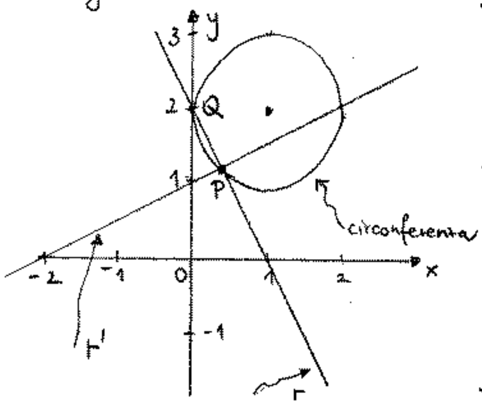
FILA (B)

1) i) $x^2 - 2x + y^2 - 4y + 4 = 0 \iff (x-1)^2 - 1 + (y-2)^2 - 4 + 4 = 0$

$\iff (x-1)^2 + (y-2)^2 = 1$ eq. circonferenza di centro $(1,2)$ e raggio 1.

$y + 2x - 2 = 0 \iff y = -2x + 2$.



Per trovare i pt. d'intersezione, risolviamo il sistema

$$\begin{cases} x^2 - 2x + (-2x+2)^2 - 4(-2x+2) + 4 = 0 \\ y = -2x+2, \end{cases}$$

ossia

$$\begin{cases} x^2 - 2x + 4x^2 \cancel{- 8x} \cancel{+ 4} \cancel{+ 8x} \cancel{- 8} \cancel{+ 4} = 0 \\ y = -2x+2; \end{cases}$$

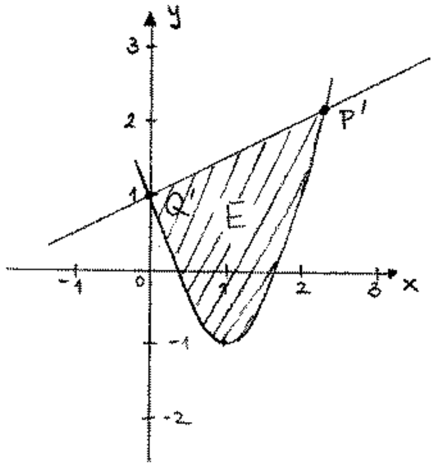
abbiamo $\begin{cases} 5x^2 - 2x = 0 \\ y = -2x+2 \end{cases} \implies \underline{Q = (0,2)}$, $\underline{P = \left(\frac{2}{5}, \frac{6}{5}\right)}$. $\square$

ii) $d(P,Q) = \sqrt{\left(0-\frac{2}{5}\right)^2 + \left(2-\frac{6}{5}\right)^2} = \underline{\sqrt{\frac{4}{5}}}$. $\square$

iii) $y = \frac{6}{5} + \frac{1}{2}\left(x - \frac{2}{5}\right) \implies \underline{y = \frac{1}{2}x + 1}$. $\square$

iv) $y - 2x^2 + 4x - 1 = 0 \iff y = 2(x^2 - 2x) + 1$

$y = 2(x-1)^2 - 1$.



I pt. di intersezione della parabola con la retta r' soddisfano il sistema

$$\begin{cases} \frac{1}{2}x + \cancel{1} = 2x^2 - 4x + \cancel{1} \\ y = \frac{1}{2}x + 1 \end{cases}$$

ossia $\begin{cases} 2x^2 - \frac{9}{2}x = 0 \\ y = \frac{1}{2}x + 1, \end{cases}$

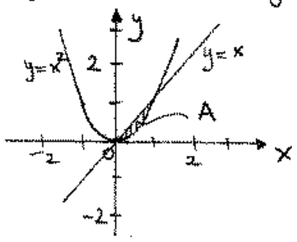
da cui $Q' = (0,1)$, $P' = \left(\frac{9}{4}, \frac{17}{8}\right)$. Allora

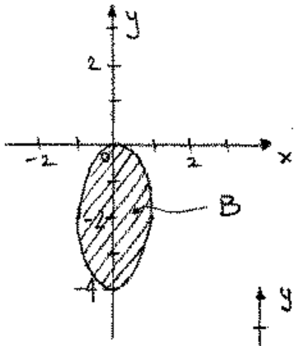
$$\text{area } E = \int_0^{9/4} \left(\frac{1}{2}x + \cancel{1} - 2x^2 + 4x - \cancel{1}\right)dx = \int_0^{9/4} \left(-2x^2 + \frac{9}{2}x\right)dx = \left[-2\frac{x^3}{3} + 9\frac{x^2}{4}\right]_0^{9/4} = \underline{\underline{\frac{1}{3}\left(\frac{9}{4}\right)^3}}.$$

$\blacksquare$

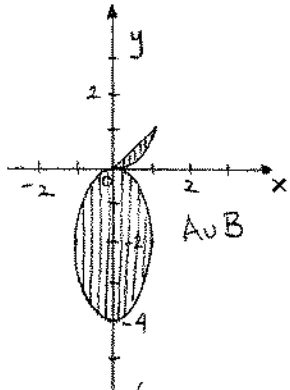
2) i) $A = \{(x,y) \in \mathbb{R}^2 : \; x^2 \leq y \leq x\}$

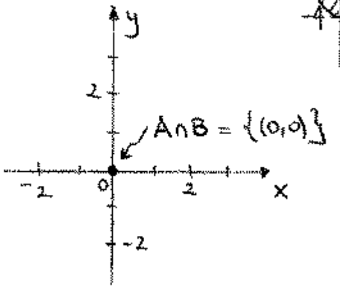
$B = \{(x,y) \in \mathbb{R}^2 : \; 4x^2 + (y+2)^2 \leq 4\} = \{(x,y) \in \mathbb{R}^2 : \; x^2 + \frac{(y+2)^2}{4} \leq 1\}$

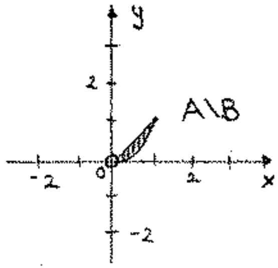




ii)







iii) a) non $(\forall (x,y) \in B, \; y \leq 0) \iff \exists (x,y) \in B : $ non $(y \leq 0)$

$\iff \exists (x,y) \in B : \; y > 0.$

b) non $(\exists (x,y) \in \mathbb{R}^2 : (x,y) \in A$ o $(x,y) \in B) \iff \forall (x,y) \in \mathbb{R}^2,$ non $((x,y) \in A$ o $(x,y) \in B)$

$\iff \forall (x,y) \in \mathbb{R}^2, \; (x,y) \notin A$ e $(x,y) \notin B.$

3) Vedi FILA A Es. 4 .

4) i) $|x+3| - 2x^2 \leq 0 \iff \begin{cases} x+3 \geq 0 \\ x+3-2x^2 \leq 0 \end{cases}$ o $\begin{cases} x+3 < 0 \\ -(x+3) - 2x^2 \leq 0 \end{cases}$

$\iff \begin{cases} x \geq -3 \\ 2x^2 - x - 3 \geq 0 \end{cases}$ o $\begin{cases} x < -3 \\ 2x^2 + x + 3 \geq 0 \end{cases}$

$\iff \begin{cases} x \geq -3 \\ x \leq -1 \text{ o } x \geq \frac{3}{2} \end{cases}$ o $\begin{cases} x < -3 \\ \mathbb{R} \end{cases}$

$\Rightarrow \; \underline{S = \,]-\infty, -1] \cup [\tfrac{3}{2}, +\infty[\,.}$

$\log_2(|x| - 2) \leq 3 \iff \log_2(|x|-2) \leq \log_2 8$

$\iff \begin{cases} |x| - 2 > 0 \\ |x| - 2 \leq 8 \end{cases} \iff \begin{cases} |x| > 2 \\ |x| \leq 10 \end{cases} \iff \begin{cases} x < -2 \text{ o } x > 2 \\ -10 \leq x \leq 10 \end{cases}$

$\Rightarrow \; \underline{S = [-10, -2[\,\cup\,]2, 10].}$

$$\left(\frac{1}{2}\right)^{x^2-x} < 2^{-4x} \iff 2^{-(x^2-x)} < 2^{-4x} \iff -(x^2-x) < -4x$$

$$\iff x^2-x > 4x \iff x^2-5x > 0$$

$$\Rightarrow \underline{\underline{S =]-\infty, 0[\cup]5, +\infty[}}.$$ $\square$

ii) $f : [0, +\infty[\to \mathbb{R} \qquad f(x) = \sqrt{x}$

$g : \mathbb{R}\setminus\{1\} \to \mathbb{R} \qquad g(x) = \frac{x}{x-1}$.

Abbiamo $f+g : [0,+\infty[\setminus\{1\} \to \mathbb{R} \qquad (f+g)(x) = \underline{\underline{\sqrt{x} + \frac{x}{x-1}}}$;

$\frac{f}{g} :]0,+\infty[\setminus\{1\} \to \mathbb{R} \qquad \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{\frac{x}{x-1}} = \underline{\underline{\frac{(x-1)\sqrt{x}}{x}}}$;

$g\circ f : [0,+\infty[\setminus\{1\} \to \mathbb{R} \qquad (g\circ f)(x) = g(f(x)) = \underline{\underline{\frac{\sqrt{x}}{\sqrt{x}-1}}}$;

$f\circ g :]-\infty, 0] \cup]1,+\infty[\to \mathbb{R} \qquad (f\circ g)(x) = f(g(x)) = \underline{\underline{\sqrt{\frac{x}{x-1}}}}$. $\square$

iii) $$\int_0^1 \left(2x^3 + \sqrt[4]{x} + \frac{1}{x+2}\right) dx = \left[2\frac{x^4}{4} + \frac{x^{5/4}}{5/4} + \log|x+2|\right]_0^1 =$$

$$= \frac{2}{4} + \frac{4}{5} + \log 3 - \log 2 = \underline{\underline{\frac{13}{10} + \log\left(\frac{3}{2}\right)}}.$$

$$\int_0^1 (3x+1)^5 dx = \frac{1}{3}\int_0^1 3(3x+1)^5 dx = \frac{1}{3}\left[\frac{(3x+1)^6}{6}\right]_0^1 = \frac{1}{3}\left(\frac{4^6}{6} - \frac{1}{6}\right) = \frac{4095}{18} = \underline{\underline{\frac{455}{2}}}.$$ $\blacksquare$

5) i) Vedi grafico pag. 3 FILA (A) Terza Prova Intermedia 7/1/08.

ii) $\underline{\underline{y = -x-1}}$ (")

iii) $$\int_2^3 2x\, e^{-x} f(x)\, dx = \int_2^3 2x\, e^{-x} \frac{e^x}{x^2-1} dx = \int_2^3 \frac{2x}{x^2-1} dx = \left[\log|x^2-1|\right]_2^3 = \underline{\underline{\log\frac{8}{3}}}.$$

iv) Oss. che f è decrescente su $[2, 1+\sqrt{2}]$ e crescente su $[1+\sqrt{2}, 3]$. Notiamo che $f(2) = \frac{e^2}{3} < f(3) = \frac{e^3}{8}$ (infatti: $\frac{e^2}{3} < \frac{e^3}{8} \iff \frac{1}{3} < \frac{e}{8} \iff \frac{8}{3} < e$) e quindi $\max_{[2,3]} f = f(3) = \underline{\underline{\frac{e^3}{8}}}$ $\underline{\underline{x = 3 \text{ pt. di max}}}$;

$\min_{[2,3]} f = f(1+\sqrt{2}) = \underline{\underline{\frac{e^{1+\sqrt{2}}}{2\sqrt{2}+2}}}$ $\underline{\underline{x = 1+\sqrt{2} \text{ pt. di min}}}$. $\blacksquare$

6) $D_{25,7} = \frac{25!}{(25-7)!} = \underline{\underline{\frac{25!}{18!}}}$. $\blacksquare$