}}$. □

iii) $A \times B = \left(\left]-\infty, 0\right[\cup \left]3, +\infty\right[\right) \times \left]-2, \tfrac{1}{2}\right]$

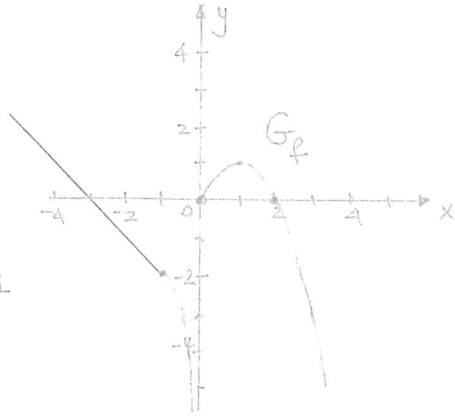


■

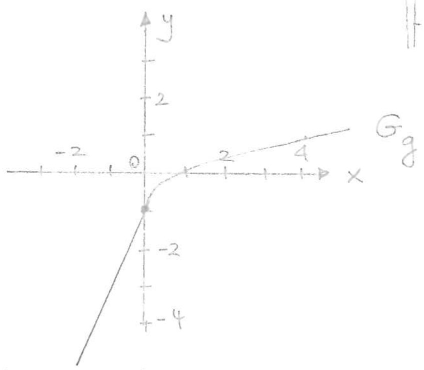
b5) i) $f, g : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} -x-3 & \text{se } x \leq -1 \\ \frac{1}{x} - 1 & \text{se } -1 < x < 0 \\ -x^2+2x & \text{se } x \geq 0 \end{cases}$$

$\hookrightarrow -(x^2-2x) = -(x-1)^2+1$



$$g(x) = \begin{cases} 2x-1 & \text{se } x<0 \\ \sqrt{x}-1 & \text{se } x \geq 0 . \end{cases}$$



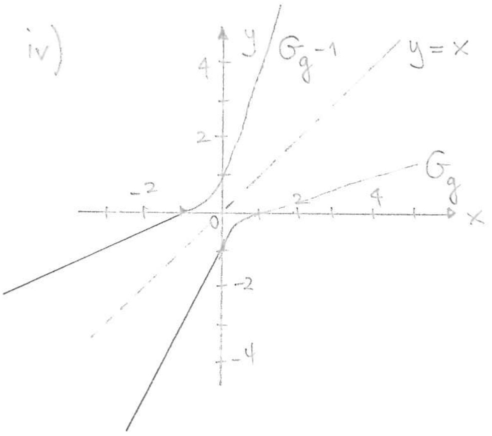
ii) f non è iniettiva: basta prendere $x_1 = 0 \neq x_2 = 2$ e si ha $f(x_1) = f(x_2)$.

iii) Se $k<0$ o $k=1$ $\exists$ono due soluzioni dell'eq. $f(x)=k$;

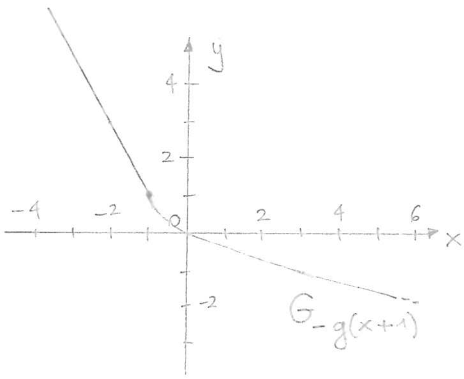
$0 \leq k < 1$ $\exists$ono tre soluzioni dell'eq. $f(x)=k$;

$k > 1$ $\exists$ una soluzione dell'eq. $f(x)=k$.

iv)

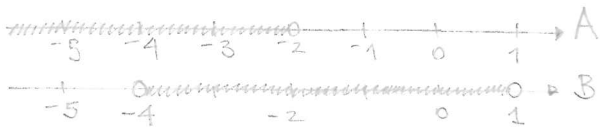


v)



FILA D

a1) $A =]-\infty, -2[$, $B =]-4, 1[$



$A \cup B =]-\infty, 1[$; $A \cap B =]-4, -2[$.

a2) non($\forall n \in \mathbb{N}$; n è pari o n è dispari) $\Leftrightarrow$ "$\exists n \in \mathbb{N}$: n non è pari e n non è dispari

a3) $\frac{1}{x^2} = 2 - a \iff 2 - a > 0 \Rightarrow a \in]-\infty, 2[$

a4) $(x^3 - 5x^2 + 4x)(x-2) = 0 \iff x(x^2 - 5x + 4)(x-2) = 0$

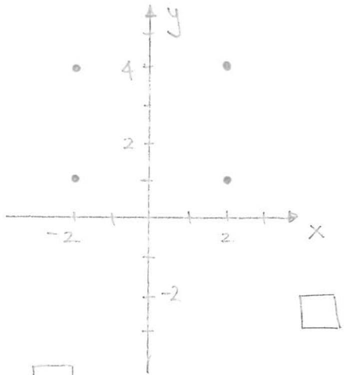
$\iff x(x-4)(x-1)(x-2) = 0$

$S = \{0, 1, 2, 4\}$.

Si ha che $S \subseteq \mathbb{N}$.

a5) $E = \{x \in \mathbb{R} : x^2 = 4\} = \{-2, 2\}$; $F = \{1, 4\}$.

$E \times F = \{(-2,1), (-2,4), (2,1), (2,4)\}$



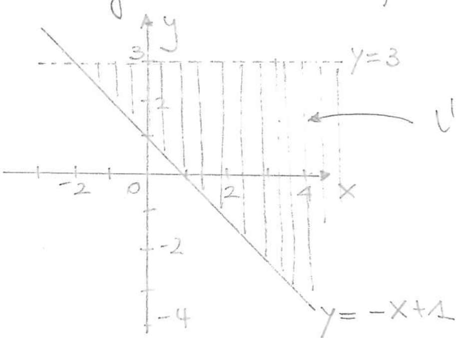
a6) Punto di intersezione delle rette di equazione $x = 2$ e $y = 3x - 1$ è $S = (2, 5)$.

a7) $\frac{x}{x-1} \geq 2 \iff \frac{-x+2}{x-1} \geq 0$ (vedi FILA A Es. a7))

$S =]1, 2]$.

a8) $S = (-1, 2)$, $m = -2$ $y - 2 = -2(x+1)$ $y = -2x$

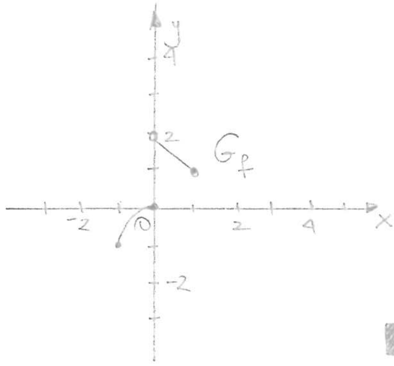
a9) $\begin{cases} y \geq -x + 1 \\ y < 3 \end{cases}$



L'insieme ricercato.

a10) $f : [-1,1] \to \mathbb{R}$

$$f(x) = \begin{cases} -x^2 & se \quad -1 \leq x \leq 0 \\ -x+2 & se \quad 0 < x \leq 1 \end{cases}$$



$f([-1,1]) = \underline{[-1,0] \cup [1,2[}$.

---

b1) i) $non\left[\exists n \in \mathbb{N} : \forall m \in \mathbb{N},\ n^2 - \frac{m^2}{2} < 0\right] \iff$ "$\forall n \in \mathbb{N}, \exists m \in \mathbb{N} : n^2 - \frac{m^2}{2} \geq 0$"

ii) È vero $\underline{non\,\mathcal{A}}$; infatti, $\forall n \in \mathbb{N}$ basta prendere $m = n$ e si ha $n^2 - \frac{n^2}{2} \geq 0$!

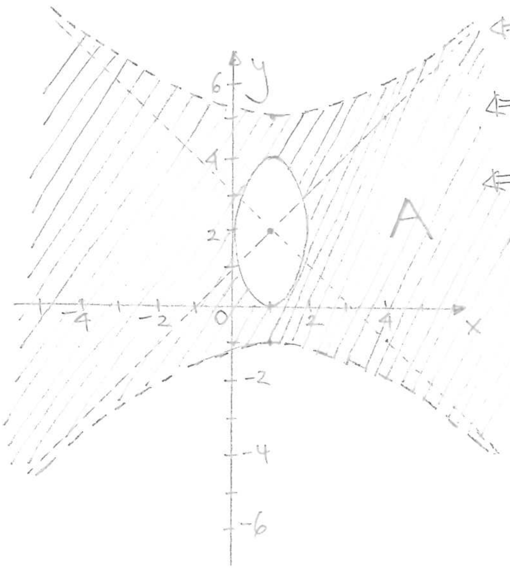
b2) $4(x-1)^2 + (y-2)^2 \geq 4 \iff (x-1)^2 + \frac{(y-2)^2}{4} \geq 1$.

$-x^2 + y^2 + 2x - 4y < 6 \iff -(x^2-2x) + y^2 - 4y < 6$

$\iff -(x-1)^2 + 1 + (y-2)^2 - 4 < 6$

$\iff -(x-1)^2 + (y-2)^2 < 9$

$\iff -\frac{(x-1)^2}{9} + \frac{(y-2)^2}{9} < 1$.



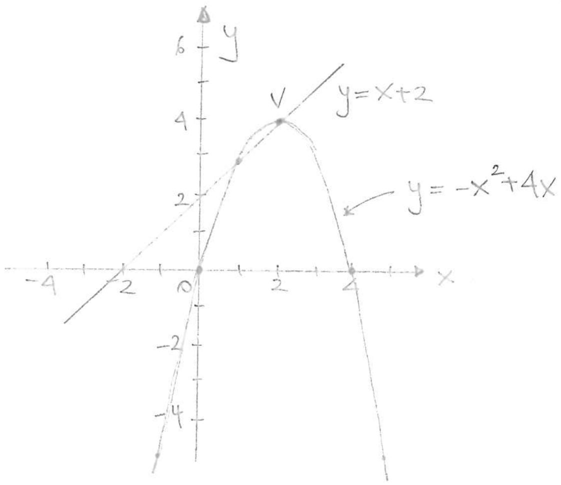
b3) i) $-x^2 + 4x \geq x + 2 \iff x^2 - 3x + 2 \leq 0$

$\iff (x-2)(x-1) \leq 0 \Rightarrow \underline{S = [1,2]}$.

$y = -x^2 + 4x$

$= -(x^2 - 4x) = -(x-2)^2 + 4 \qquad V = (2,4)$

<u>Interpretazione geom. della diseq. $-x^2+4x \geq x+2$</u> : la soluzione



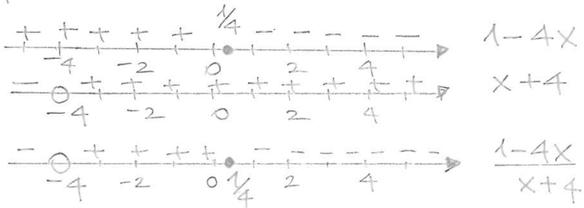
corrisponde alle coordinate x dei pt. (x,y) appartenenti alla parabola $y=-x^2+4x$, per i quali $y=-x^2+4x$ è maggiore o uguale a $y=x+2$, che intuitivamente significa le x per cui la parabola "sta sopra o interseca" la retta $y=x+2$.

ii) $y-4=-(x-2)$ (retta passante per $V=(2,4)$ con pendenza $m=-1$)

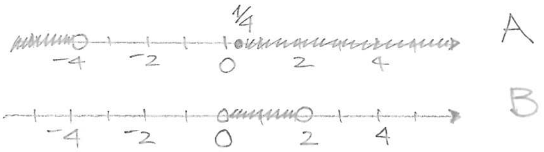
$y=-x+6$.

b4) i) $A=\left\{x\in\mathbb{R}:\ \dfrac{1+x^2}{x+4}\le x\right\}=\left\{x\in\mathbb{R}:\ \dfrac{1+\cancel{x^2}-\cancel{x^2}-4x}{x+4}\le 0\right\}$

$=\left\{x\in\mathbb{R}:\ \dfrac{1-4x}{x+4}\le 0\right\}=\ ]-\infty,-4[\ \cup\ [\tfrac14,+\infty[\,.$



$B=\left\{x\in\mathbb{R}:\ \dfrac{(2x-x^2)\,x^2}{x^2+1}>0\right\}=\ ]0,2[\,.$

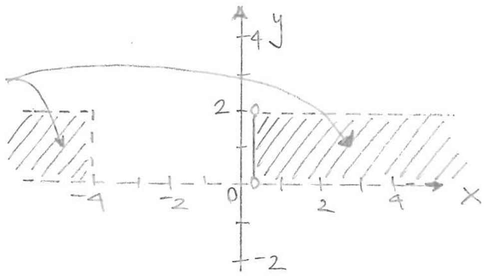


A non è un intervallo, B lo è.

ii) A non è limitato, mentre B lo è.

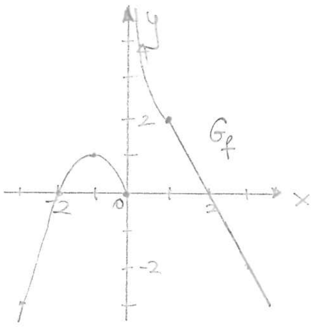
$\mathbb{R}\setminus A=[-4,\tfrac14[$ $\qquad \min(\mathbb{R}\setminus A)=-4 \qquad \nexists\ \max(\mathbb{R}\setminus A)$.

iii) $A\times B$

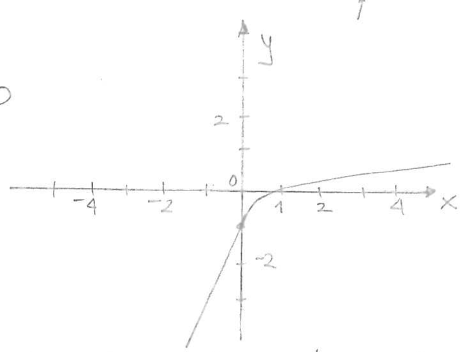


b5) i) $f, g : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} \underbrace{-x^2 - 2x}_{-(x+1)^2+1} & \text{se } x \leq 0 \\ \frac{1}{x} + 1 & \text{se } 0 < x < 1 \\ -2x + 4 & \text{se } x \geq 1 \end{cases}$$



$$g(x) = \begin{cases} 2x - 1 & \text{se } x < 0 \\ \sqrt[3]{x} - 1 & \text{se } x \geq 0 \end{cases}$$



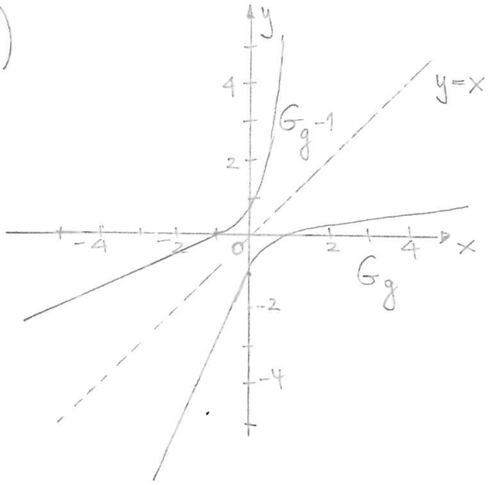
ii) f non è iniettiva; basta prendere $x_1 = -2 \neq x_2 = 0$ e si ha $f(x_1) = f(x_2)$.

iii) Se $k < 0$ o $k = 1$, ∃ono due soluzioni dell'eq. $f(x) = k$;

$0 \leq k < 1$, ∃ono tre soluzioni dell'eq. $f(x) = k$;

$k > 1$, ∃ una soluzione dell'eq. $f(x) = k$.

iv)



v)