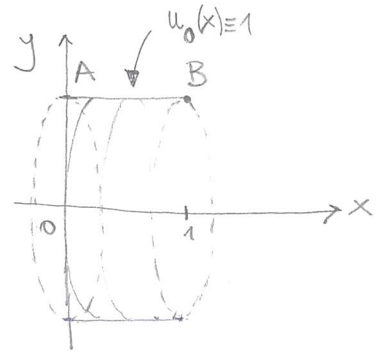
Commento alla lez. del 13/03/2019 (vedi anche [catenarie per A=(0,1) e B=(1,1).pdf])

"Confrontiamo $S(u_0)$ con $S(u_2)$, dove $\boxed{u_0(x)\equiv 1 \text{ su } [0,1]}$, $\boxed{u_2(x)= \frac{\cosh(x\cosh d_2+d_2)}{\cosh d_2} \text{ su } [0,1]}$

dove $\boxed{u_2(x)}$ è la catenaria passante da $A=(0,1)$ e $B=(1,1)$" con $-0.6<d_2<-0.5$



$$\boxed{S(u_0)} = 2\pi\int_0^1 u_1(x)\sqrt{1+u_1'^2(x)}\,dx = 2\pi\int_0^1 dx = \boxed{2\pi}$$

$$S(u) = 2\pi\int_0^1 u(x)\sqrt{1+u'^2(x)}\,dx = \frac{2\pi}{\cosh d}\int_0^1 \cosh^2(x\cosh d+d)\,dx$$

$\boxed{u(x)=\frac{\cosh(x\cosh d+d)}{\cosh d} \quad \text{catenaria passante per } A=(0,1)}$

$\boxed{\text{Ricordo che } \cosh^2 x-\sinh^2 x=1 \quad \forall x\in\mathbb{R}}$

$$u'(x)=\sinh(x\cosh d+d)$$

$$\sqrt{1+u'^2}=\sqrt{1+\sinh^2(x\cosh d+d)} = \cosh(x\cosh d+d)$$

Notiamo che

$$\int\cosh^2(x\cosh d+d)\,dx = \cosh(x\cosh d+d)\frac{\sinh(x\cosh d+d)}{\cosh d}-\int\sinh^2(x\cosh d+d)\,dx$$

$$= \cosh(x\cosh d+d)\frac{\sinh(x\cosh d+d)}{\cosh d}-\int\left[\cosh^2(x\cosh d+d)-1\right]dx$$

$$\Rightarrow \int\cosh^2(x\cosh d+d)\,dx = \frac{\cosh(x\cosh d+d)\sinh(x\cosh d+d)}{2\cosh d}+\frac{x}{2}+c$$

Quindi

$$S(u)=\frac{2\pi}{\cosh d}\left[\frac{\cosh(x\cosh d+d)\sinh(x\cosh d+d)}{2\cosh d}+\frac{x}{2}\right]_0^1$$

$$=\frac{\pi}{\cosh d}\left[\frac{\cosh(\cosh d+d)\sinh(\cosh d+d)}{\cosh d}+1-\frac{\cancel{\cosh d}\sinh d}{\cancel{\cosh d}}\right]$$

Consideriamo ora $u_2(x) = \dfrac{\cosh(x\cosh d_2 + d_2)}{\cosh d_2}$ (*) e si ha

$$S(u_2) = \frac{\pi}{\cosh d_2}\Bigg[\underbrace{\frac{\cosh(\cosh d_2 + d_2)}{\cosh d_2}}_{1}\sinh(\cosh d_2 + d_2) + 1 - \sinh d_2\Bigg]$$

$(u_2(1)=1 \;!!)$

$$= \pi\left[\frac{\sinh(\cosh d_2 + d_2) - \sinh d_2 + 1}{\cosh d_2}\right]$$

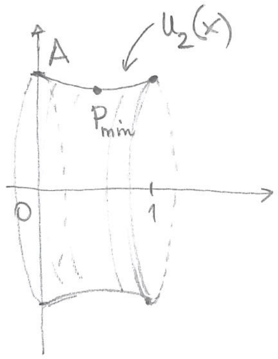
Poniamo $h(d) \doteq \dfrac{\sinh(\cosh d + d) - \sinh d + 1}{\cosh d}$

e quindi $\boxed{S(u_2) = \pi h(d_2)}$

(*) [Ricordiamo che $u(x) = \dfrac{\cosh(x\cosh d + d)}{\cosh d}$ catenaria passante da $A=(0,1)$ passa per $B=(1,1)$ se $1 = u(1) = \dfrac{\cosh(\cosh + d)}{\cosh d} \doteq g(d)$

Dalla rappresentazione grafica in $①_D$ possiamo dedurre che esiste $-0.6 < d_2 < -0.5$ ($\bar{e}$ il d_2 per cui $u_2(x)$ risulterà essere punto di minimo) tale che $g(d_2)=1$]. Per tale d_2 si ha $\boxed{h(d_2) < 2}$ (rappresentazione grafica in $②_D$).

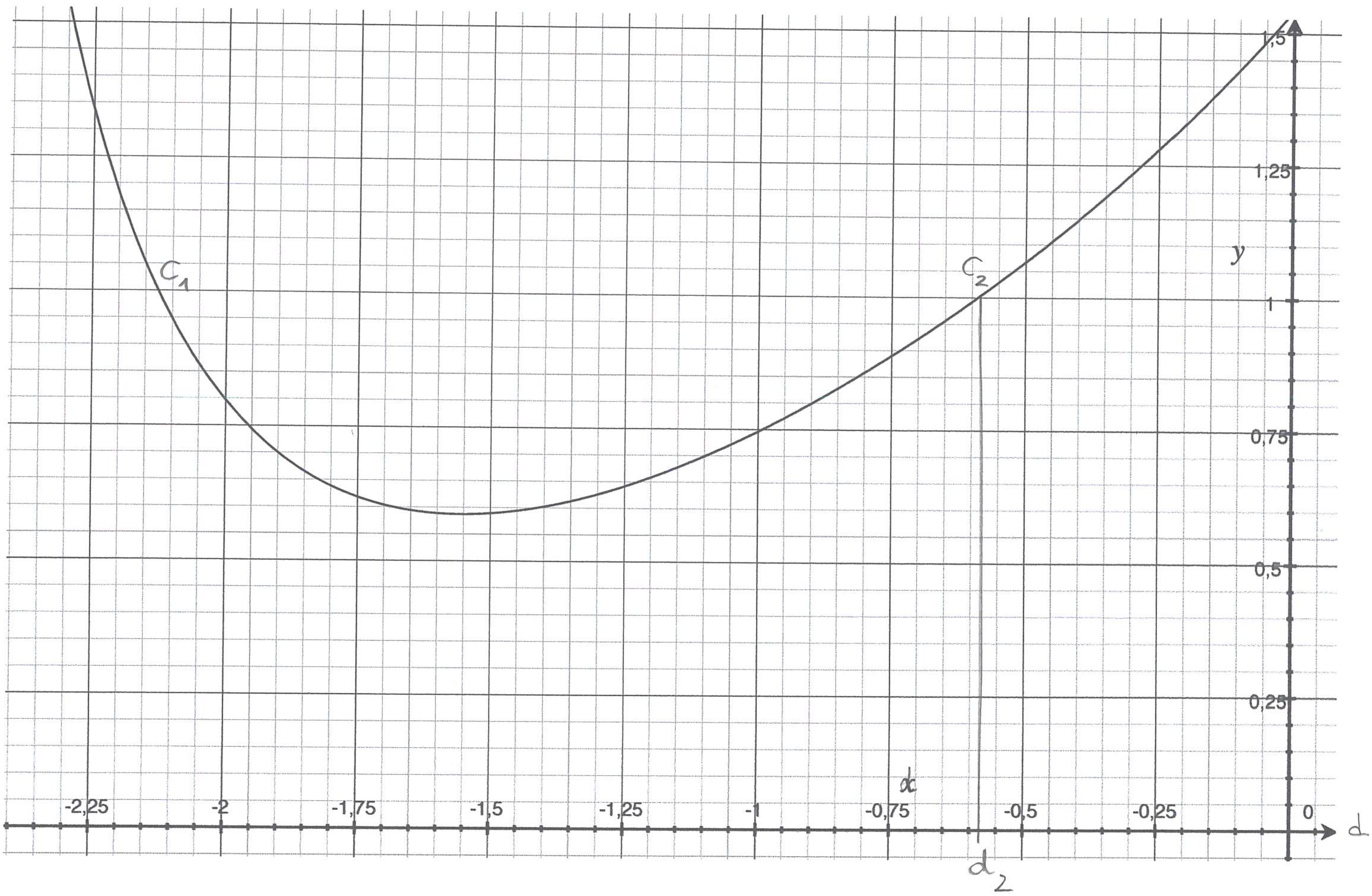
Quindi per $u_2(x) = \dfrac{\cosh(x\cosh d_2 + d_2)}{\cosh d_2}$ si ha $S(u_2) = \pi h(d_2) < 2\pi = S(u_0)$. □



$P_{min} = \left(\dfrac{-d_2}{\cosh d_2}, \dfrac{1}{\cosh d_2}\right)$ pt di minimo di $u_2(x)$

$\sim (0.5,\ 0.8)$

$$g(d) = \frac{\cosh(\cosh d + d)}{\cosh d}$$



②D

$$h(d) = \frac{\sinh(\cosh d + d) - \sinh d + 1}{\cosh d}$$