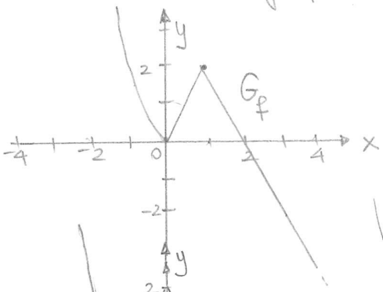
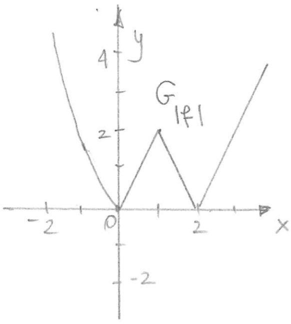
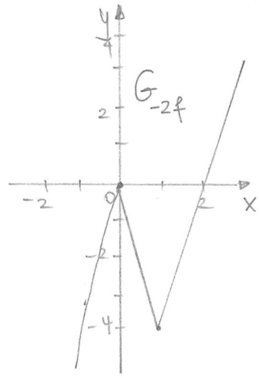
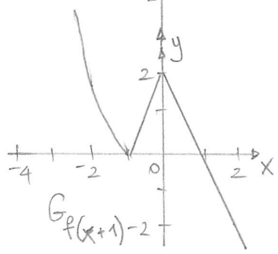
Università di Trento – Dip. di Ingegneria e Scienza dell'Informazione
CdL in Informatica, Ingegneria dell'informazione e delle comunicazioni e Ingegneria dell'informazione e organizzazione d'impresa
a.a. 2017–18 – PIAZZA 6 – "... funzioni elementari – grafici – trasformazioni ..."

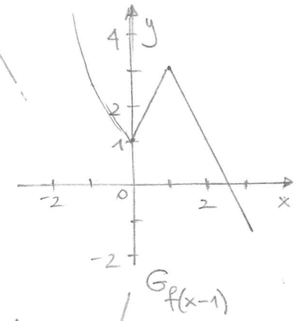
1.1) $f(x) = \begin{cases} e^{-x} - 1 & \text{se } x \le 0 \\ -2|x-1| + 2 & \text{se } x > 0 \end{cases}$



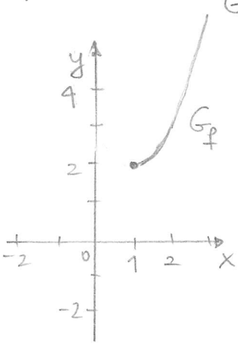








1.2) $f : [1, +\infty[\to \mathbb{R}$, $f(x) = x^2 - 2x + 3 = (x-1)^2 + 2$,



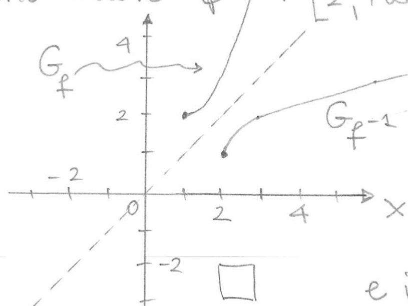
a) $f([1, +\infty[) = \underline{\underline{[2, +\infty[}}$.

b) $y = x^2 - 2x + 3 \iff x^2 - 2x + 3 - y = 0$
$\forall y \ge 2 \iff x_{1/2} = \dfrac{2 \pm \sqrt{4 - 4(3-y)}}{2}$

$x_{1/2} = 1 \pm \sqrt{y-2} \rightsquigarrow x = 1 + \sqrt{y-2}$
(poiché deve essere $x \ge 1$).

Abbiamo allora $f^{-1} : [2, +\infty[\to [1, +\infty[$ $\quad \underline{\underline{f^{-1}(x) = 1 + \sqrt{x-2}}}$

c)

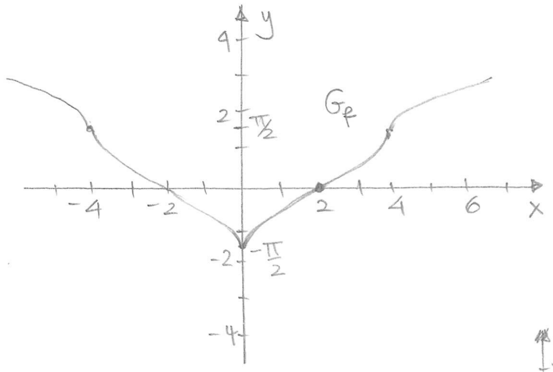


c) $x^2 - 2x + 3 = 1 + \sqrt{x-2}$ non ha soluzione poiché il grafico G_f e il grafico $G_{f^{-1}}$ non hanno punti di intersezione.

1.3) i) $f(x) = \begin{cases} \arcsin\left(\left|\frac{1}{2}x\right| - 1\right) & \text{se } -4 \le x \le 4 \\ \sqrt{|x| - 4} + \frac{\pi}{2} & \text{se } x < -4 \text{ o } x > 4 \end{cases}$

• funzione pari

$\arcsin\left(\left|\frac{1}{2}x\right| - 1\right) = \arcsin\left(\frac{1}{2}(|x| - 2)\right)$ e per $x \ge 0$ risulta $\arcsin\left(\frac{1}{2}(x-2)\right)$.



• f limitata inferiormente ($f(x) \geq -\frac{\pi}{2}\ \forall x \in \mathbb{R}$)

• $f|_{]-\infty,0]}$ ↘ $\quad f|_{[0,+\infty[}$ ↗

• $\inf_{\mathbb{R}} f = \min_{\mathbb{R}} f = -\frac{\pi}{2}$ $\quad \sup_{\mathbb{R}} f = +\infty$.

□

ii) $f(x) = -|\log|x+1|| + 2$

$x > -1,\ x > 0$
$\log(x+1) = 2 \iff x = e^2 - 1$
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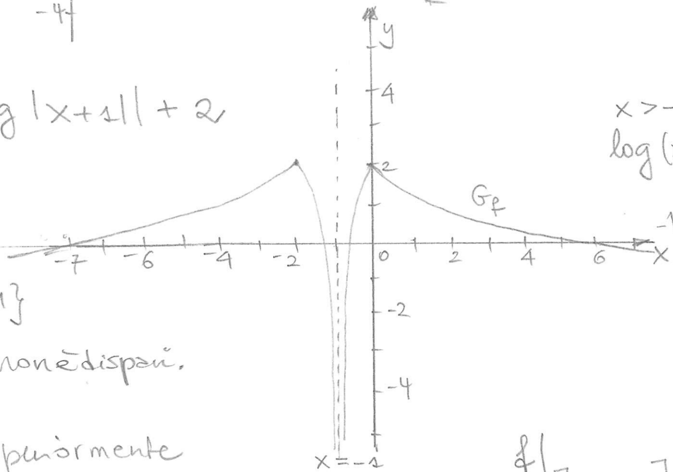
$-1 < x < 0$
$\log(x+1) = -2 \iff x = \frac{1}{e^2} - 1$



• $\operatorname{dom} f = \mathbb{R} \setminus \{-1\}$

• f non è pari, non è dispari.

• limitata superiormente

($f(x) \leq 2 \quad \forall x \in \mathbb{R} \setminus \{-1\}$)

• $\inf_{\mathbb{R}\setminus\{-1\}} f = -\infty$; $\quad \sup_{\mathbb{R}\setminus\{-1\}} f = \max_{\mathbb{R}\setminus\{-1\}} f = 2$.

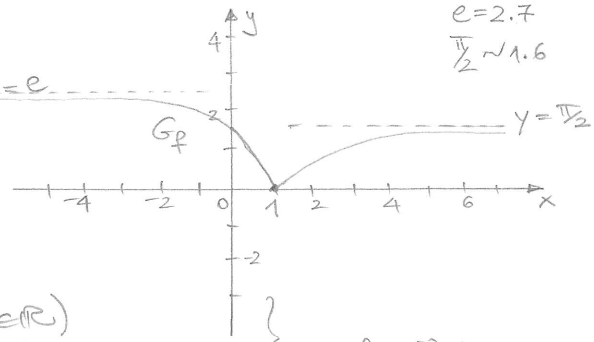
$f|_{]-\infty,-2]}$ ↗ $\quad f|_{[-2,-1[}$ ↘

$f|_{]-1,0]}$ ↗ $\quad f|_{[0,+\infty[}$ ↘ .

□

iii) $f(x) = \begin{cases} -e^x + e & \text{se } x < 1 \\ \operatorname{arctg}(x-1) & \text{se } x \geq 1 \end{cases}$

$e = 2.7$
$\frac{\pi}{2} \sim 1.6$



• nessuna simmetria

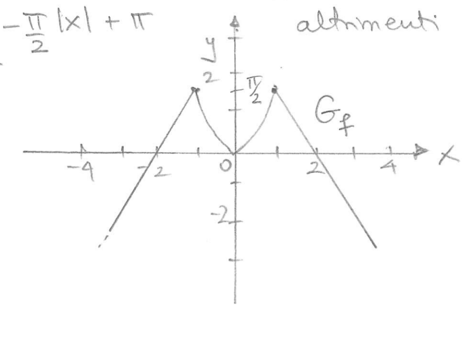
• f limitata inferiorm. ($f(x) \geq 0\ \forall x \in \mathbb{R}$)
limitata superiormente ($f(x) \leq e \quad \forall x \in \mathbb{R}$) $\Big\}$ ⇒ f è limitata.

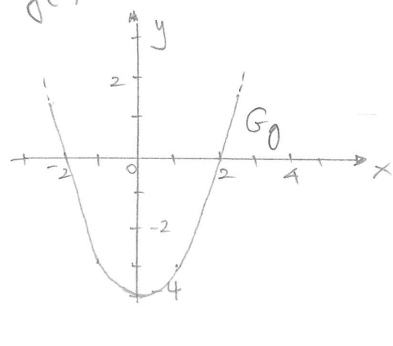
• $f|_{]-\infty,1]}$ ↘ $\quad f|_{[1,+\infty[}$ ↗ .

• $\inf_{\mathbb{R}} f = \min_{\mathbb{R}} f = 0$; $\quad \sup_{\mathbb{R}} f = e$.

1.4) $f(x) = \begin{cases} |\arcsin x| & \text{se } -1 \leq x \leq 1 \\ -\frac{\pi}{2}|x| + \pi & \text{altrimenti} \end{cases}$ $\qquad g(x) = x^2 - 4$

a)





b) $$(g\circ f)(x)=\begin{cases}(\arcsin x)^2-4 & -1\le x\le 1\\ \left(-\frac{\pi}{2}|x|+\pi\right)^2-4 & \text{altrimenti}\end{cases}.$$

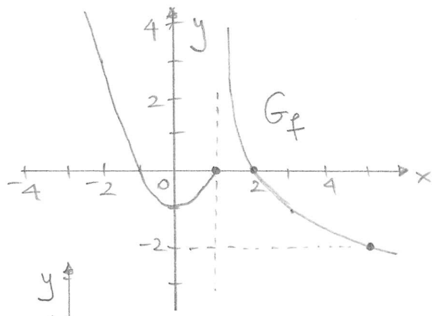
$$(f\circ g)(x)=\begin{cases}|\arcsin g(x)| & \text{se } -1\le g(x)\le 1\\ -\frac{\pi}{2}|g(x)|+\pi & \text{altrimenti}\end{cases}$$

Ora $-1\le \underbrace{x^2-4}_{g(x)}\le 1 \iff 3\le x^2\le 5 \iff x\in[-\sqrt5,-\sqrt3]\cup[\sqrt3,\sqrt5]$

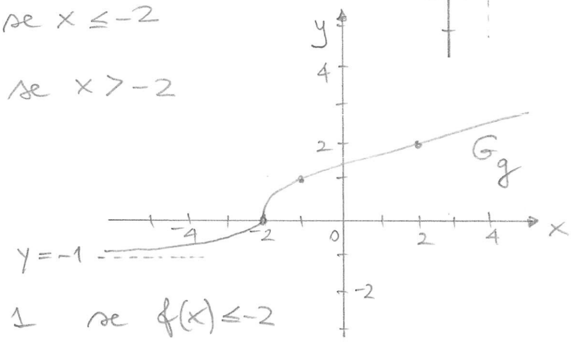
Quindi $$(f\circ g)(x)=\begin{cases}|\arcsin(x^2-4)| & \text{se } x\in[-\sqrt5,-\sqrt3]\cup[\sqrt3,\sqrt5]\\ -\frac{\pi}{2}|x^2-4|+\pi & \text{altrimenti.}\end{cases}$$

$\blacksquare$

1.5) $$f(x)=\begin{cases}x^2-1 & \text{se } x\le 1\\ \log_{1/2}(x-1) & \text{se } x>1\end{cases}$$



$$g(x)=\begin{cases}e^{x+2}-1 & \text{se } x\le -2\\ \sqrt{x+2} & \text{se } x>-2\end{cases}$$



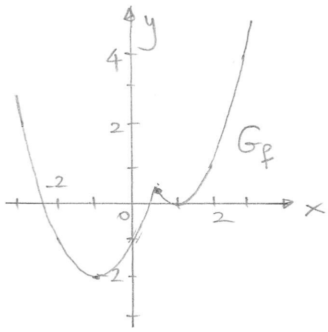
$$(g\circ f)(x)=\begin{cases}e^{f(x)+2}-1 & \text{se } f(x)\le -2\\ \sqrt{f(x)+2} & \text{se } f(x)>-2\end{cases}$$

$$=\begin{cases}e^{\log_{1/2}(x-1)+2}-1 & \text{se } x\geqslant 5\\ \sqrt{\log_{1/2}(x-1)+2} & \text{se } 1<x<5\\ \sqrt{(x^2-1)+2} & \text{se } x\le 1\end{cases}$$

$$(f\circ g)(x)=\begin{cases}g^2(x)-1 & \text{se } g(x)\le 1\\ \log_{1/2}(g(x)-1) & \text{se } g(x)>1.\end{cases}=\begin{cases}\left(e^{x+2}-1\right)^2-1 & \text{se } x\le -2\\ \left(\sqrt{x+2}\right)^2-1 & -2<x\le -1\\ \log_{1/2}\left(\sqrt{x+2}-1\right) & \text{se } x>-1\end{cases}$$

$\blacksquare$

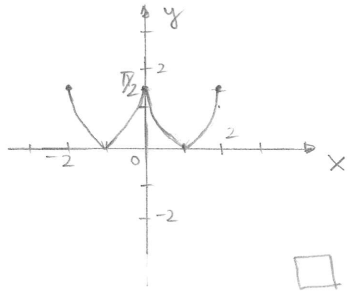
1.6) i) $f(x) = x^2 - |2x-1| = \begin{cases} x^2-2x+1 = (x-1)^2 & \text{se } x \geq \frac{1}{2} \\ x^2+2x-1 = (x+1)^2 - 2 & \text{se } x < \frac{1}{2} \end{cases}$



$g(x) = \left|\arccos(|x|-1) - \frac{\pi}{2}\right|$ funzione pari

$-1 \leq |x| - 1 \leq 1$

$0 \leq |x| \leq 2 \iff -2 \leq x \leq 2$



□

ii) se $k < -2$ $\nexists$ sono soluzioni di $f(x) = k$

$k = -2$ $\exists$ una soluzione

$-2 < k < 0$ oppure $\frac{1}{4} < k$ $\exists$ sono 2 soluzioni dell'eq. $f(x) = k$

$k = 0$ oppure $k = \frac{1}{4}$ $\exists$ sono 3 "

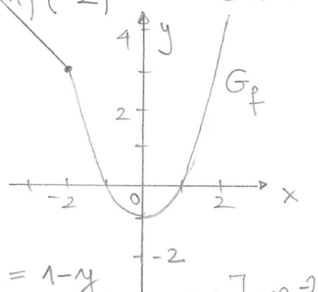
$0 < k < \frac{1}{4}$ $\exists$ sono 4 soluzioni dell'eq. $f(x) = k$. □

iii) $\min\limits_{[-2,2]} g = 0$ $x = -1, 1$ pt. di minimo.

$\max\limits_{[-2,2]} g = \frac{\pi}{2}$ $x = -2, 0, 2$ pt. di massimo. ■

1.7) i) $f(x) = \frac{1}{2}(x-1)(x+|x+2|) = \begin{cases} \frac{1}{2}(x-1)(2x+2) & \text{se } x \geq -2 \\ \frac{1}{2}(x-1)(-2) & \text{se } x < -2 \end{cases}$

$= \begin{cases} x^2 - 1 & \text{se } x \geq -2 \\ -x+1 & \text{se } x < -2 \end{cases}$



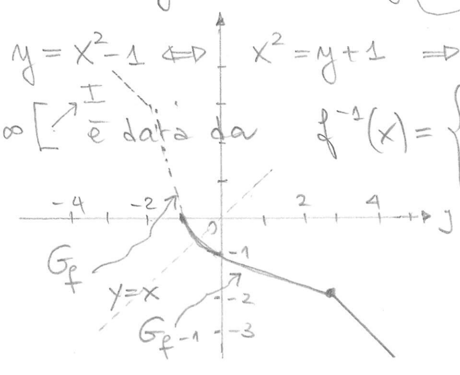
$f(I) = [-1, +\infty[$

ii) $I =]-\infty, 0]$, $f|_I$ è iniettiva.

$\forall y > 3$ si ha $-x+1 = y \iff x = 1-y$ $\to x \in]-\infty, 2[$

$\forall -1 \leq y \leq 3$ si ha $y = x^2 - 1 \iff x^2 = y+1 \Rightarrow x = -\sqrt{y+1}$ (poiché $x \in I$!) $x \in [-2, 0]$

Quindi $f^{-1} : [-1, +\infty[\to I$ è data da $f^{-1}(x) = \begin{cases} -\sqrt{x+1} & \text{se } -1 \leq x \leq 3 \\ 1 - x & \text{se } x > 3. \end{cases}$



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