$$\boxed{f(x) = \begin{cases} 0 & \text{se } x \in [-\pi, 0] \\ 3 & \text{se } x \in]0, \pi[\end{cases}}$$

e poi estesa con periodicità 2π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} 3\,dx = 3$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\int_0^{\pi} 3\cos(nx)\,dx = \frac{3}{\pi}\left[\frac{\sin(nx)}{n}\right]_0^{\pi} = 0 \qquad \forall n = 1, 2, \dots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \frac{1}{\pi}\int_0^{\pi} 3\sin(nx)\,dx = \frac{3}{\pi}\left[-\frac{\cos(nx)}{n}\right]_0^{\pi} =$$

$$= -\frac{3}{\pi}\left[\frac{\cos(n\pi)}{n} - \frac{1}{n}\right]$$

$$= -\frac{3}{\pi}\left[\frac{(-1)^n}{n} - \frac{1}{n}\right] = \begin{cases} -\frac{3}{\pi}\left[-\frac{2}{n}\right] & \text{se } n \text{ dispari} \\ 0 & \text{se } n \text{ è pari} \end{cases} \qquad \forall n = 1, 2, \dots$$

Quindi

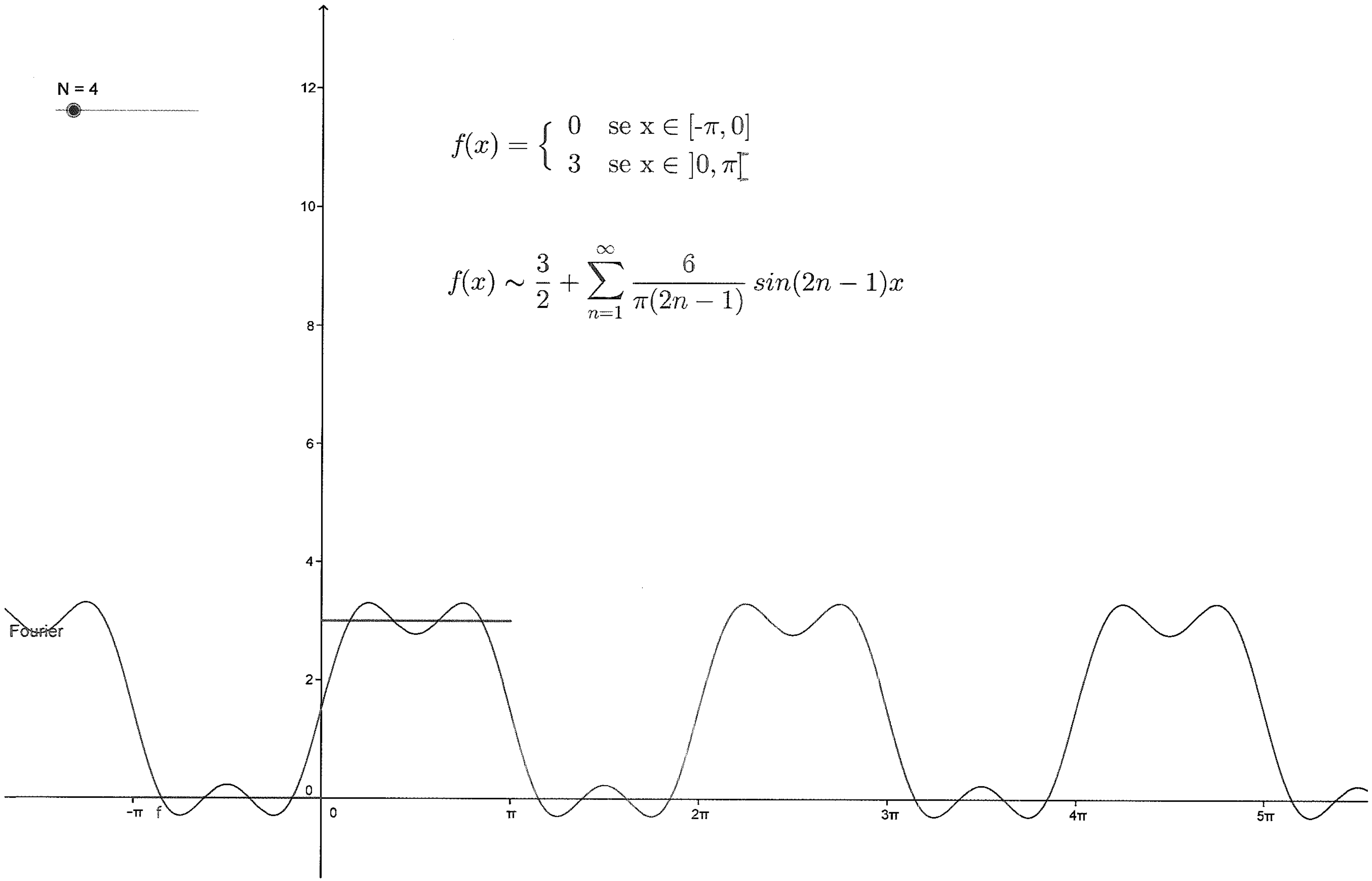
$$\boxed{f(x) \sim \frac{3}{2} + \sum_{n=1}^{\infty} \frac{6}{\pi(2n-1)} \operatorname{sen}(2n-1)x}$$

Fourier onda quadra

N = 4

$$f(x) = \begin{cases} 0 & \text{se x} \in [-\pi, 0] \\ 3 & \text{se x} \in]0, \pi[\end{cases}$$

$$f(x) \sim \frac{3}{2} + \sum_{n=1}^{\infty} \frac{6}{\pi(2n-1)} sin(2n-1)x$$



Fourier onda quadra

$$f(x) = \begin{cases} 0 & \text{se x} \in [-\pi, 0] \\ 3 & \text{se x} \in]0, \pi[\end{cases}$$

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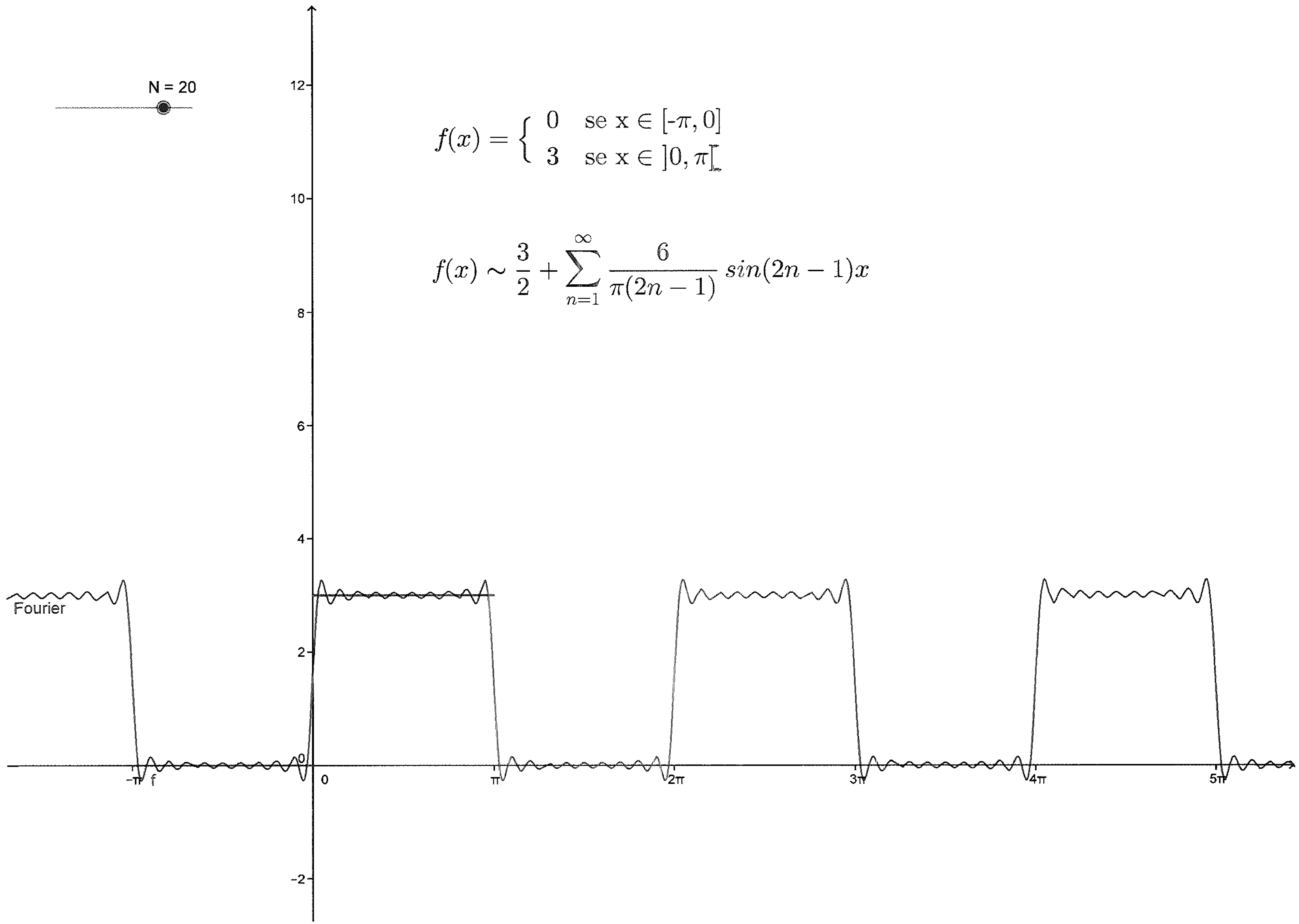
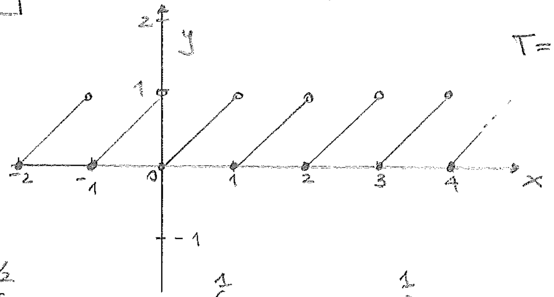


$$\boxed{f(x) = x - [x] \quad \text{in } \mathbb{R}}$$

$[x]$: parte intera di x

funzione 1-periodica

$T = 1$



$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\,dx \underset{T=1}{=} 2\int_{-1/2}^{1/2} f(x)\,dx \underset{f \text{ 1-periodica}}{=} 2\int_0^1 f(x)\,dx = 2\int_0^1 x\,dx = 2\left[\frac{x^2}{2}\right]_0^1 = \cancel{2}\cdot\frac{1}{\cancel{2}} = 1$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\cos\left(\frac{2\pi n}{T}x\right)dx \underset{T=1}{=} 2\int_{-1/2}^{1/2} f(x)\cos(2\pi n x)\,dx \underset{\substack{\text{funzione integranda}\\ \text{1-periodica}}}{=} 2\int_0^1 f(x)\cos(2\pi n x)\,dx$$

$$= 2\int_0^1 x\cos(2\pi n x)\,dx =$$

$$= 2\left[\cancel{x\,\frac{\sin(2\pi n x)}{2\pi n}\Big|_0^1} - \int_0^1 \frac{\sin(2\pi n x)}{2\pi n}\,dx\right] =$$

$$= -2\left[\frac{\cos(2\pi n x)}{(2\pi n)^2}\right]_0^1 = 0 \qquad \forall n = 1, 2, \ldots$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\sin\left(\frac{2\pi n x}{T}\right)dx = 2\int_{-1/2}^{1/2} f(x)\sin(2\pi n x)\,dx =$$

$$= 2\int_0^1 f(x)\sin(2\pi n x)\,dx = 2\int_0^1 x\sin(2\pi n x)\,dx =$$

$$= 2\left[-x\,\frac{\cos(2\pi n x)}{2\pi n}\Big|_0^1 + \int_0^1 \frac{\cos(2\pi n x)}{2\pi n}\,dx\right] =$$

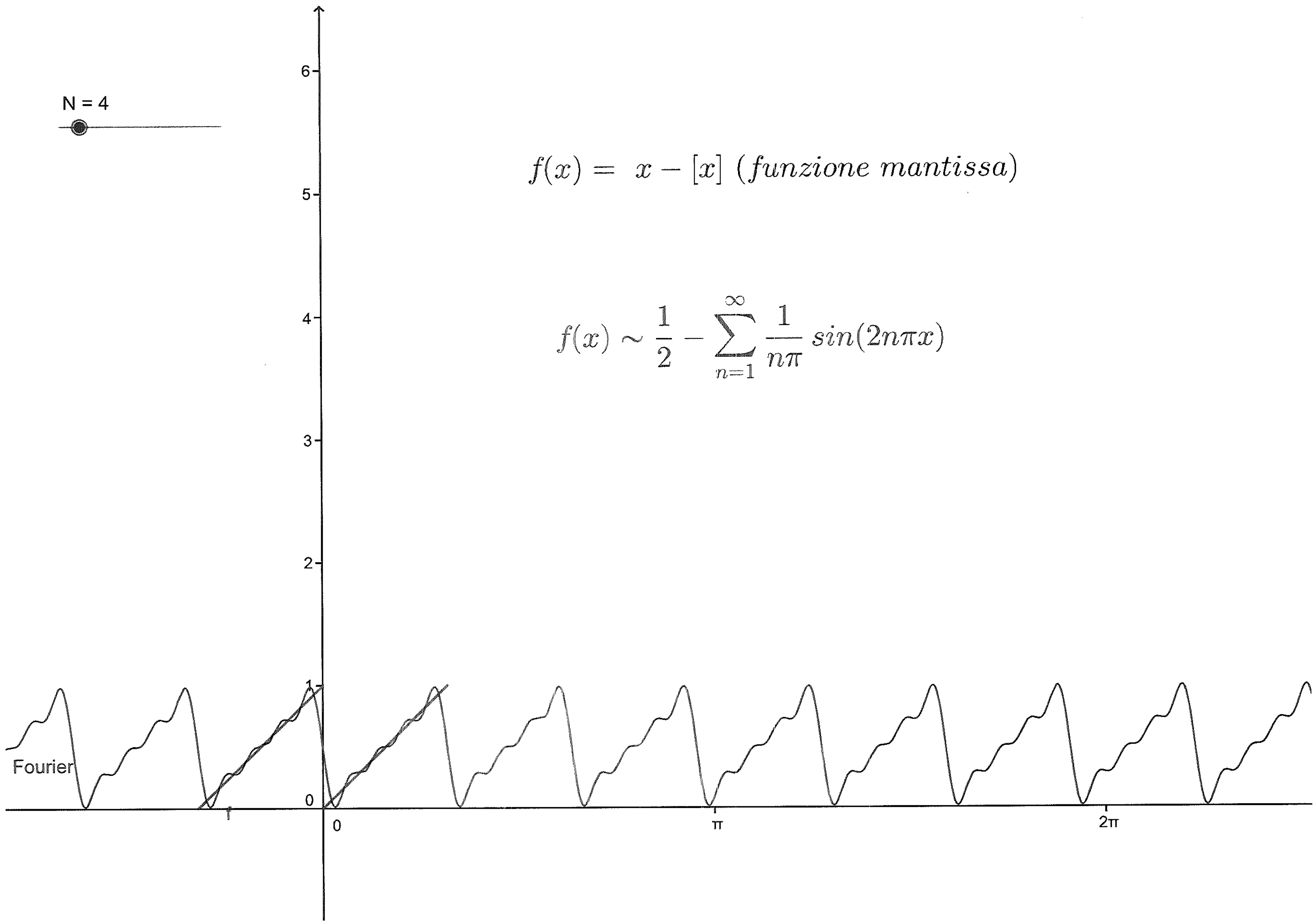
$$= 2\left[-\frac{1}{2\pi n} + \cancel{\frac{\sin(2\pi n x)}{(2\pi n)^2}\Big|_0^1}\right] = -\frac{1}{\pi n} \qquad \forall n = 1, 2, \ldots$$

Quindi

$$\boxed{f(x) \sim \frac{1}{2} - \sum_{n=1}^{\infty} \frac{1}{\pi n}\sin(2\pi n x)}$$

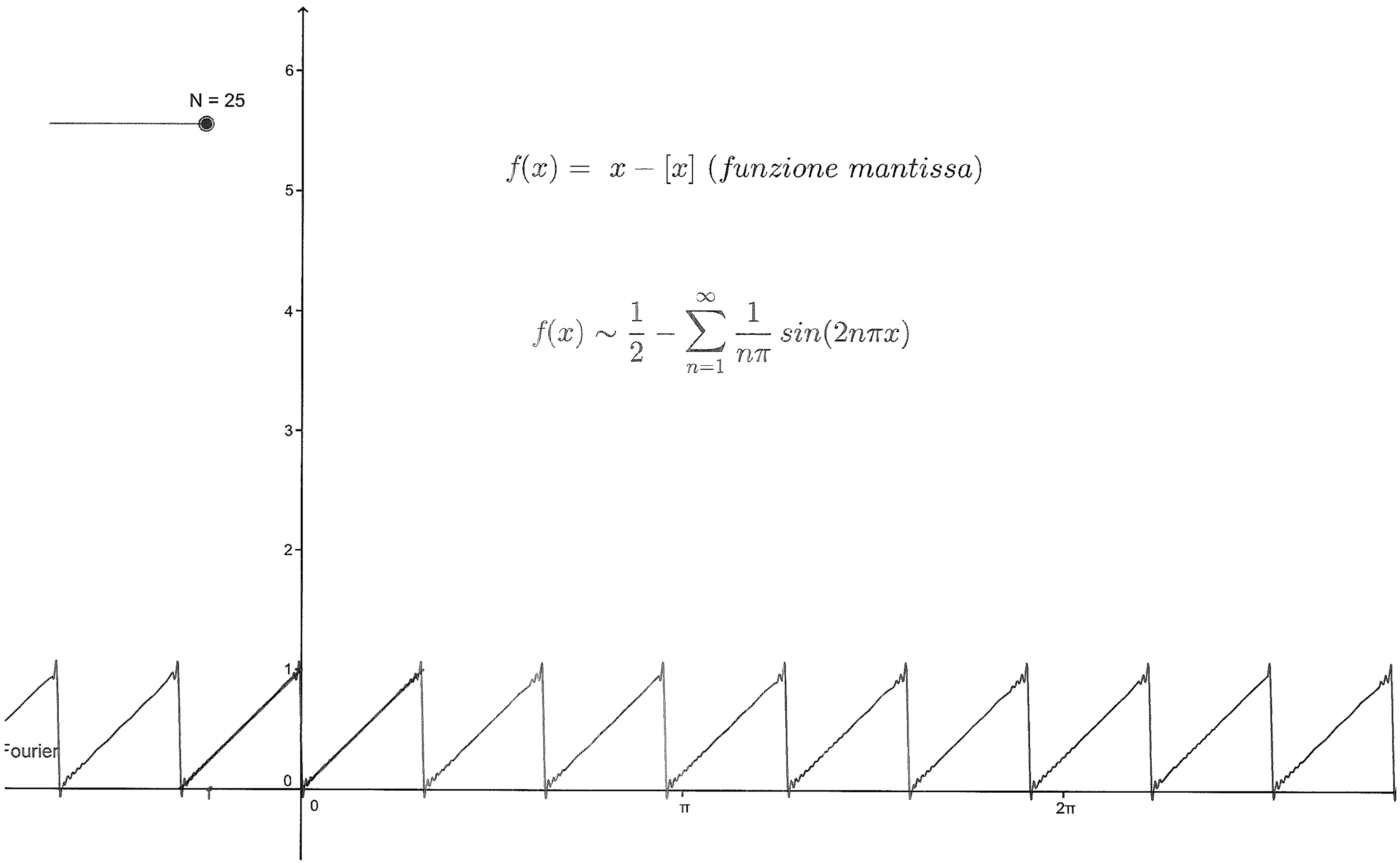
$$f(x) = x - [x] \ (funzione\ mantissa)$$

$$f(x) \sim \frac{1}{2} - \sum_{n=1}^{\infty} \frac{1}{n\pi} sin(2n\pi x)$$



$$f(x) = x - [x] \ (funzione\ mantissa)$$

$$f(x) \sim \frac{1}{2} - \sum_{n=1}^{\infty} \frac{1}{n\pi} sin(2n\pi x)$$



$$\boxed{f(x) = x^2 \quad \text{su } [-\pi, \pi]}$$ poi estesa con periodicità 2π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx = \frac{1}{\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos(nx)\,dx =$$

$$= \frac{1}{\pi}\left[\cancel{x^2\frac{\sin(nx)}{n}\Big|_{-\pi}^{\pi}} - 2\int_{-\pi}^{\pi} x\,\frac{\sin(nx)}{n}\,dx\right] = -\frac{2}{\pi n}\int_{-\pi}^{\pi} x\sin(nx)\,dx$$

$$= -\frac{2}{\pi n}\left[-x\frac{\cos(nx)}{n}\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi}\frac{\cos(nx)}{n}\,dx\right] =$$

$$= -\frac{2}{\pi n}\left[-\pi\frac{\cos(n\pi)}{n} - \pi\frac{\cos(n\pi)}{n}\right] - \frac{2}{\pi n^2}\cancel{\left[\frac{\sin(nx)}{n}\right]_{-\pi}^{\pi}}$$

$$= \frac{4}{\cancel{\pi} n^2}\cancel{\pi}(-1)^n = \frac{4}{n^2}(-1)^n \qquad \forall n = 1, 2, \dots$$

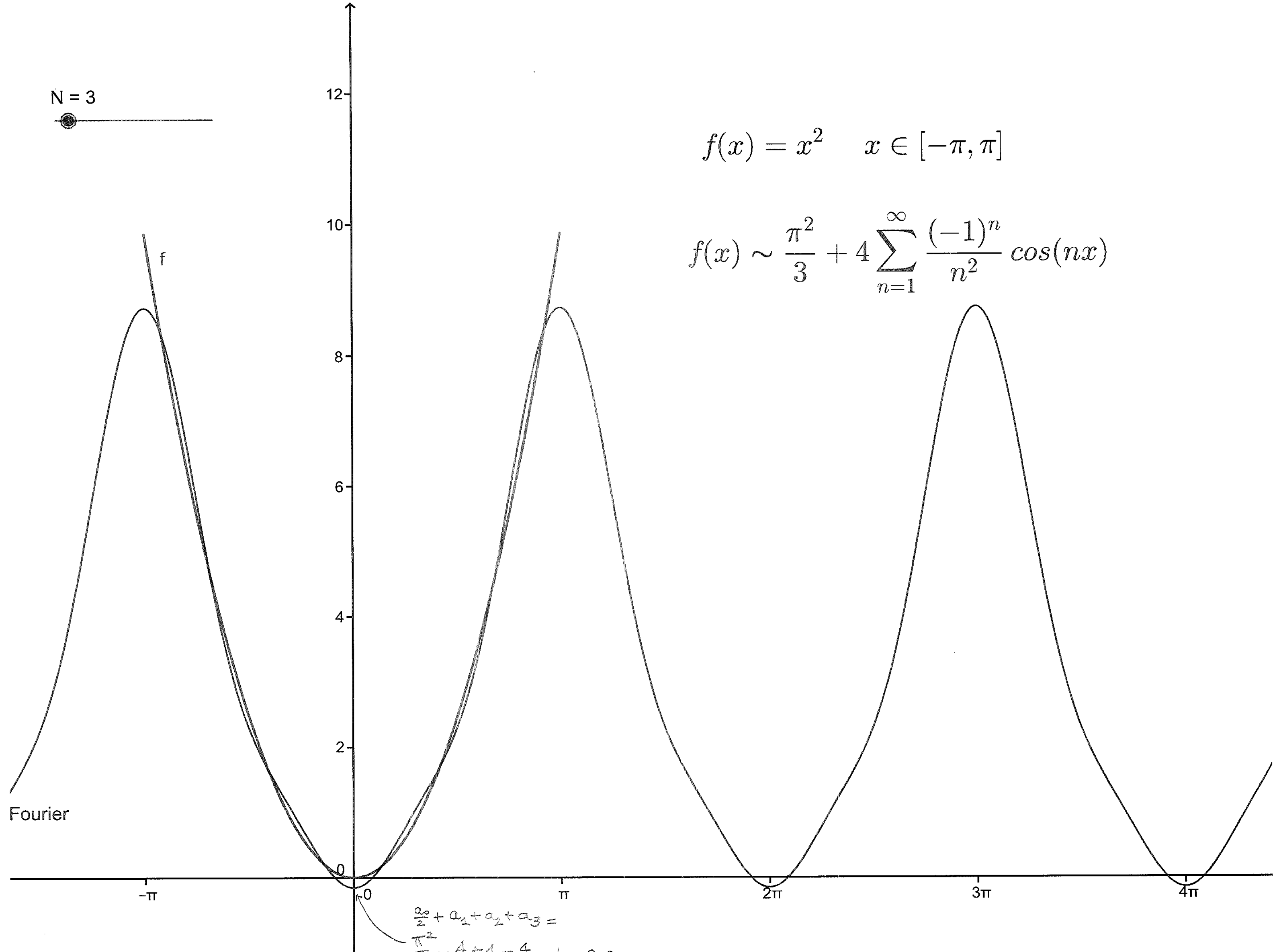
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{x^2\sin(nx)}\,dx = 0 \qquad \forall n = 1, 2, \dots$$

funzione integranda dispari su $[-\pi, \pi]$

Quindi
$$\boxed{f(x) \sim \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(nx)}$$

$$f(x) = x^2 \quad x \in [-\pi, \pi]$$

$$f(x) \sim \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} cos(nx)$$

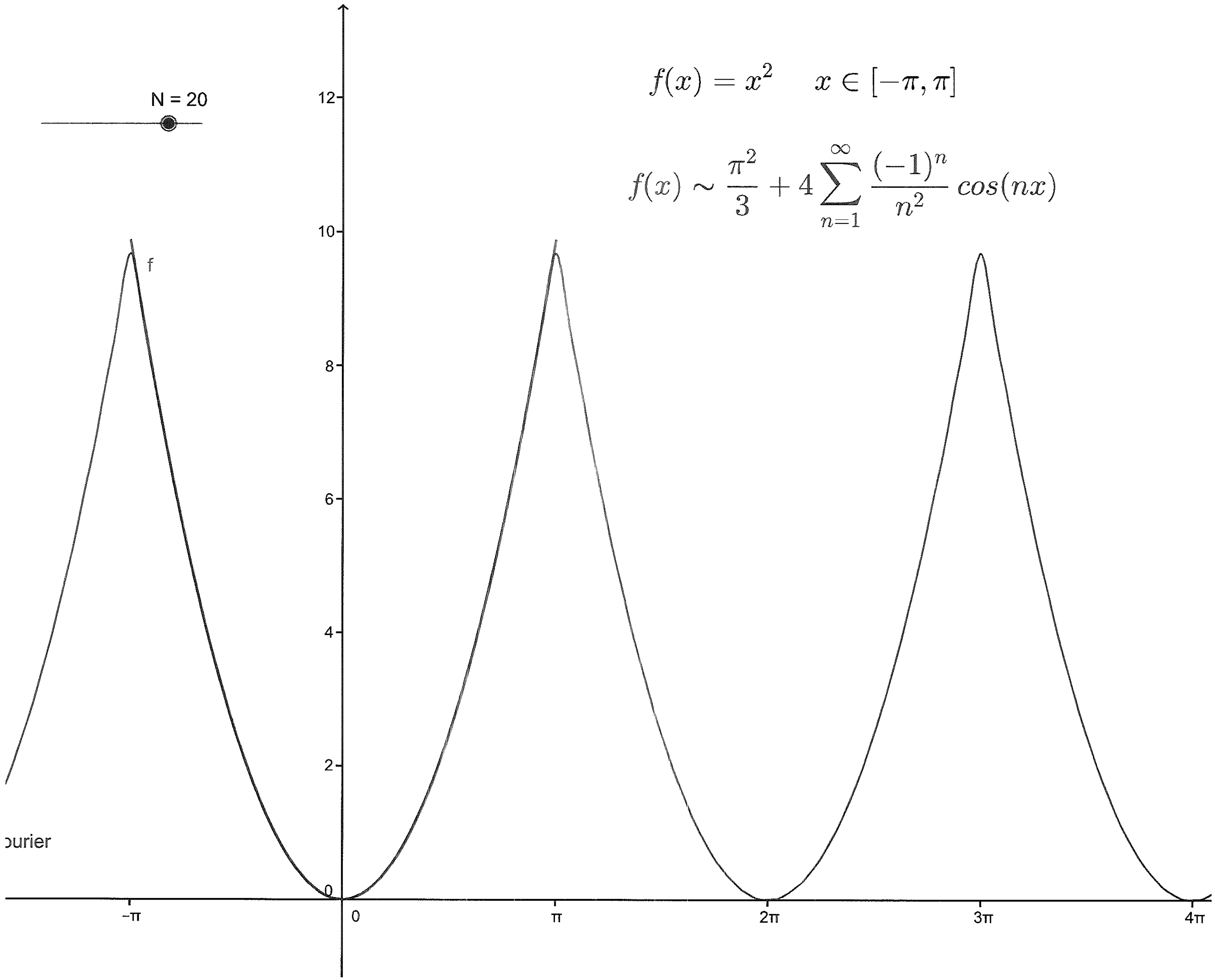


$\frac{a_0}{2} + a_1 + a_2 + a_3 =$

$\frac{\pi^2}{3} - 4 + 1 - \frac{4}{9} \sim -0.2$

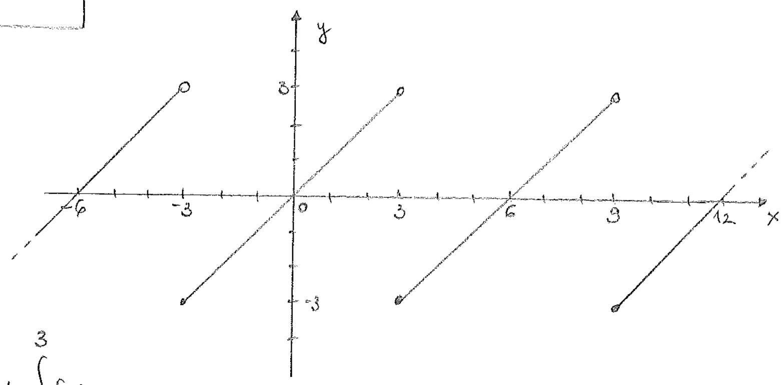
$$f(x) = x^2 \qquad x \in [-\pi, \pi]$$

$$f(x) \sim \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\, cos(nx)$$



$$\boxed{f(x) = x \text{ su } [-3,3[}$$

estesa con periodicità $T=6$ su tutto $\mathbb{R}$,



$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\,dx = \frac{1}{3}\int_{-3}^{3} f(x)\,dx = 0$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\cos\left(\frac{2\pi n}{T}x\right)dx = \frac{2}{6}\int_{-3}^{3} \underbrace{x\cos\left(\frac{\pi}{3}n x\right)}_{\text{funzione integranda dispari}}dx = 0 \qquad \forall n = 1,2,\dots$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\sin\left(\frac{2\pi n}{T}x\right)dx = \frac{2}{6}\int_{-3}^{3} x\sin\left(\frac{\pi}{3}n x\right)dx$$

$$= \frac{1}{3}\left[-x\,\frac{\cos\left(\frac{\pi n}{3}x\right)}{\frac{\pi n}{3}}\Bigg|_{-3}^{3} + \int_{-3}^{3}\frac{\cos\left(\frac{\pi}{3}n x\right)}{\frac{\pi n}{3}}dx\right]$$

$$= \frac{2\cdot 3(-1)^{n+1}}{\pi n} + \cancel{\frac{1}{\pi n}\,\frac{\sin\left(\frac{\pi}{3}n x\right)}{\frac{\pi n}{3}}\Bigg|_{-3}^{3}} \qquad \forall n = 1,2,\dots$$

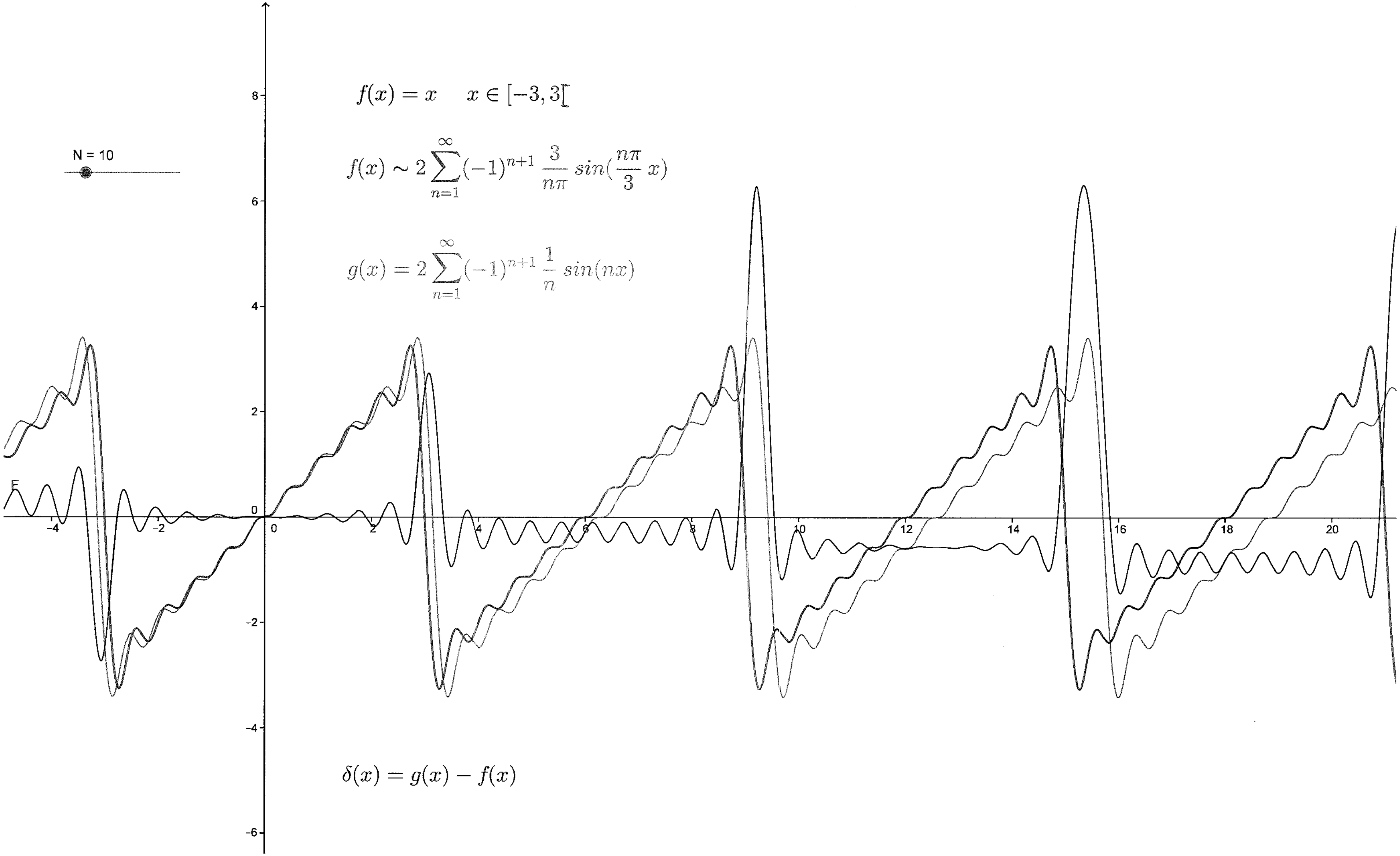
Quindi

$$\boxed{f(x) \sim 2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3}{\pi n}\sin\left(\frac{n\pi}{3}x\right)}$$

<u>NOTA</u>: $\frac{\pi}{3} \sim 1$; quindi $g(x) = 2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}\sin(nx)$

dà una buona approssimazione di $f(x)$ nell'intervallo $[-3,3[$, mentre non è ottimale negli intervalli $[-3+6k, 3+6k[$ per $k \in \mathbb{Z}\setminus\{0\}$.

Fourier Dente Sega Dispari



$$f(x) = x \quad x \in [-3, 3[$$

$$f(x) \sim 2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3}{n\pi}\,sin(\frac{n\pi}{3}\,x)$$

$$g(x) = 2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}\,sin(nx)$$

$$\delta(x) = g(x) - f(x)$$

Fourier Dente Sega Dispari

$$f(x) = x \qquad x \in [-3, 3[$$

$$f(x) \sim 2\sum_{n=1}^{\infty}(-1)^{n+1}\,\frac{3}{n\pi}\,sin(\frac{n\pi}{3}\,x) \qquad\qquad g(x) = 2\sum_{n=1}^{\infty}(-1)^{n+1}\,\frac{1}{n}\,sin(nx)$$