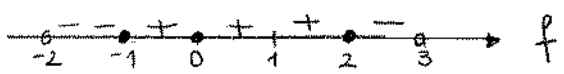
Dodicesima verifica settimanale di ANALISI MATEMATICA – CdL in STPCA –
a.a. 2007/08 – Rovereto, 10-14 dicembre 2007

1) $\displaystyle\sum_{i=1}^{5}(-i)^3 = -\sum_{i=1}^{5} i^3 = -(1+8+27+64+125) = \underline{\underline{-225.}}$

$$\sum_{m=2}^{5}\frac{m}{m-1} = 2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4} = \frac{2\cdot 12+3\cdot 6+16+15}{12} = \frac{24+18+16+15}{12} = \underline{\underline{\frac{73}{12}}}$$

$$\sum_{k=2}^{100}\left(\frac{1}{k^2}-\frac{1}{(k-1)^2}\right) = \left(\frac{1}{2^2}-\frac{1}{1^2}\right)+\left(\frac{1}{3^2}-\frac{1}{2^2}\right)+\dots+\left(\frac{1}{100^2}-\frac{1}{99^2}\right) = \underline{\underline{-1+\frac{1}{100^2}}}. \quad \blacksquare$$

2) i)



ii) F è decrescente in $[-2,1]$;

F è crescente in $[1,3]$. $\blacksquare$

3) i) $\displaystyle\int_{-2}^{1}(2^x+3x)\,dx = \left[\frac{2^x}{\log 2}+\frac{3x^2}{2}\right]_{-2}^{1} = \left(\frac{2}{\log 2}+\frac{3}{2}\right)-\left(\frac{1}{4\log 2}+6\right) = \underline{\underline{\frac{7}{4\log 2}-\frac{9}{2}}}.$

$$\int_{1}^{e}\left(x^2+\frac{1}{x}\right)dx = \left[\frac{x^3}{3}+\log|x|\right]_{1}^{e} = \left(\frac{e^3}{3}+\log e\right)-\left(\frac{1}{3}-0\right) = \frac{e^3}{3}+1-\frac{1}{3} = \underline{\underline{\frac{e^3}{3}+\frac{2}{3}}}.$$

$$\int_{-2}^{0}\frac{x^2-4}{x-2}\,dx = \int_{-2}^{0}\frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}\,dx = \int_{-2}^{0}(x+2)\,dx = \left[\frac{x^2}{2}+2x\right]_{-2}^{0} = -\left(\frac{4}{2}-4\right) = \underline{\underline{2.}} \quad \square$$

ii) $\displaystyle\int_{1}^{2}\left(\sqrt[4]{x}-\sqrt[3]{x}\right)dx = \int_{1}^{2}\left(x^{1/4}-x^{1/3}\right)dx = \left[\frac{x^{5/4}}{5/4}-\frac{x^{4/3}}{4/3}\right]_{1}^{2} = \left(\frac{4}{5}\cdot 2^{5/4}-\frac{3}{4}\cdot 2^{4/3}\right)-$

$$-\left(\frac{4}{5}-\frac{3}{4}\right) = \left(\frac{4}{5}\cdot 2\sqrt[4]{2}-\frac{3}{4}\cdot 2\sqrt[3]{2}\right)-\frac{1}{20} = \underline{\underline{\frac{8}{5}\sqrt[4]{2}-\frac{3}{2}\sqrt[3]{2}-\frac{1}{20}}}.$$

$$\int_{-2}^{-1}\frac{2x^2+3x^4}{x^4}\,dx = \int_{-2}^{-1}\left(\frac{2}{x^2}+3\right)dx = \left[-2x^{-1}+3x\right]_{-2}^{-1} = (2-3)-(1-6) = \underline{\underline{4}}.$$

$$\int_{1}^{2}\left(e^x+x^{-3}\right)dx = \left[e^x-\frac{x^{-2}}{2}\right]_{1}^{2} = \left(e^2-\frac{1}{8}\right)-\left(e-\frac{1}{2}\right) = \underline{\underline{e^2-e+\frac{3}{8}}}. \quad \square$$

iii) $\displaystyle\int_{0}^{2}\left(e^{3x}+(2x)^{1/3}\right)dx = \frac{1}{3}\int_{0}^{2}3e^{3x}\,dx+\frac{1}{2}\int_{0}^{2}2(2x)^{1/3}\,dx = \frac{1}{3}\left[e^{3x}\right]_{0}^{2}+\frac{1}{2}\left[\frac{(2x)^{4/3}}{4/3}\right]_{0}^{2}$

$$= \frac{1}{3}\left(e^6-1\right)+\frac{1}{2}\left(\frac{3}{4}\cdot 4^{4/3}-0\right) = \frac{1}{3}\left(e^6-1\right)+\frac{1}{2}\cdot\frac{3}{\cancel{4}}\cdot\cancel{4}\sqrt[3]{4} =$$

$$= \underline{\underline{\frac{1}{3}\left(e^6-1\right)+\frac{3}{2}\sqrt[3]{4}}}.$$

$$\int_0^1 \left((3x+1)^5 + \frac{1}{3x+1}\right) dx = \frac{1}{3}\int_0^1 3(3x+1)^5 dx + \frac{1}{3}\int_0^1 \frac{3}{3x+1} dx =$$

$$= \frac{1}{3}\left[\left(\frac{3x+1)^6}{6}\right)\right]_0^1 + \frac{1}{3}\left[\log|3x+1|\right]_0^1 = \frac{1}{3}\left(\frac{4^6}{6}-\frac{1}{6}\right) + \frac{1}{3}\left(\log 4 - \log 1\right)$$

$$= \underline{\underline{\frac{1}{3}\left(\frac{4^6}{6} - \frac{1}{6} + \log 4\right)}}.$$

$$\int_0^4 \frac{4x}{x^2+1} dx = 2\int_0^4 \frac{2x}{x^2+1} dx = 2\left[\log(x^2+1)\right]_0^4 = 2\left(\log 17 - \log 1\right) =$$

$$= \underline{2\log 17}. \qquad \square$$

iv) $$\int_1^2 \frac{x^3+1}{x} dx = \int_1^2 \left(x^2 + \frac{1}{x}\right) dx = \left[\frac{x^3}{3} + \log|x|\right]_1^2 = \left(\frac{8}{3} + \log 2\right) - \left(\frac{1}{3} + \log 1\right) =$$

$$= \underline{\underline{\frac{7}{3} + \log 2}}.$$

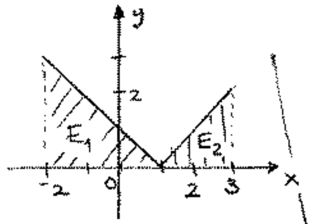
$$\int_0^1 x e^{x^2+1} dx = \frac{1}{2}\int_0^1 2x e^{x^2+1} dx = \frac{1}{2}\left[e^{x^2+1}\right]_0^1 = \underline{\underline{\frac{1}{2}\left(e^2 - e\right)}}.$$

$$\int_1^2 \frac{e^{3x}+1}{e^x} dx = \int_1^2 \left(e^{2x} + e^{-x}\right) dx = \frac{1}{2}\int_1^2 2e^{2x} dx - \int_1^2 -e^{-x} dx = \frac{1}{2}\left[e^{2x}\right]_1^2 - \left[e^{-x}\right]_1^2 =$$

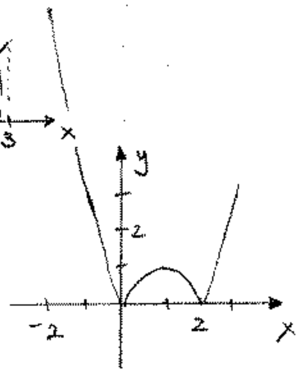
$$= \frac{1}{2}\left(e^4 - e^2\right) - \left(e^{-2} - e^{-1}\right) = \underline{\underline{\frac{1}{2}\left(e^4 - e^2\right) - \left(\frac{1}{e^2} - \frac{1}{e}\right)}}. \qquad \square$$

v) $$\int_{-2}^3 |x-1| dx = \text{area } E_1 + \text{area } E_2 =$$

$$= \frac{3\cdot 3}{2} + \frac{2\cdot \cancel{2}}{\cancel{2}} = \underline{\underline{\frac{13}{2}}}$$



$$\int_{-2}^1 \left|(x-1)^2 - 1\right| dx =$$

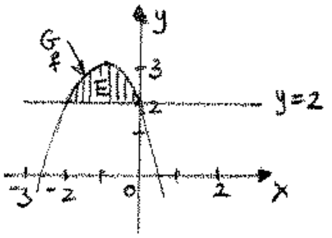


$$= \int_{-2}^0 \left((x-1)^2 - 1\right) dx + \int_0^1 -\left[(x-1)^2 - 1\right] dx$$

$$= \int_{-2}^0 \left(x^2 - 2x\right) dx + \int_0^1 \left(-x^2 + 2x\right) dx = \left[\frac{x^3}{3} - x^2\right]_{-2}^0 + \left[-\frac{x^3}{3} + x^2\right]_0^1 =$$

$$= -\left(-\frac{8}{3} - 4\right) + \left(-\frac{1}{3} + 1\right) = \frac{7}{3} + 5 = \underline{\underline{\frac{22}{3}}} \qquad \blacksquare$$

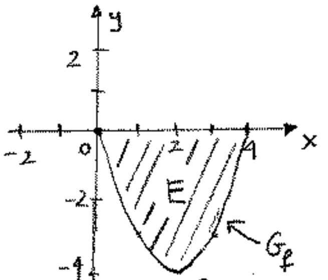
4) i) $f(x) = -(x+1)^2+3$



$$\text{area } E = \int_{-2}^{0}\left(-(x+1)^2+3-2\right)dx = \int_{-2}^{0}\left(-(x+1)^2+1\right)dx$$

$$= \left[-\frac{(x+1)^3}{3}\right]_{-2}^{0} + \left[x\right]_{-2}^{0} = -\frac{1}{3}-\frac{1}{3}+2 = \underline{\underline{\frac{4}{3}}}.$$

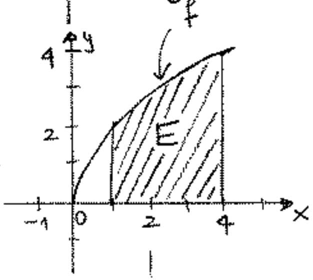
ii) $f(x) = x^2-4x = (x-2)^2-4$



$$\text{area } E = -\int_{0}^{4}(x^2-4x)\,dx = -\left[\frac{x^3}{3}-2x^2\right]_0^4 =$$

$$= -\left[\frac{64}{3}-32\right] = \underline{\underline{\frac{32}{3}}}$$

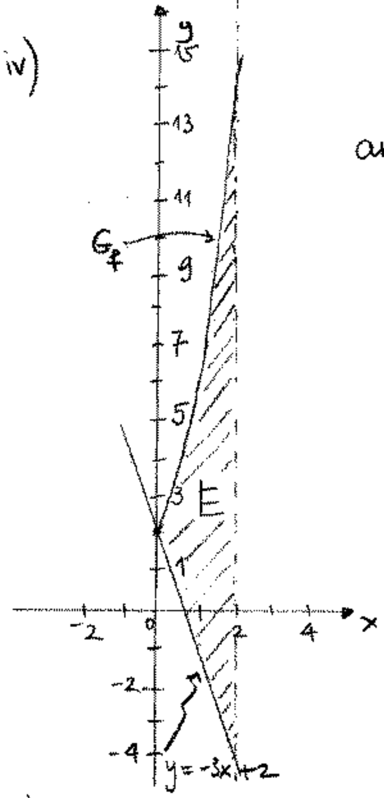
iii)



$$\text{area } E = \int_{1}^{4} 2\sqrt{x}\,dx = 2\int_{1}^{4} x^{1/2}\,dx = 2\left[\frac{x^{3/2}}{3/2}\right]_1^4 =$$

$$= 2\left(\frac{2}{3}\cdot 4^{3/2} - \frac{2}{3}\right) = 2\left(\frac{2}{3}\cdot 8 - \frac{2}{3}\right) = \underline{\underline{\frac{28}{3}}}.$$

iv)

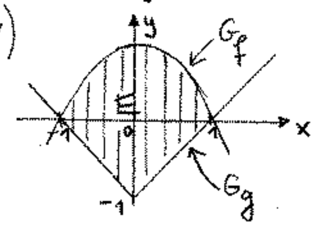


$$\text{area } E = \int_{0}^{2}\left(2e^x-(-3x+2)\right)dx = \int_{0}^{2}\left(2e^x+3x-2\right)dx =$$

$$= \left[2e^x+3\frac{x^2}{2}-2x\right]_0^2 = (2e^2+6-4)-(2) =$$

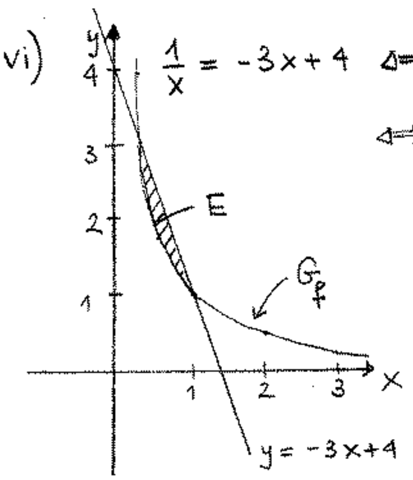
$$= \underline{\underline{2e^2}}$$

v)



$$\text{area } E = \int_{-1}^{1}\left(-x^4+1-|x|+1\right)dx = 2\int_{0}^{1}\left(-x^4-x+2\right)dx$$

$$= 2\left[-\frac{x^5}{5}-\frac{x^2}{2}+2x\right]_0^1 = 2\left(-\frac{1}{5}-\frac{1}{2}+2\right) = \underline{\underline{\frac{13}{5}}}$$

vi) 

$\frac{1}{x} = -3x+4 \iff \frac{1}{x} + 3x - 4 = 0 \iff \frac{3x^2-4x+1}{x} = 0$

$\iff 3x^2-4x+1=0 \qquad x_{1/2} = \frac{4 \pm \sqrt{16-12}}{6} = \frac{4\pm 2}{6} \left\langle \begin{matrix} 1 \\ \frac{1}{3} \end{matrix} \right.$

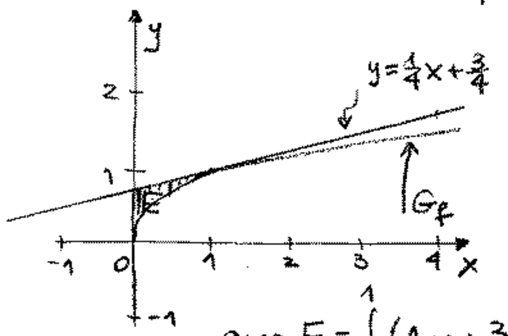
$$\text{area } E = \int_{\frac{1}{3}}^{1} \left(-3x+4-\frac{1}{x}\right)dx =$$

$$= \left[-3\frac{x^2}{2} + 4x - \log|x|\right]_{\frac{1}{3}}^{1} =$$

$$= \left(-\frac{3}{2} + 4 - \log 1\right) - \left(-\frac{\cancel{3}}{2}\cdot\frac{1}{\cancel{9}_3} + \frac{4}{3} - \log\frac{1}{3}\right)$$

$$= \frac{5}{3} - \frac{7}{6} + \log\frac{1}{3} = \underline{\underline{\frac{1}{2} + \log\frac{1}{3}}}.$$

5) i) $f(x) = \sqrt[4]{x}$ $\qquad f'(x) = \frac{1}{4}x^{-3/4}$ $\qquad$ eq. retta tg. al grafico di f nel pt. (1,1):



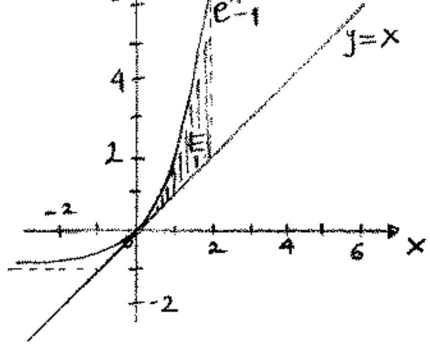
$y = 1 + \frac{1}{4}(x-1)$

$\underline{\underline{y = \frac{1}{4}x + \frac{3}{4}}}$

$$\text{area } E = \int_0^1 \left(\frac{1}{4}x + \frac{3}{4} - \sqrt[4]{x}\right)dx = \left[\frac{1}{4}\frac{x^2}{2} + \frac{3}{4}x - \frac{x^{5/4}}{5/4}\right]_0^1$$

$$= \frac{1}{8} + \frac{3}{4} - \frac{4}{5} = \frac{5+30-32}{40} = \underline{\underline{\frac{3}{40}}}.$$

ii) $f(x) = e^x - 1$ $\qquad f'(x) = e^x$ $\qquad$ eq. retta tg. al grafico di f nel pt. (0,0):



$y = 0 + 1(x-0)$

$y = x$

$$\text{area } E = \int_0^2 (e^x - 1 - x)\,dx = \left[e^x - x - \frac{x^2}{2}\right]_0^2$$

$$= (e^2 - 2 - 2) - 1 = \underline{\underline{e^2 - 5}}.$$

6) i) $P_7 = 7!$ $\qquad P_7^{2,2,3} = \frac{7!}{2!\,2!\,3!}$ $\qquad D_{6,2} = \frac{6!}{4!}$ $\qquad C_{8,3} = \frac{8!}{3!\,5!}$

ii) $P_5^{2,1,1,1} = \frac{5!}{2!} = 5\cdot 4\cdot 3 = 60$ $\qquad$ iii) $C_{50,24} = \frac{50!}{24!\,26!}$

iv) $D_{12,3} = \frac{12!}{9!} = \frac{12\cdot 11\cdot 10\cdot \cancel{9!}}{\cancel{9!}} = 1320.$