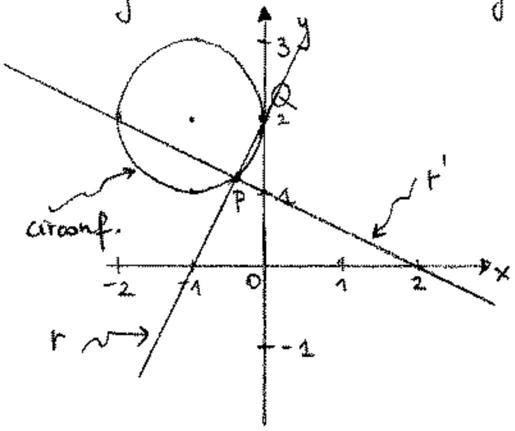
Primo Appello di ANALISI MATEMATICA – CdL in STPCA –
a.a. 2007/08 – Rovereto, 7 gennaio 2008

FILA (A)

1) i) $x^2 + 2x + y^2 - 4y + 4 = 0 \iff (x+1)^2 - 1 + (y-2)^2 - 4 + 4 = 0$

$\iff (x+1)^2 + (y-2)^2 = 1$ eq. circonferenza di centro $C = (-1, 2)$ e raggio 1.

$y - 2x - 2 = 0 \iff y = 2x + 2.$



Per trovare i pt. d'intersezione, risolviamo il sistema

$$\begin{cases} x^2 + 2x + (2x+2)^2 - 4(2x+2) + 4 = 0 \\ y = 2x + 2, \end{cases}$$

ossia

$$\begin{cases} x^2 + 2x + 4x^2 + \cancel{8x} + \cancel{4} - \cancel{8x} - \cancel{8} + \cancel{4} = 0 \\ y = 2x + 2; \end{cases}$$

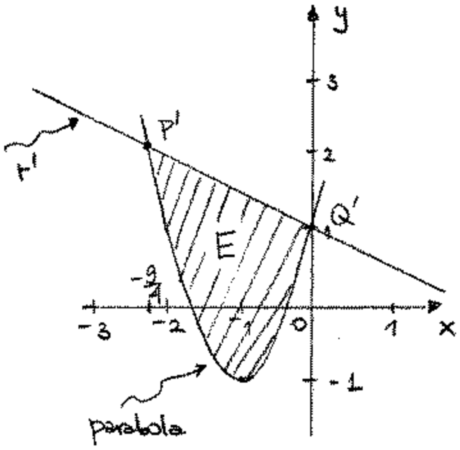
abbiamo $\begin{cases} 5x^2 + 2x = 0 \\ y = 2x + 2 \end{cases} \implies \underline{Q = (0, 2)}$, $\underline{P = \left(-\frac{2}{5}, \frac{6}{5}\right)}$. $\square$

ii) $d(P, Q) = \sqrt{\left(0 + \frac{2}{5}\right)^2 + \left(2 - \frac{6}{5}\right)^2} = \sqrt{\frac{4}{25} + \frac{16}{25}} = \sqrt{\frac{20}{25}} = \underline{\sqrt{\frac{4}{5}}}$. $\square$

iii) $y = \frac{6}{5} - \frac{1}{2}\left(x + \frac{2}{5}\right) \implies \underline{y = -\frac{1}{2}x + 1}$ $\square$

iv) $y - 2x^2 - 4x - 1 = 0 \iff y = 2(x^2 + 2x) + 1$

$\iff y = 2(x+1)^2 - 1.$



I pt. di intersezione della parabola con la retta r' soddisfano il sistema

$$\begin{cases} -\frac{1}{2}x + 1 = 2x^2 + 4x + 1 \\ y = -\frac{1}{2}x + 1 \end{cases},$$

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$$\begin{cases} 2x^2 + \frac{9}{2}x = 0 \\ y = -\frac{1}{2}x + 1 \end{cases},$$

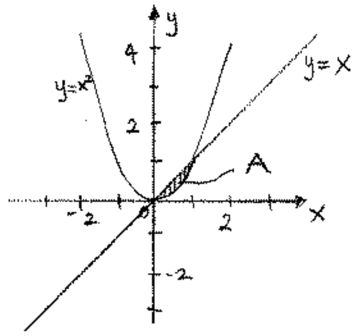
da cui $Q' = (0, 1)$, $P' = \left(-\frac{9}{4}, \frac{17}{8}\right)$.

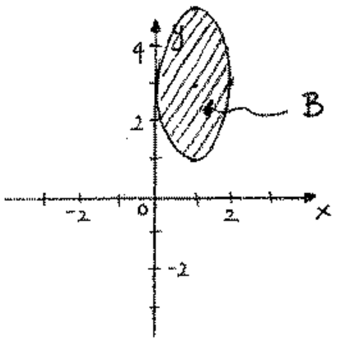
$$\text{area } E = \int_{-9/4}^{0} \left(-\tfrac{1}{2}x + \cancel{1} - 2x^2 - 4x - \cancel{1}\right) dx = \int_{-9/4}^{0} \left(-2x^2 - \tfrac{9}{2}x\right) dx =$$

$$= \left[-\frac{2x^3}{3} - \frac{9x^2}{4}\right]_{-9/4}^{0} = \frac{2}{3}\left(-\frac{9}{4}\right)^3 + \frac{9}{4}\left(-\frac{9}{4}\right)^2 = -\frac{2}{3}\cdot\left(\frac{9}{4}\right)^3 + \left(\frac{9}{4}\right)^3 = \underline{\underline{\frac{1}{3}\left(\frac{9}{4}\right)^3}}.$$

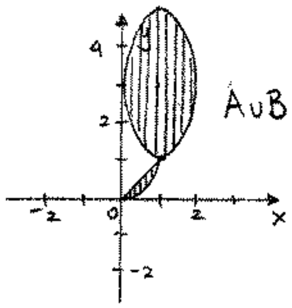
2) i) $A = \{(x,y) \in \mathbb{R}^2 : \; x^2 \le y \le x\}$

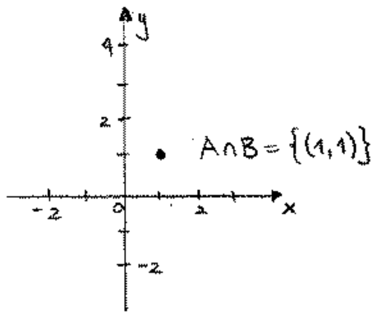
$B = \{(x,y) \in \mathbb{R}^2 : \; 4(x-1)^2 + (y-3)^2 \le 4\} = \{(x,y) \in \mathbb{R}^2 : \; (x-1)^2 + \frac{(y-3)^2}{4} \le 1\}$

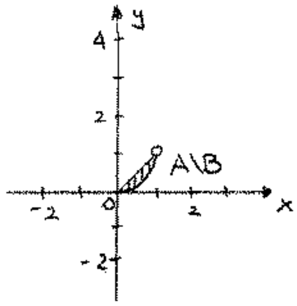




ii)







iii) a) $\text{non}\left(\forall (x,y) \in A \;, \; y \ge x^2\right) \iff \exists (x,y) \in A \; : \; \text{non}\left(y \ge x^2\right)$

$\iff \exists (x,y) \in A \; : \; y < x^2.$

b) $\text{non}\left(\exists (x,y) \in \mathbb{R}^2 : \; (x,y) \in A \text{ e } (x,y) \in B\right) \iff \forall (x,y) \in \mathbb{R}^2, \; \text{non}\left((x,y) \in A \text{ e } (x,y) \in B\right)$

$\iff \forall (x,y) \in \mathbb{R}^2, \; (x,y) \notin A \text{ o } (x,y) \notin B.$

3) i) $|x+2| - x^2 \le 0 \iff \begin{cases} x+2 \ge 0 \\ x+2-x^2 \le 0 \end{cases} \text{ o } \begin{cases} x+2 < 0 \\ -(x+2) - x^2 \le 0 \end{cases}$

$\iff \begin{cases} x \ge -2 \\ x^2 - x - 2 \ge 0 \end{cases} \text{ o } \begin{cases} x < -2 \\ x^2 + x + 2 \ge 0 \end{cases}$

$\iff \begin{cases} x \ge -2 \\ x \le -1 \text{ o } x \ge 2 \end{cases} \text{ o } \begin{cases} x < -2 \\ \mathbb{R} \end{cases}$

$\Rightarrow \underline{\underline{S = \left]-\infty, -1\right] \cup \left[2, +\infty\right[.}}$

$\log_2(|x| - 1) \le 2 \iff \log_2(|x| - 1) \le \log_2 4$

$$\Leftrightarrow \begin{cases} |x|-1>0 \\ |x|-1\le 4 \end{cases} \Leftrightarrow \begin{cases} |x|>1 \\ |x|\le 5 \end{cases} \Leftrightarrow \begin{cases} x<-1 \text{ o } x>1 \\ -5\le x\le 5 \end{cases}$$

$$\Rightarrow \underline{\underline{S=[-5,-1[\,\cup\,]1,5]}}. \qquad \square$$

$$\left(\tfrac{1}{2}\right)^{x^2+x} > 2^{-3x} \Leftrightarrow 2^{-(x^2+x)} > 2^{-3x} \Leftrightarrow -(x^2+x) > -3x$$

$$\Leftrightarrow x^2+x<3x \Leftrightarrow x^2-2x<0 \Rightarrow \underline{\underline{S=]0,2[}}. \qquad \square$$

ii) $f:[0,+\infty[\,\to\mathbb{R} \qquad f(x)=\sqrt{x}$

$g:\mathbb{R}\setminus\{-2\}\to\mathbb{R} \qquad g(x)=\dfrac{x}{x+2}$.

Abbiamo $f+g:[0,+\infty[\,\to\mathbb{R} \qquad (f+g)(x)=\underline{\underline{\sqrt{x}+\dfrac{x}{x+2}}}$;

$\dfrac{f}{g}:\,]0,+\infty[\,\to\mathbb{R} \qquad \left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x}}{\frac{x}{x+2}}=\underline{\underline{\dfrac{(x+2)\sqrt{x}}{x}}}$;

$g\circ f:[0,+\infty[\,\to\mathbb{R} \qquad (g\circ f)(x)=g(f(x))=\underline{\underline{\dfrac{\sqrt{x}}{\sqrt{x}+2}}}$;

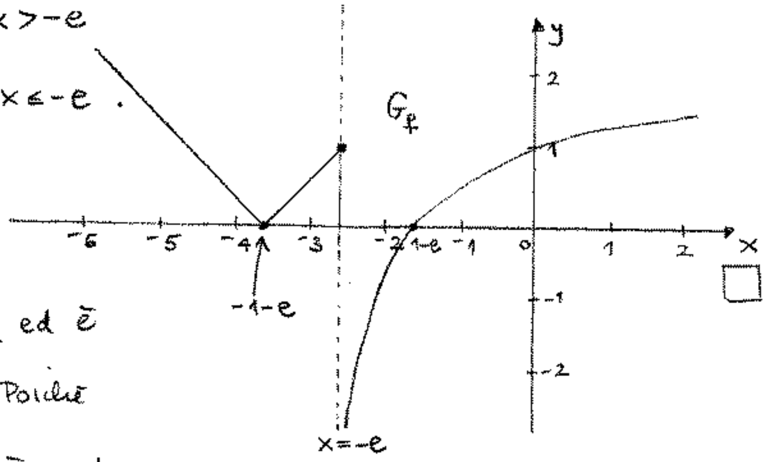
$f\circ g:\,]-\infty,-2[\,\cup[0,+\infty[\,\to\mathbb{R} \qquad (f\circ g)(x)=f(g(x))=\underline{\underline{\sqrt{\dfrac{x}{x+2}}}}$. $\square$

iii) $$\int_0^1\left(x^4+\sqrt[3]{x}+\frac{1}{x+1}\right)dx=\left[\frac{x^5}{5}+\frac{x^{4/3}}{4/3}+\log|x+1|\right]_0^1=$$

$$=\frac{1}{5}+\frac{3}{4}+\log 2=\underline{\underline{\frac{19}{20}+\log 2}}.$$

$$\int_0^1(2x+1)^4dx=\frac{1}{2}\int_0^1 2(2x+1)^4dx=\frac{1}{2}\left[\frac{(2x+1)^5}{5}\right]_0^1=\frac{1}{2}\left(\frac{3^5}{5}-\frac{1}{5}\right)=\frac{1}{2}\cdot\frac{242}{5}=\underline{\underline{\frac{121}{5}}}.$$ $\blacksquare$

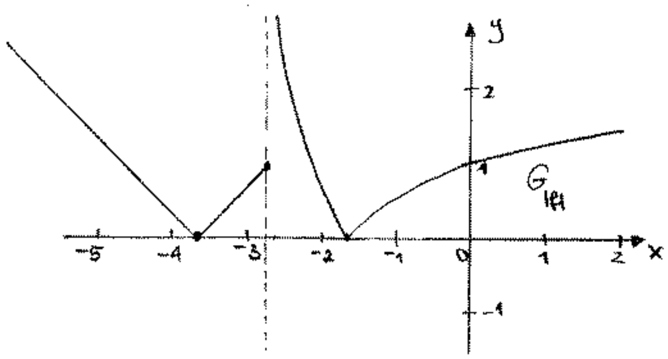
4) i) $$f(x)=\begin{cases}\log(e+x) & \text{se } x>-e \\ |1-|x+e|| & \text{se } x\le -e\end{cases}.$$



$\square$

ii) f è continua in $]-\infty,-e[$ ed è continua in $]-e,+\infty[$. Poiché $\lim_{x\to -e^+}f(x)=-\infty$, $x=-e$ è un pt. di discontinuità per f. $\square$

iii) $x=-1-e$ pt. di min. locale per f; $x=-e$ pt. di massimo locale per f. $\square$



iv) se $k<0$ $\nexists$ soluzioni di $|f(x)|=k$;

se $k=0$ $\exists$ sono 2 soluzioni " ;

se $0<k\leq 1$ $\exists$ sono 4 soluzioni " ;

se $k>1$ $\exists$ sono 3 soluzioni " .

5) i) Osserva che $f(x)=\dfrac{e^x}{1-x^2}=-\dfrac{e^x}{x^2-1}$ e quindi il grafico di questa funzione è il simmetrico rispetto all'asse x del grafico rappresentato a pag. 3 FILA Ⓐ, Terza Prova Intermedia 7/1/08).

ii) $\underline{\underline{y=x+1}}$ eq. della retta tangente al grafico di f in $(0,1)$.

iii) $$\int_{-3}^{-2} xe^{-x} f(x)\,dx = \int_{-3}^{-2} xe^{-x}\frac{e^x}{1-x^2}\,dx = \int_{-3}^{-2}\frac{x}{1-x^2}\,dx = -\frac{1}{2}\int_{-3}^{-2}\frac{-2x}{1-x^2}\,dx =$$

$$= -\frac{1}{2}\Big[\log|1-x^2|\Big]_{-3}^{-2} = -\frac{1}{2}\left(\log 3-\log 8\right)=\frac{1}{2}\left(\log 8-\log 3\right)= \underline{\underline{\log\sqrt{\frac{8}{3}}}}\,.$$

iv) Essendo f decrescente su $[-3,-2]$, abbiamo

$$\max_{[-3,-2]} f = f(-3) = \frac{e^{-3}}{1-9} = \underline{\underline{-\frac{e^{-3}}{8}}} \qquad \underline{\underline{x=-3 \text{ pt. di max}}};$$

$$\min_{[-3,-2]} f = f(-2) = \frac{e^{-2}}{1-4} = \underline{\underline{-\frac{e^{-2}}{3}}} \qquad \underline{\underline{x=-2 \text{ pt. di min}}}.$$

6) $$D_{24,8} = \frac{24!}{(24-8)!} = \underline{\underline{\frac{24!}{16!}}}\,.$$