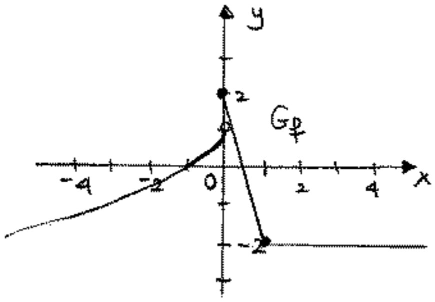
Settima verifica settimanale di <u>ANALISI MATEMATICA</u> – CdL in STPCA –
a.a. 2007/08 – Rovereto, 5 – 9 novembre 2007

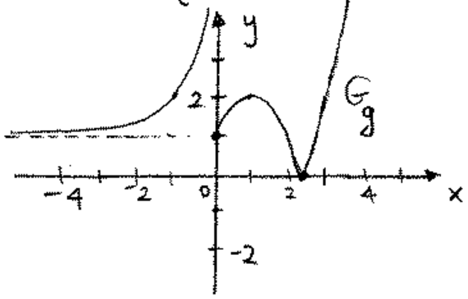
1) i) $f(x)=\begin{cases} \sqrt[3]{x}+1 & \text{se } x<0 \\ -2x+2|x-1| & \text{se } x\geq 0; \end{cases}$ osserviamo che $|x-1|=\begin{cases} x-1 & \text{se } x\geq 1 \\ -(x-1) & \text{se } x<1 \end{cases}$

allora

$$f(x)=\begin{cases} \sqrt[3]{x}+1 & \text{se} \quad x<0 \\ -4x+2 & \text{se} \quad 0\leq x\leq 1 \\ -2 & \text{se} \quad x>1\,. \end{cases}$$



$$g(x)=\begin{cases} -\frac{1}{x}+1 & \text{se} \quad x<0 \\ |-(x-1)^2+2| & \text{se } x\geq 0\,. \end{cases}$$

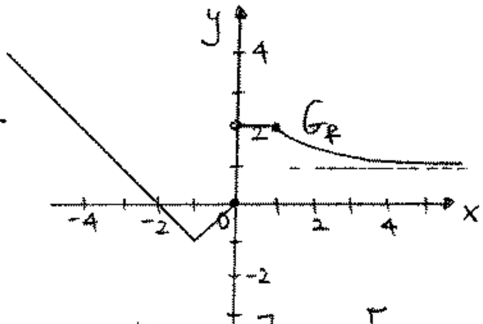


Oss. che $-(x-1)^2+2=0 \iff x_{1/2}=1\pm\sqrt{2}$

ii) se $k<-2$ $\exists$ una soluz. dell'eq. $f(x)=k$;
$k=-2$ $\exists$ sono infinite soluz. " ;
$-2<k<1$ $\exists$ sono 2 soluz. " ;
$1\leq k\leq 2$ $\exists$ una soluzione " ;
$k>2$ $\nexists$ soluzione dell'eq. $f(x)=k$.

iii) f è crescente su $]-\infty,0]$; f è decrescente deb. su $[0,+\infty[$.
g è crescente su $]-\infty,0[$, su $[0,1]$, su $[1+\sqrt{2},+\infty[$;
è decrescente su $[1,1+\sqrt{2}]$.

2) i) $f(x)=\begin{cases} |x+1|-1 & \text{se} \quad x\leq 0 \\ 2 & \text{se} \quad 0<x<1 \\ \frac{1}{x^2}+1 & \text{se} \quad x\geq 1\,. \end{cases}$

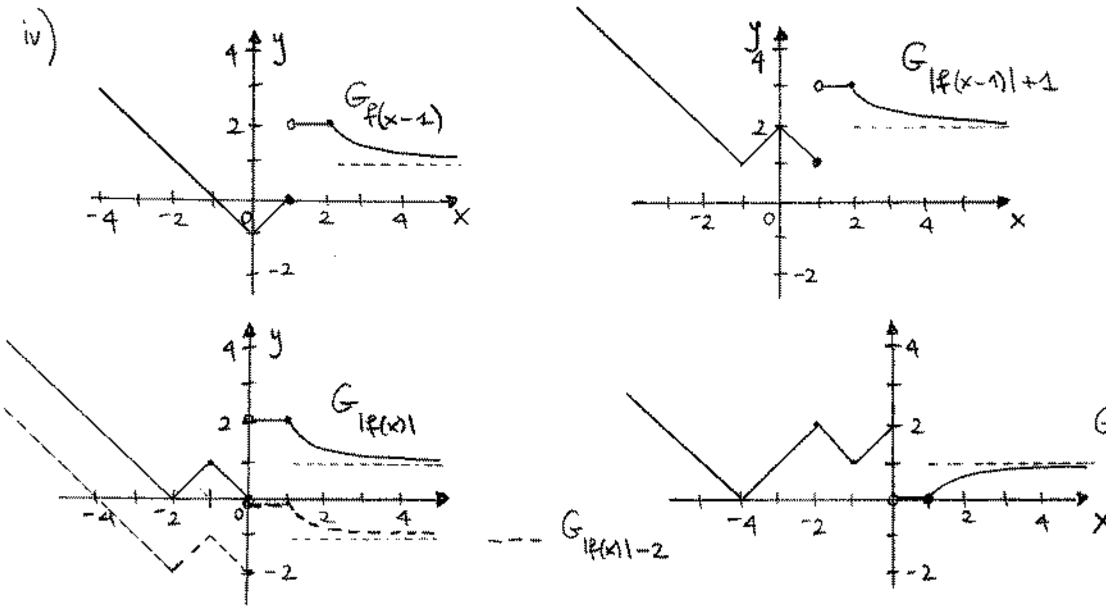


ii) f è decrescente su $]-\infty,-1]$, decrescente deb. su $]0,+\infty[$;
è crescente su $[-1,0]$.

iii) $\min\limits_{[-1,1[} f=-1$ pt. di minimo $x=-1$

$\max\limits_{[-1,1[} f = 2$ pt. di massimo $x \in]0,1[$.

iv)



3) $2|x-2|>6 \iff |x-2|>3 \iff x-2<-3$ o $x-2>3$

$\iff x<-1$ o $x>5$

$\implies \underline{\underline{S =]-\infty,-1[\cup]5,+\infty[}}$

$2|-x|+|x|+x^2<4 \underset{|-x|=|x|}{\iff} 3|x|+x^2<4 \iff$

$\begin{cases} x \geq 0 \\ 3x+x^2<4 \end{cases}$ oppure $\begin{cases} x<0 \\ -3x+x^2<4 \end{cases}$

$\iff \begin{cases} x \geq 0 \\ x^2+3x-4<0 \end{cases}$ oppure $\begin{cases} x<0 \\ x^2-3x-4<0 \end{cases}$

$\iff \begin{cases} x \geq 0 \\ -4<x<1 \end{cases}$ oppure $\begin{cases} x<0 \\ -1<x<4 \end{cases}$

$\implies \underline{\underline{S =]-1,1[}}$.

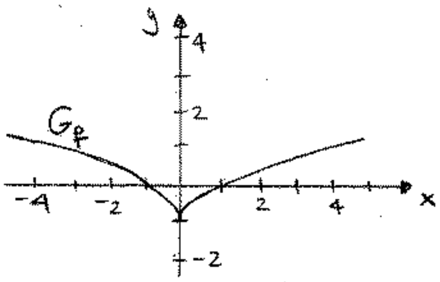
$|2x-3| \leq 3 \iff -3 \leq 2x-3 \leq 3 \iff 0 \leq 2x \leq 6$

$\iff 0 \leq x \leq 3 \implies \underline{\underline{S = [0,3]}}$

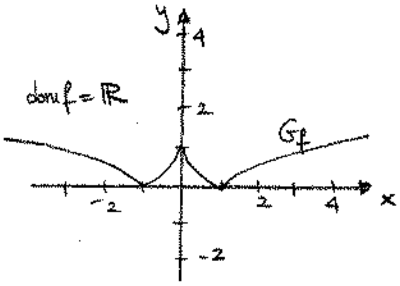
$|x^2+4x|>0 \iff x^2+4x \neq 0 \iff x \neq 0,\ x \neq -4$

$\underline{\underline{S = \mathbb{R} \setminus \{-4,0\}}}$

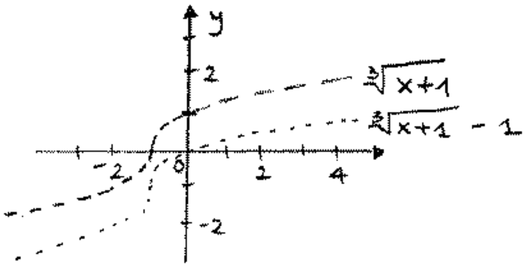
4) $\sqrt[4]{|x|} - 1$ $\qquad$ $\operatorname{dom} f = \mathbb{R}$

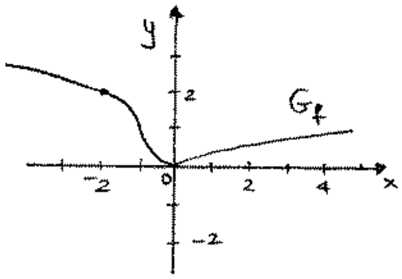


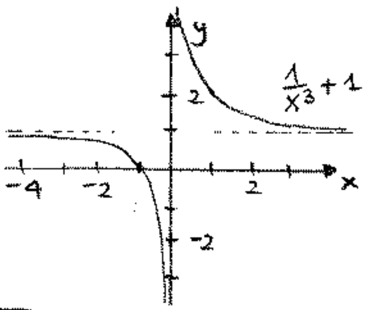
$\left|\sqrt[4]{|x|} - 1\right|$



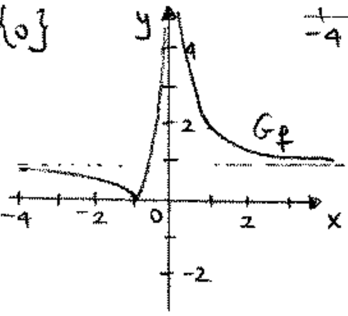
$\left|\sqrt[3]{x+1} - 1\right|$ $\quad$ $\operatorname{dom} f = \mathbb{R}$







$\left|\frac{1}{x^3} + 1\right|$ $\qquad$ $\operatorname{dom} f = \mathbb{R}\setminus\{0\}$



5) i) $\log_{1/2} 32 = \underline{\underline{-5}}$ $\quad \left(\Leftrightarrow \quad \left(\frac{1}{2}\right)^{-5} = 2^5 = 32\right)$;

$\log_2 \frac{1}{16} = \underline{\underline{-4}}$ $\quad \left(\Leftrightarrow \quad 2^{-4} = \frac{1}{2^4} = \frac{1}{16}\right)$;

$\log e^{-3} = \underline{\underline{-3}}$

ii) $2^x = 64 \quad \Leftrightarrow \quad 2^x = 2^6 \quad \Rightarrow \quad \underline{\underline{x = 6}}$;

$8^x = -\frac{1}{64} \quad \Rightarrow \quad \underline{\underline{S = \emptyset}}.$;

$\log_3 x = -2 \quad \Leftrightarrow \quad 3^{-2} = x \quad \Rightarrow \quad \underline{\underline{x = \frac{1}{9}}}$;

$\log x^2 = 1 \quad \Leftrightarrow \quad x^2 = e \quad \Rightarrow \quad \underline{\underline{x \in \{-\sqrt{e}, \sqrt{e}\}}}.$

iii) $2^{|x|} \geq \frac{1}{2} \quad \Leftrightarrow \quad 2^{|x|} \geq 2^{-1} \quad \Leftrightarrow \quad |x| \geq -1 \quad \Rightarrow \quad \underline{\underline{S = \mathbb{R}}}$;

$e^{x^2-1} \leq 1 \quad \Leftrightarrow \quad x^2 - 1 \leq 0 \quad \Leftrightarrow \quad x^2 \leq 1 \quad \Rightarrow \quad \underline{\underline{S = [-1, 1]}}$;

$\log(x+1) < 0 \quad \Leftrightarrow \quad 0 < x+1 < 1 \quad \Rightarrow \quad \underline{\underline{S =]-1, 0[}}.$