

Università di Trento - Dip. di Ingegneria e Scienza dell'Informazione
 CdL in Informatica, Ingegneria dell'Informazione e delle Comunicazioni e
 Ingegneria dell'Informazione e Organizzazione d'Impresa
 a.a. 2017-18 - PIAZZA 2 - "Disegnatori"

$$1.1)^{*} \log_4 \frac{1}{16} = x \iff 4^x = \frac{1}{16} \iff 4^x = 4^{-2} \Rightarrow \underline{x = -2}.$$

$$\log_2 \sqrt[4]{2} = x \iff 2^x = 2^{\frac{1}{4}} \Rightarrow \underline{x = \frac{1}{4}}.$$

$$\log_{\sqrt{5}} 125 = x \iff (\sqrt{5})^x = 5^3 \iff 5^{\frac{x}{2}} = 5^3 \Rightarrow \underline{x = 6}.$$

$$\log_{\frac{1}{4}} \frac{4}{64} = x \iff \left(\frac{1}{4}\right)^x = \frac{1}{16} \iff \left(\frac{1}{4}\right)^x = \left(\frac{1}{4}\right)^2 \Rightarrow \underline{x = 2}.$$

$$1.2) x^2 - x + 2 = y \iff \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 2 = y \iff \left(x - \frac{1}{2}\right)^2 + \frac{7}{4} = y \\ \iff \left(x - \frac{1}{2}\right)^2 = y - \frac{7}{4}.$$

Allora, se $y < \frac{7}{4}$ $\nexists$ soluz. dell'eq. $x^2 - x + 2 = y$;

se $y = \frac{7}{4}$ $\exists!$ soluzione dell'eq. $x^2 - x + 2 = y$ ($x = \frac{1}{2}$);

se $y > \frac{7}{4}$ Sono due soluzioni dell'eq. $x^2 - x + 2 = y$
 $\left(x_1 = \frac{1}{2} - \sqrt{y - \frac{7}{4}}, x_2 = \frac{1}{2} + \sqrt{y - \frac{7}{4}}\right).$

$$1.3) a) \sqrt{|x-1|+2x} \text{ è ben definito } \forall x \in \mathbb{R}; |x-1|+2x \geq 0.$$

$$\text{Ora } |x-1|+2x \geq 0 \iff |x-1| \geq -2x.$$

Poiché $|x-1| \geq 0 \forall x \in \mathbb{R}$, $|x-1| \geq -2x$ è sempre vero
 $\forall x \geq 0$ (essendo in questo caso $-2x \leq 0$).

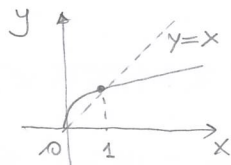
D'altra parte, se $x < 0$, allora $|x-1| \geq -2x \iff$

$$x-1 \leq 2x \text{ o } x-1 \geq -2x \text{ ossia}$$

$$x \geq -1 \text{ o } x \geq \frac{1}{3}. \text{ In conclusione, } \sqrt{|x-1|+2x}$$

è ben definito $\forall x \in [-1, +\infty[$.

$$b) \sqrt[3]{\log(x-\sqrt{x})} \text{ è ben definito } \forall x \in \mathbb{R}: \begin{cases} x \geq 0 \\ x - \sqrt{x} > 0 \end{cases} \\ \iff \begin{cases} x \geq 0 \\ x > \sqrt{x} \end{cases} \Rightarrow \underline{]1, +\infty[}$$



* Si può anche scrivere

$$\log_4 \frac{1}{16} = \log_4 4^{-2} = -2 \log_4 4 = \underline{-2}; \quad \log_2 \sqrt[4]{2} = \log_2 2^{\frac{1}{4}} = \frac{1}{4} \log_2 2 = \underline{\frac{1}{4}}; \quad \log_{\sqrt{5}} 125 = \log_{\sqrt{5}} (\sqrt{5})^6 = 6 \log_{\sqrt{5}} \sqrt{5} = \underline{6}.$$

$$\log_{\frac{1}{4}} \frac{4}{64} = \log_{\frac{1}{4}} \frac{1}{16} = \log_{\frac{1}{4}} \left(\frac{1}{4}\right)^2 = 2 \log_{\frac{1}{4}} \frac{1}{4} = \underline{2}.$$

c) $\sqrt[4]{1-\log x}$ $\bar{e}$ ben definito $\forall x \in \mathbb{R}$: $\begin{cases} x > 0 \\ 1-\log x \geq 0 \end{cases}$

$\Leftrightarrow \begin{cases} x > 0 \\ \log x \leq 1 \\ \parallel \\ \log e \end{cases} \Rightarrow \underline{\underline{[0, e]}}$.

d) $\log|e^x - \frac{1}{e}|$ $\bar{e}$ ben definito $\forall x \in \mathbb{R}$: $e^x - \frac{1}{e} \neq 0 \Leftrightarrow e^x \neq e^{-1} \Rightarrow \underline{\underline{\mathbb{R} \setminus \{-1\}}}$. ■

1.4) i) $2 \log_{\frac{1}{2}}(x-1) - \log_{\frac{1}{2}} x^2 > 0 \Leftrightarrow \begin{cases} x-1 > 0 \\ x \neq 0 \\ \log_{\frac{1}{2}}(x-1)^2 > \log_{\frac{1}{2}} x^2 \end{cases}$

$\Leftrightarrow \begin{cases} x > 1 \\ (x-1)^2 < x^2 \end{cases} \Leftrightarrow \begin{cases} x > 1 \\ x^2 - 2x + 1 < x^2 \end{cases}$

$\Leftrightarrow \begin{cases} x > 1 \\ 2x > 1 \end{cases} \Rightarrow \underline{\underline{S =]1, +\infty[}}$. □

ii) $\log_3(2x + \sqrt{x^2 - 1}) < \log_3 2x \Leftrightarrow \begin{cases} x^2 - 1 \geq 0 \\ x > 0 \\ 2x + \sqrt{x^2 - 1} > 0 \\ 2x + \sqrt{x^2 - 1} < 2x \end{cases}$

$\Leftrightarrow \begin{cases} x \geq 1 \\ \sqrt{x^2 - 1} < 0 \end{cases} \Rightarrow \underline{\underline{S = \emptyset}}$. □

iii) $\log(x - \sqrt{x}) < 1 \Leftrightarrow \begin{cases} x \geq 0 \\ x - \sqrt{x} > 0 \\ \log(x - \sqrt{x}) < \log e \end{cases} \Leftrightarrow \begin{cases} x \geq 0 \\ x > \sqrt{x} \\ x - \sqrt{x} < e \end{cases}$

$\Leftrightarrow \begin{cases} x \geq 0 \\ x > 1 \\ x - \sqrt{x} - e < 0 \end{cases} \rightsquigarrow \sqrt{x} = t \quad \begin{matrix} t^2 - t - e < 0 \\ t_{1/2} = \frac{1 \pm \sqrt{1+4e}}{2} \end{matrix}$

$\frac{1 - \sqrt{1+4e}}{2} < t < \frac{1 + \sqrt{1+4e}}{2} \rightsquigarrow \underline{\underline{S =]1, \left(\frac{1 + \sqrt{1+4e}}{2}\right)^2[}}$. □

1.5) i) $|1 - |x^2 - 1|| \leq 2 \Leftrightarrow -2 \leq 1 - |x^2 - 1| \leq 2$

$\Leftrightarrow -3 \leq -|x^2 - 1| \leq 1$

$$\Leftrightarrow -1 \leq |x^2 - 1| \leq 3$$

sempre soddisfatta

$$\Leftrightarrow -3 \leq x^2 - 1 \leq 3$$

$$\Leftrightarrow -2 \leq x^2 \leq 4$$

sempre soddisfatta

$$\Rightarrow S = [-2, 2].$$

□

$$e^{|x|} e^{1-x^2} > e \Leftrightarrow e^{|x|+1-x^2} > e^1 \Leftrightarrow |x|+1-x^2 > 1$$

$$\Leftrightarrow \begin{cases} x \geq 0 \\ x - x^2 > 0 \end{cases} \circ \begin{cases} x < 0 \\ -x - x^2 > 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq 0 \\ x^2 - x < 0 \end{cases} \circ \begin{cases} x < 0 \\ x^2 + x < 0 \end{cases}$$

$$\Rightarrow x \in]0, 1[\quad \downarrow \quad]-1, 0[$$

$$\Rightarrow \underline{S =]-1, 1[\setminus \{0\}}.$$

□

$$ii) (2-|x|) e^{x^3-1} < 0$$

$$\Leftrightarrow 2-|x| < 0 \Leftrightarrow |x| > 2$$

(e^{x³-1} > 0)
∀ x ∈ ℝ

$$\underline{S =]-\infty, -2[\cup]2, +\infty[.}$$

□

$$(2-|x|) \log(x^2-1) < 0 \Leftrightarrow \begin{cases} x^2-1 > 0 \\ 2-|x| < 0 \\ \log(x^2-1) > 0 \end{cases} \circ \begin{cases} x^2-1 > 0 \\ 2-|x| > 0 \\ \log(x^2-1) < 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x < -1 \text{ o } x > 1 \\ x < -2 \text{ o } x > 2 \\ x^2-1 > 1 \end{cases} \circ \begin{cases} x < -1 \text{ o } x > 1 \\ -2 < x < 2 \\ x^2-1 < 1 \end{cases}$$

$$\Leftrightarrow x \in]-\infty, -2[\cup]2, +\infty[\quad x \in]-\sqrt{2}, -1[\cup]1, \sqrt{2}[$$

$$\Rightarrow \underline{S =]-\infty, -2[\cup]-\sqrt{2}, -1[\cup]1, \sqrt{2}[\cup]2, +\infty[.}$$

□

$$iii) t^2 - (\log_2 4)t + \log_3 \frac{1}{3} \geq 0 \Leftrightarrow t^2 - 2t - 1 \geq 0$$

$$\text{oss. } t^2 - 2t - 1 = 0 \Leftrightarrow t_{1/2} = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

$$\Rightarrow \underline{S =]-\infty, 1-\sqrt{2}] \cup [1+\sqrt{2}, +\infty[.}$$

□

$$e^{2x} - (\log_2 4) e^x + \log_3 \frac{1}{3} < 0 \Leftrightarrow 1 - \sqrt{2} < e^x < 1 + \sqrt{2}$$

$$\text{sempre} \Rightarrow \underline{S =]-\infty, \log(1+\sqrt{2})[.}$$

□

1.6) a) $\forall y \in \mathbb{R}, \exists x \in \mathbb{R} : e^x = y$ (F) $e^x > 0 \forall x \in \mathbb{R}$; quindi $\forall y \leq 0$
 $\nexists x \in \mathbb{R} : e^x = y.$ $\square$

b) $\exists y \in \mathbb{R} : \forall x \in \mathbb{R}, e^x > y$ (V) Per esempio, $y = 0.$ $\square$

c) $\exists x \in \mathbb{R}, x > 0 : x^6 = 4$ (V) $x = \sqrt[6]{4}.$ $\square$

d) $\exists x \in \mathbb{R}, x < 0 : 2^x = 3^{-1}$ (V) $2^x = \frac{1}{3} \Leftrightarrow 2^x = 2^{\log_2 \frac{1}{3}}$
 $\Leftrightarrow 2^x = 2^{-\log_2 3}$
 $\Rightarrow x = -\log_2 3$ $\blacksquare$

1.7) i) $\log(x^2 - 1) = \log(x+1) + \log(x-1)$
 $\forall x \in]-\infty, -1[\cup]1, +\infty[$ (F) $\square$

ii) $\log(x^2 - 1) = \log(x+1) + \log(x-1)$
 $\forall x \in]1, +\infty[$ (V) $\begin{cases} x^2 - 1 > 0 \\ x+1 > 0 \\ x-1 > 0 \end{cases} \Leftrightarrow x > 1$ $\square$

iii) $\log(x^2 - 1) = [\log(x+1)][\log(x-1)]$
 $\forall x \in]1, +\infty[$ (F) $\left(\begin{matrix} x=2 & \log(3) \neq (\log 3)(\log 1) = 0 \\ \text{Falso} \end{matrix} \right)$ $\blacksquare$

1.8) $A = \{x_n = \frac{3}{n} - \frac{1}{n^2} : n \in \mathbb{N}, n \geq 1\} = \{2, \frac{3}{2} - \frac{1}{4}, 1 - \frac{1}{9}, \frac{3}{4} - \frac{1}{16}, \dots\}$

i) $x_n = \frac{3}{n} - \frac{1}{n^2} \geq \frac{3}{n} - \frac{1}{n} = \frac{2}{n} > 0 \forall n$

$x_n = \frac{3}{n} - \frac{1}{n^2} \leq 2 \Leftrightarrow 3n - 1 \leq 2n^2 \Leftrightarrow 2n^2 - 3n + 1 \geq 0$

Or $2n^2 - 3n + 1 = 0 \Leftrightarrow n_{1/2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4}$

$= \begin{cases} 1 \\ \frac{1}{2} \end{cases}$

Quindi $2n^2 - 3n + 1 \geq 0 \Leftrightarrow n \geq 1$ $\square$

ii) $\inf A = 0$. Abbiamo $0 < x_n \forall n \in \mathbb{N}, n \geq 1$. Dobbiamo provare che

$\forall \varepsilon > 0 \exists x_n \in A : x_n < 0 + \varepsilon.$ Or $\frac{3}{n} - \frac{1}{n^2} < \varepsilon$

$\Leftrightarrow 0 < \varepsilon n^2 - 3n + 1 \Leftrightarrow n > \frac{3 + \sqrt{9-4\varepsilon}}{2\varepsilon}.$ Basta prendere

$\varepsilon n^2 - 3n + 1 = 0 \quad n_{1/2} = \frac{3 \pm \sqrt{9-4\varepsilon}}{2\varepsilon}$

$n > \frac{3}{\varepsilon}.$

$\sup A = \max A = 2$ ($x_1 = \max A$). $\square$

iii) $\inf A$ non è $\min A$; $\sup A = \max A = 2.$ $\blacksquare$