Immagine e Funzione Inversa

Es. 1: $f(x) = e^{2x} + e^{x} \quad : \mathbb{R} \to \mathbb{R}$

a) f è iniettiva $\quad \left(\begin{array}{l} e^{2x} \text{ strett. cresc.} \\ e^{x} \text{ strett. cresc.} \end{array} \right\} \Rightarrow f(x)$ strett. cresc. $\Rightarrow f$ è iniettiva.

b) det. l'immagine di f (im f)

c) det. $f^{-1} : \text{im} f \to \mathbb{R}$

b) Vogliamo det. per quali $y \in \mathbb{R}$, l'eq. $f(x) = y$ ha almeno una soluz. in $\mathbb{R}$

$$e^{2x} + e^{x} = y \iff e^{2x} + e^{x} - y = 0$$

$$\underset{e^{x} = t \ (>0)}{\iff} t^2 + t - y = 0$$

$$t_{1/2} = \frac{-1 \pm \sqrt{1+4y}}{2}$$

deve essere:

altrimenti $\nexists$ soluz. di $t^2 + t - y = 0$ $\leadsto$

$t > 0$ perché $t = e^x$

$$\begin{cases} 1 + 4y \geq 0 \\ \dfrac{-1 + \sqrt{1+4y}}{2} > 0 \end{cases} \iff$$

$$\iff \begin{cases} y \geq -1/4 \\ \underbrace{\sqrt{1+4y} > 1} \end{cases} \iff \begin{cases} y \geq -1/4 \\ y > 0 . \end{cases}$$

$$\left[\begin{array}{l} \iff \cancel{1} + 4y > \cancel{1} \\ \iff y > 0 \end{array} \right]$$

Quindi $\forall y>0$ sappiamo risolvere l'eq. e otteniamo

$$e^x = \frac{-1+\sqrt{1+4y}}{2}$$

ossia

$$\underbrace{x = \log\left(-\frac{1}{2}+\frac{\sqrt{1+4y}}{2}\right)}_{x=f^{-1}(y)}$$

scambio $y \longleftrightarrow x$

$\Rightarrow \operatorname{im} f =]0,+\infty[$ e

la funzione inversa

$$f^{-1}(x) = \log\left(-\frac{1}{2}+\frac{\sqrt{1+4x}}{2}\right), \quad x>0.$$

$\blacksquare$

Funzione composte di funz. definite a tratti

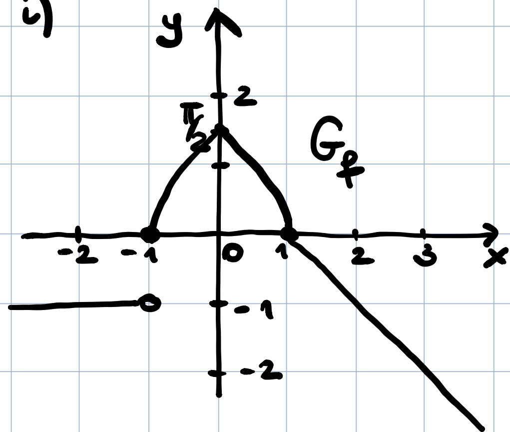
Es.2: $f: \mathbb{R} \to \mathbb{R}$ $\qquad$ $g: \mathbb{R} \to \mathbb{R}$

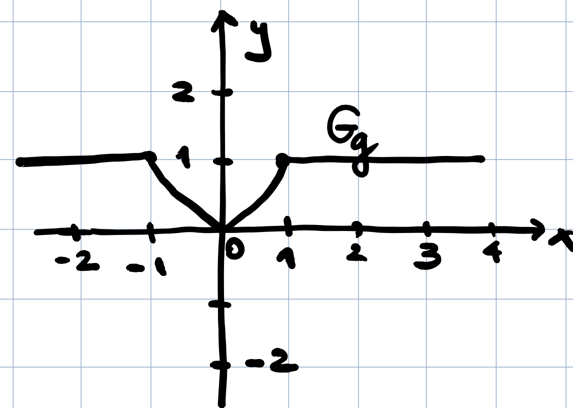
$$f(x) = \begin{cases} -1 & \text{se } x<-1 \\ \arccos|x| & \text{se } |x|\leq 1 \\ -x+1 & \text{se } x>1 \end{cases}$$

$$g(x) = \begin{cases} \frac{2}{\pi}|\arcsin x| & \text{se } |x|\leq 1 \\ 1 & \text{se } |x|>1 \end{cases}$$

i) Rappresentare G_f, G_g $\qquad$ ii) determinare $(f\circ g)$.

i)





$\operatorname{im} f = \left]-\infty, \frac{\pi}{2}\right]$ $\qquad$ $\operatorname{im} g = [0, 1]$

ii) nessun pbm. sul $\operatorname{dom}(f \circ g) : \mathbb{R} \to \mathbb{R}$

$$(f \circ g)(x) = f(g(x)) = \begin{cases} -1 & \cancel{\text{se } g(x) < -1} \\ \arccos|g(x)| & \text{se } |g(x)| \leq 1 \\ -g(x)+1 & \cancel{\text{se } g(x) > 1} \end{cases}$$

MAI !!

$$= \begin{cases} \arccos 1 & \text{se } x < -1 \text{ o } x > 1 \\ \arccos\left|\frac{2}{\pi}|\arcsin x|\right| & \text{se } -1 \leq x \leq 1 \end{cases}$$

$$= \begin{cases} 0 & \text{se } x < -1 \text{ o } x > 1 \\ \arccos\left[\frac{2}{\pi}|\arcsin x|\right] & \text{se } -1 \leq x \leq 1 \end{cases}$$

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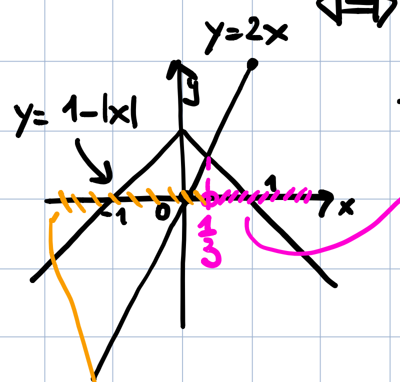
Disequazioni con funzione arcoseno/arcocoseno

Es. Risolvere la diseq.

ⓐ $\boxed{\arcsin(1-|x|) < \arcsin 2x}$



$$\iff \begin{cases} \left.\begin{array}{l} -1 \le 1-|x| \le 1 \\ -1 \le 2x \le 1 \end{array}\right\} & \text{(dominio!)} \\ 1-|x| < 2x & (\arcsin x \uparrow) \end{cases}$$

$1-x=2x$

$\iff x = 1/3$

$$\begin{cases} \overset{\text{sempre vero}}{0 \le |x| \le 2} \\ -1/2 \le x \le 1/2 \\ x > 1/3 \end{cases} \iff \begin{cases} -2 \le x \le 2 \\ -1/2 \le x \le 1/2 \end{cases}$$

$$\Longrightarrow \quad S = \left]1/3, \tfrac{1}{2}\right]. \quad \square$$

ⓑ $\boxed{\arccos(1-|x|) < \arccos 2x}$

$$\iff \begin{cases} \left.\begin{array}{l} -1 \le 1-|x| \le 1 \\ -1 \le 2x \le 1 \end{array}\right\} & \text{(dominio!)} \\ 1-|x| > 2x & (\arccos x \searrow) \end{cases}$$

$$\Longrightarrow \quad S = \left[-1/2, \tfrac{1}{3}\right[\quad \blacksquare$$