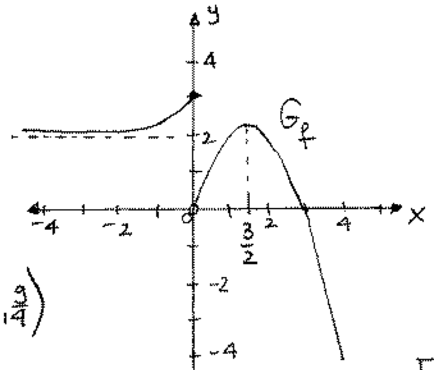
Sesta verifica settimanale di <u>ANALISI MATEMATICA</u> – CdL in STPCA –
a.a. 2007/08 – Rovereto, 22 – 26 ottobre 2007

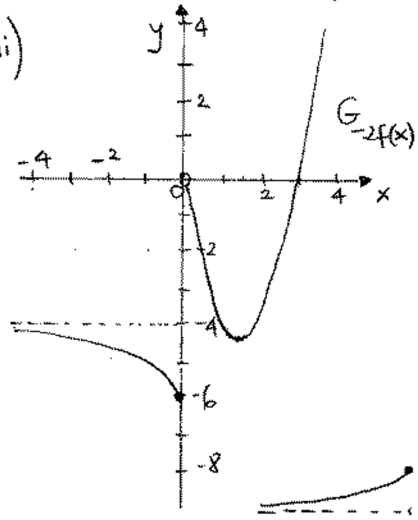
1) i)
$$f(x) = \begin{cases} \dfrac{1}{(x-1)^2} + 2 & \text{se } x \leq 0 \\ -x^2+3x & \text{se } x > 0 \end{cases}$$

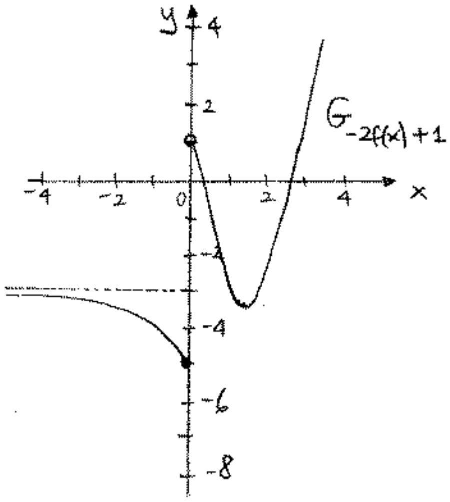


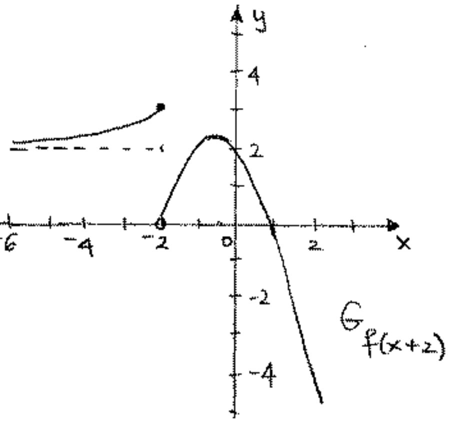
Oss. che $-x^2+3x = -(x^2-3x) =$
$= -\left(x-\frac{3}{2}\right)^2 + \frac{9}{4} \qquad V=\left(\frac{3}{2},\frac{9}{4}\right)$

ii) se $k \leq 0$ $\exists!$ soluzione di $f(x)=k$;
se $0<k\leq 2$ $\exists$ono 2 soluzioni dell'eq. $f(x)=k$;
se $2<k<\frac{9}{4}$ $\exists$ono 3 soluzioni dell'eq. " ;
se $k=\frac{9}{4}$ $\exists$ono 2 soluzioni dell'eq. " ;
se $\frac{9}{4}<k\leq 3$ $\exists!$ soluzione dell'eq. $f(x)=k$;
se $k>3$ $\nexists$ono soluzioni.

iii)

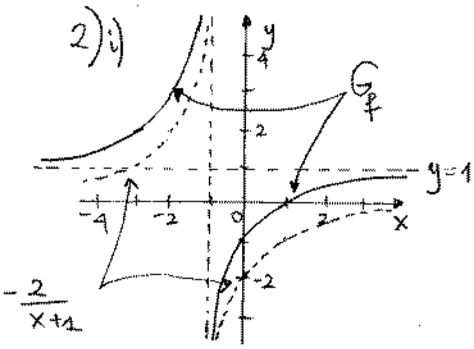


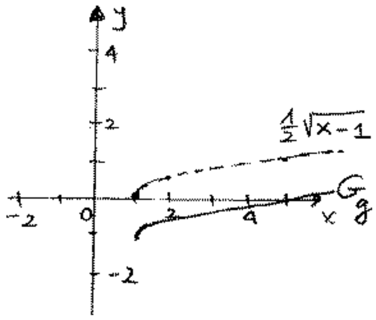


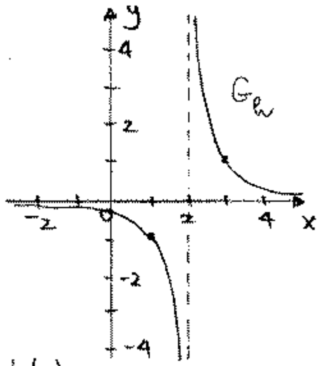


iv) $f(\mathbb{R}) = \underline{\underline{]-\infty, 3]}}$.

2) i)







$f(x) = -\frac{2}{x+1} + 1$ $\qquad$ $g(x) = \frac{1}{2}\sqrt{x-1} - 1$ $\qquad$ $h(x) = \frac{1}{(x-2)^3}$

$\mathrm{dom}\, f = \mathbb{R}\setminus\{-1\}$ $\qquad$ $\mathrm{dom}\, g = [1,+\infty[$ $\qquad$ $\mathrm{dom}\, h = \mathbb{R}\setminus\{2\}$

ii) f non è limitata; g è limitata inferiormente $\left(g(x) \geq -1 \ \forall x \in [1,+\infty[\right)$,
h non è limitata. □

iii) $\min\limits_{[3,4]} f = f(3) = \frac{1}{2}$ $\quad$ $x=3$ pt. di minimo; $\quad$ $\max\limits_{[3,4]} f = f(4) = \frac{3}{5}$ $\quad$ $x=4$ pt. di massimo;

$\min\limits_{[3,4]} g = g(3) = \frac{\sqrt{2}}{2} - 1$ $\quad$ $x=3$ pt. di minimo; $\quad$ $\max\limits_{[3,4]} g = g(4) = \frac{\sqrt{3}}{2} - 1$ $\quad$ $x=4$ pt. di massimo;

$\min\limits_{[3,4]} h = h(4) = \frac{1}{8}$ $\quad$ $x=4$ pt. di minimo; $\quad$ $\max\limits_{[3,4]} h = h(3) = 1$ $\quad$ $x=3$ pt. di massimo.

■

3) i) $f(x) = \sqrt[3]{x-1}$ $\quad$ $\mathrm{dom}\, f = \mathbb{R}$ $\quad$ $f(\mathbb{R}) = \mathbb{R}$;

$g(x) = (x-2)^3$ $\quad$ $\mathrm{dom}\, g = \mathbb{R}$ $\quad$ $g(\mathbb{R}) = \mathbb{R}$:

$$(g \circ f)(x) \underset{\substack{\uparrow \\ x\in\mathbb{R}}}{=} g(f(x)) = \left(\sqrt[3]{x-1} - 2\right)^3 \qquad (f \circ g)(x) \underset{\substack{\uparrow \\ x\in\mathbb{R}}}{=} f(g(x)) = \sqrt[3]{(x-2)^3 - 1}$$

□

ii) $f(x) = \sqrt{x}$ $\quad$ $\mathrm{dom}\, f = [0,+\infty[$ $\quad$ $f([0,+\infty[) = [0,+\infty[$;

$g(x) = -x^2 + 1$ $\quad$ $\mathrm{dom}\, g = \mathbb{R}$ $\quad$ $g(\mathbb{R}) =]-\infty, 1]$:

$$(g \circ f)(x) \underset{\substack{\uparrow \\ x\in[0,+\infty[}}{=} g(f(x)) = -(\sqrt{x})^2 + 1 = -x + 1$$

$$(f \circ g)(x) \underset{\substack{\uparrow \\ \{x:\, g(x) \geq 0\} \\ = \{x:\, -x^2+1 \geq 0\} = [-1,1]}}{=} f(g(x)) \underset{\uparrow}{=} \sqrt{-x^2 + 1}$$

□

b) i) $(f+g)(2) = \sqrt[3]{2-1} + (2-2)^3 = \underline{\underline{1}}$;

$(fg)(-2) = \left(\sqrt[3]{-2-1}\right)\cdot(-2-2)^3 = +\sqrt[3]{3}\cdot 4^3 = \underline{\underline{64\sqrt[3]{3}}}$;

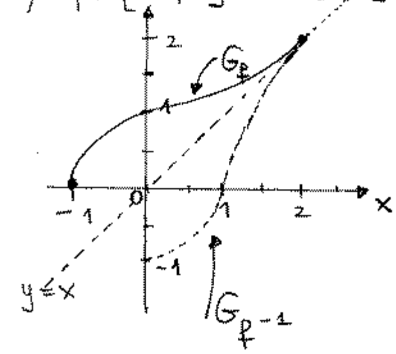
$\left(\frac{g}{f}\right)(1) \nexists$ poiché $f(1) = 0$.

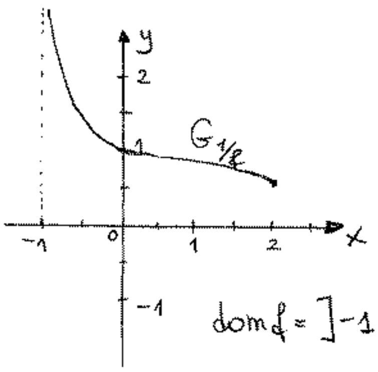
ii) $(f+g)(2) = \sqrt{2} - 4 + 1 = \underline{\underline{\sqrt{2}-3}}$;

$(fg)(-2) \nexists$ poiché $f(-2)$ non è definito ;

$\left(\frac{g}{f}\right)(1) = \frac{-1+1}{1} = \underline{\underline{0}}$.

4) $f : [-1, 2] \longrightarrow [0, 2]$





$\frac{1}{f} : \,]-1, 2] \longrightarrow \left[\frac{1}{2}, +\infty\right[$.

dom $f = \,]-1, 2]$

dom $f^{-1} : [0, 2] \longrightarrow [-1, 2]$

5) $f(x) = \begin{cases} 3x+1 & \text{se } x<0 \\ x^2+1 & \text{se } x \geq 0 \end{cases}$; $\qquad g(x) = \begin{cases} -\sqrt[3]{x-1} & \text{se } x \leq 1 \\ -x+1 & \text{se } x > 1 \end{cases}$.