Università degli Studi di Trento – Facoltà di Scienze Cognitive – CdL in STPCA
Esame scritto di ANALISI MATEMATICA – Rovereto, 10 settembre 2008

FILA (A)

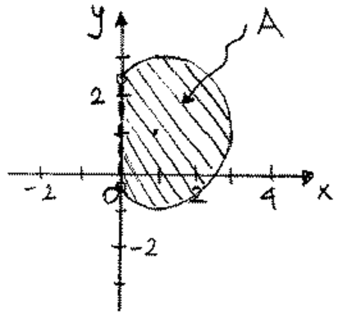
1) i)

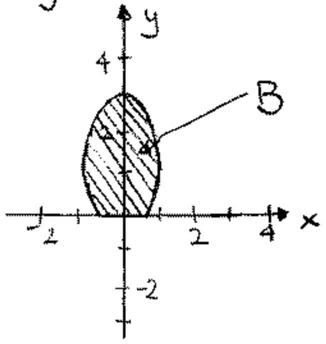
$$A=\left\{(x,y)\in\mathbb{R}^2 : x>0,\ (x-1)^2+(y-1)^2\le 4\right\}$$

$$x^2-2x+y^2-2y-2\le 0$$
$$(x-1)^2-1+(y-1)^2-1-2\le 0$$

$$4x^2+y^2-2y-3\le 0$$
$$4x^2+(y-1)^2-1-3\le 0$$
$$x^2+\frac{(y-1)^2}{4}\le 1$$

$$B=\left\{(x,y)\in\mathbb{R}^2 : y\ge 0,\ x^2+\left(\frac{y-1}{4}\right)^2\le 1\right\}$$





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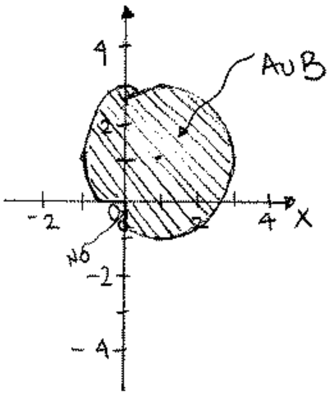
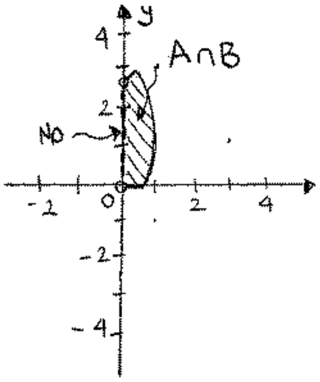
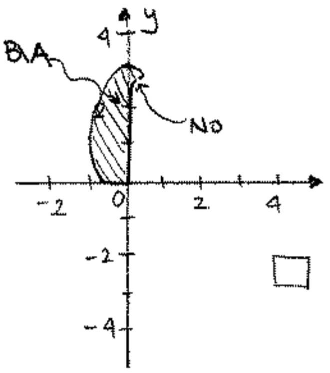
ii)







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iii) per esempio $A\cup B\subset B_4(0)$ (ma ci sono anche $r<4$ che vanno bene; sicuramente tutti gli $r>4$ vanno bene).

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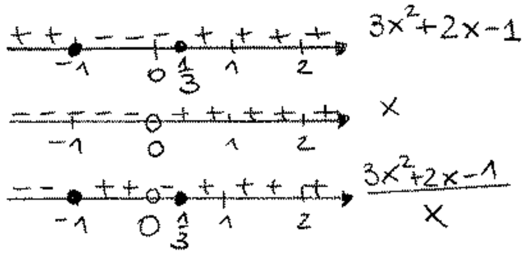
2) a) $\frac{1}{x}\le 3x+2 \iff \frac{1}{x}-3x-2\le 0 \iff$

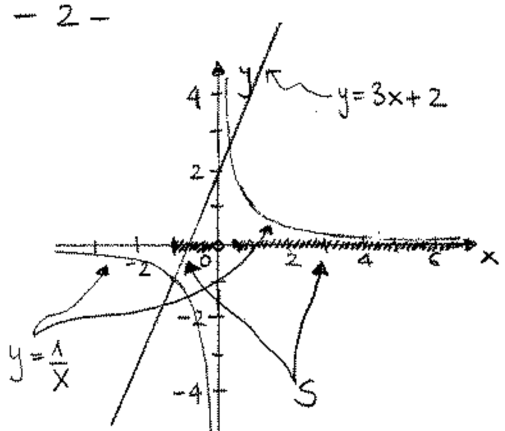
$$\iff \frac{-3x^2-2x+1}{x}\le 0$$

$$\iff \frac{3x^2+2x-1}{x}\ge 0$$

$$\Rightarrow S=[-1,0[\ \cup\ [\tfrac{1}{3},+\infty[\ .$$



Interpretazione geometrica:



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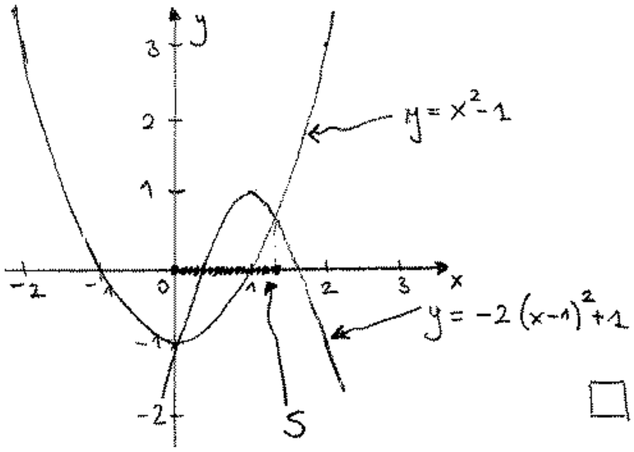
b) $x^2-1 \le -2(x-1)^2+1 \iff x^2-1 \le -2x^2+4x-2+1$

$\iff x^2-\cancel{1} \le -2x^2+4x-\cancel{1}$

$\iff 3x^2-4x \le 0 \iff x(3x-4)\le 0$

$\Rightarrow \underline{\underline{S=\left[0,\frac{4}{3}\right]}}.$



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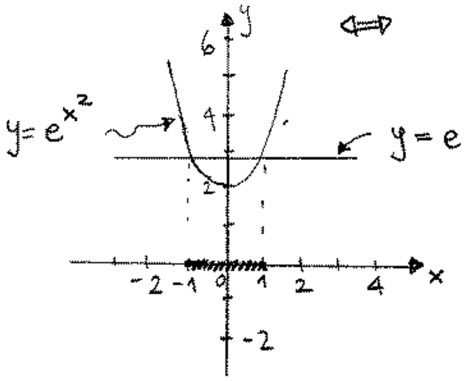
c) $e^{x^2} \le e \iff x^2 \le 1$

$\iff -1 \le x \le 1 \Rightarrow \underline{\underline{S=[-1,1]}}.$



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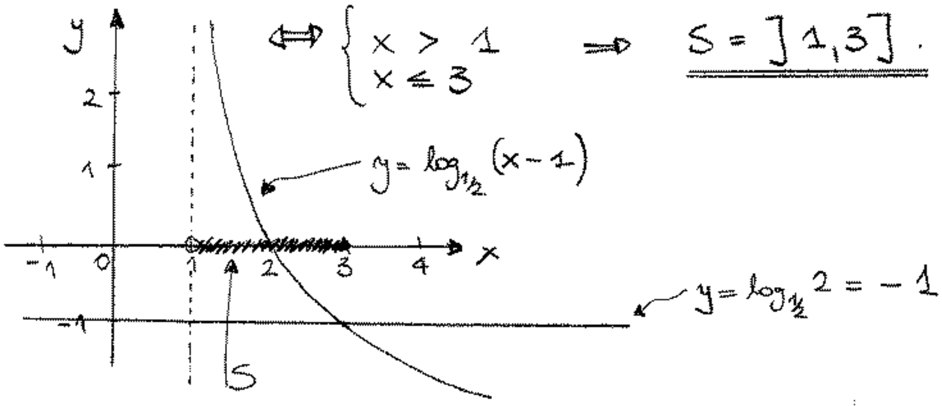
d) $\log_{\frac{1}{2}}(x-1) \geqslant \log_{\frac{1}{2}} 2 \iff \begin{cases} x-1>0 \\ x-1 \le 2 \end{cases}$ (altrimenti non è def. log) ($\log_{\frac{1}{2}} x$ è decrescente)

$\iff \begin{cases} x > 1 \\ x \le 3 \end{cases} \Rightarrow \underline{\underline{S=]1,3]}}.$



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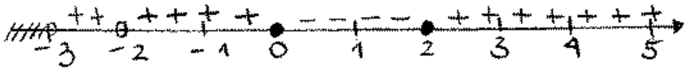
3) i) $\lim_{x\to -2^-} f(x) = 1$ $\quad \lim_{x\to -2^+} f(x) = -\infty$ $\quad \lim_{x\to +\infty} f(x) = 1$

Si ha che $x=-2$ è un asintoto verticale, mentre $y=1$ è un asintoto orizzontale. ☐

ii) f ha un pt. di discontinuità in $x=-2$; infatti $\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x) \neq f(-2)$. ☐

iii) Segno di $f'(x)$



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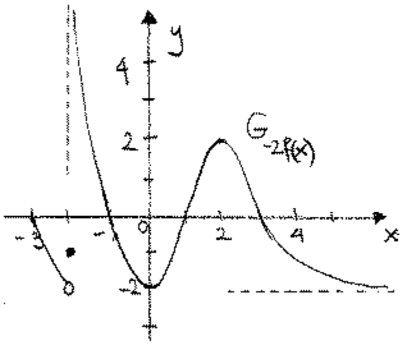
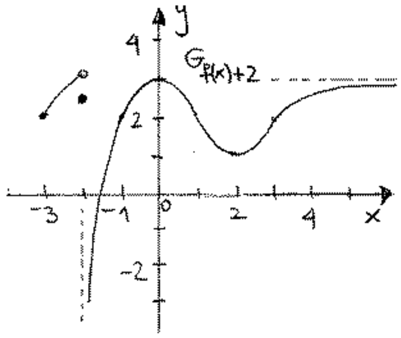
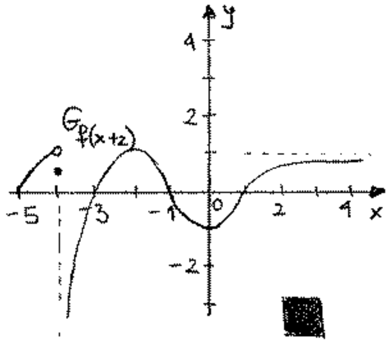
iv) $\text{dom}\left(-2f(x)\right) = [-3,+\infty[$ $\quad \text{dom}\left(f(x)+2\right) = [-3,+\infty[$ $\quad \text{dom}\left(f(x+2)\right) = [-5,+\infty[$







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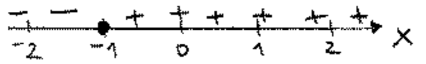
4) i)

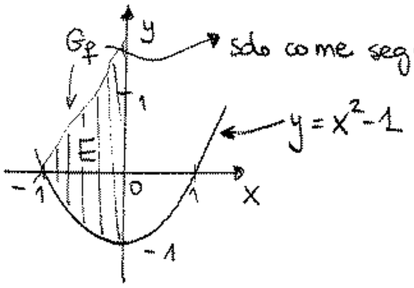


Oss. che $e^{2x^2+4x} > 0 \quad \forall x \in \mathbb{R}$, quindi il segno di f è determinato dal segno di $4x+4$. ☐

ii) $F(x) = e^{2x^2+4x}$ è derivabile su tutto $\mathbb{R}$ (composizione di funzioni derivabili) e $F'(x) = (4x+4)\,e^{2x^2+4x} = f(x) \quad \forall x \in \mathbb{R}$.

Quindi $F(x)$ è una primitiva di $f(x)$ su tutto $\mathbb{R}$. ☐

iii)



solo come segno!! cioè $f(x)$ è una funzione positiva per $x > -1$.

Quindi

$$\text{area}(E) = \int_{-1}^{0} \left(f(x) - x^2 + 1\right) dx =$$

$$= \Big[F(x)\Big]_{-1}^{0} + \left[-\frac{x^3}{3} + x\right]_{-1}^{0} =$$

$$= \Big[e^{2x^2+4x}\Big]_{-1}^{0} - \left[\frac{1}{3} - 1\right] = \left(1 - e^{-2}\right) + \frac{2}{3}$$

$$= \underline{\underline{\frac{5}{3} - \frac{1}{e^2}}}.$$

▨

5) i) $f(x)=\log\left(\frac{x^2}{x^2+2}\right)$

- dom $f=\mathbb{R}\setminus\{0\}$ (osserviamo che f è una funzione pari su $\mathbb{R}\setminus\{0\}$ i.e. $f(x)=f(-x)\ \forall x\in\mathbb{R}\setminus\{0\}$, quindi basterebbe studiare la funzione f su $]0,+\infty[$ oppure su $]-\infty,0[$).

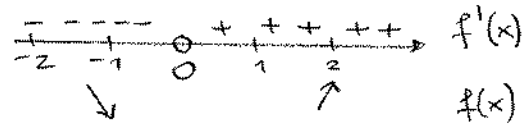
- segno : notiamo che $\frac{x^2}{x^2+2}<1\quad\forall x\in\mathbb{R}\setminus\{0\}$ e quindi $f(x)$ è negativa su $\mathbb{R}\setminus\{0\}$.

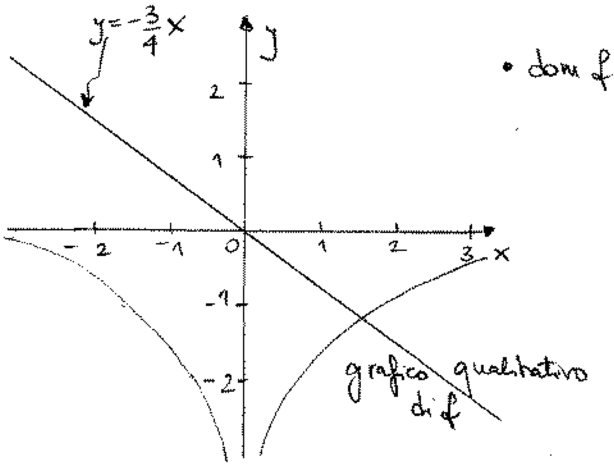
- $\lim_{x\to-\infty}f(x)=0$; $\lim_{x\to0^-}f(x)=-\infty$; $\lim_{x\to0^+}f(x)=-\infty$; $\lim_{x\to+\infty}f(x)=0$.

$\Rightarrow$ $x=0$ asintoto verticale
  $y=0$ asintoto orizzontale (sia per $x\to-\infty$, che per $x\to+\infty$).

- dom $f'=\mathbb{R}\setminus\{0\}$

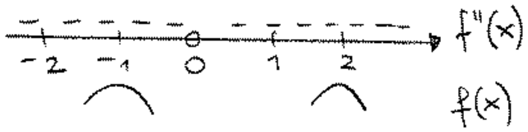
$$f'(x)=\frac{1}{\frac{x^2}{x^2+2}}\cdot\frac{2x(x^2+2)-x^2(2x)}{(x^2+2)^2}$$
$$=\frac{\cancel{x^2+2}}{x^{\cancel{2}}}\cdot\frac{4\cancel{x}}{(x^2+2)^{\cancel{2}}}$$
$$=\frac{4}{x(x^2+2)}$$





- dom $f''=\mathbb{R}\setminus\{0\}$

$$f''(x)=\frac{-4\left[(x^2+2)+x\cdot2x\right]}{x^2(x^2+2)^2}$$
$$=\frac{-4\left[3x^2+2\right]}{x^2(x^2+2)^2}$$



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ii) $y=f(1)+f'(1)(x-1)$ $\Rightarrow$ $y=\log\left(\frac{1}{3}\right)+\frac{4}{3}(x-1)$

$\Rightarrow$ $\underline{\underline{y=\frac{4}{3}x-\frac{4}{3}-\log3.}}$ ☐

iii) $P=\left(2,\frac{4}{3}-\log3\right)$ $\in$ alla retta tangente determinata in ii) se soddisfa l'eq. della retta tg. OK! appartiene ☐

iv) $\underline{y=-\frac{3}{4}x}$ è la retta perpendicolare alla retta tg. in ii) e pass. per (0,0). ▨

6) $\underline{\underline{C_{13,2}\cdot C_{11,3}\cdot C_{8,3}\cdot C_{5,3}}}$ (oppure $C_{13,2}\cdot C_{11,2}\cdot C_{9,3}\cdot C_{6,3}$). ▨