Nona verifica settimanale di <u>ANALISI MATEMATICA</u> – CdL in STPCA – a.a. 2007/08 – Rovereto, 19 – 23 novembre 2007

1) i) $\lim_{x\to 2}(1+4x^3) = \underline{33};\qquad \lim_{x\to 2^+}\left[(x+1)^2-\sqrt{x+1}\right] = \underline{9-\sqrt{3}}$;

$\lim_{x\to -1^-}\dfrac{1}{\underbrace{|x|-1}_{\text{infinitesimo positivo}}} = \underline{\underline{+\infty}}$; $\qquad \lim_{x\to -1^+}\dfrac{1}{\underbrace{|x|-1}_{\text{infinitesimo neg}}} = \underline{\underline{-\infty}}$; $\square$

ii) $\lim_{x\to 0^-}\dfrac{1}{x^2+2x} = \lim_{x\to 0^-}\dfrac{1}{x(x+2)} = \underline{\underline{-\infty}}$;

$\lim_{x\to 0^+}\dfrac{1}{x^2+2x} = \lim_{x\to 0^+}\dfrac{1}{x(x+2)} = \underline{\underline{+\infty}}$;

$\lim_{x\to 0}\dfrac{1}{x^2+x^3} = \lim_{x\to 0}\dfrac{1}{x^2(1+x)} = \underline{\underline{+\infty}}$; $\qquad \lim_{x\to 0}\dfrac{1}{x+x^2} = \lim_{x\to 0}\dfrac{1}{x(1+x)}$ $\nexists$; $\square$

iii) $\lim_{x\to 2^-}\dfrac{1}{\underbrace{\sqrt{x+2}-2}_{\text{infinit. neg}}} = \underline{\underline{-\infty}}$; $\qquad \lim_{x\to 2^+}\dfrac{1}{\underbrace{\sqrt{x+2}-2}_{\text{infinit. positivo}}} = \underline{\underline{+\infty}}$;

$\lim_{x\to 1^-}\dfrac{1}{\underbrace{\log x}_{\text{infinit. neg.}}} = \underline{\underline{-\infty}}$; $\qquad \lim_{x\to 1^+}\dfrac{1}{\underbrace{\log x}_{\text{infinit. positivo}}} = \underline{\underline{+\infty}}$; $\square$

iv) $\lim_{x\to +\infty}\dfrac{1}{3^x-1} = \underline{\underline{0}}$; $\qquad \lim_{x\to +\infty}\dfrac{1}{3^{-x}-1} = \underline{\underline{-1}}$; $\qquad \lim_{x\to 0^-}\dfrac{1}{\underbrace{3^x-1}_{\text{infinit. neg.}}} = \underline{\underline{-\infty}}$;

$\lim_{x\to 0^+}\dfrac{1}{\underbrace{3^x-1}_{\text{infinit. positivo}}} = \underline{\underline{+\infty}}$. $\blacksquare$

2) i) $\lim_{x\to -\infty}\dfrac{3-x}{2+2x} = \underline{\underline{-\tfrac{1}{2}}}$: $\qquad \dfrac{\cancel{x}(\overbrace{3/x}^{\to 0}-1)}{\cancel{x}(\underbrace{2/x}_{\to 0}+2)} \longrightarrow -\dfrac{1}{2}$.

$\lim_{x\to +\infty}\dfrac{3-x}{2+2x} = \underline{\underline{-\tfrac{1}{2}}}$: $\qquad$ //

$\lim_{x\to -\infty}\dfrac{x-x^3}{x^2+1} = \underline{\underline{+\infty}}$: $\qquad \dfrac{x^{\cancel{3}}(\overbrace{\tfrac{1}{x^2}}^{\to 0}-1)}{\cancel{x^2}(1+\underbrace{1/x^2}_{\to 0})} \longrightarrow +\infty$.

$\lim_{x\to +\infty}\dfrac{x-x^3}{x^2+1} = \underline{\underline{-\infty}}$ $\square$

ii) $\lim_{x\to -\infty}\dfrac{x-x^3}{x^3+1} = \underline{\underline{-1}}$: $\qquad \dfrac{\cancel{x^3}(\overbrace{\tfrac{1}{x^2}}^{\to 0}-1)}{\cancel{x^3}(1+\underbrace{1/x^3}_{\to 0})} \longrightarrow -1$.

$\lim_{x\to +\infty}\dfrac{x^2-x}{x+x^4} = 0$: $\qquad \dfrac{\cancel{x^2}(1-\overbrace{\tfrac{1}{x}}^{\to 0})}{x^{\cancel{4}\,2}(\underbrace{\tfrac{1}{x^3}}_{\to 0}+1)} \longrightarrow 0$.

$\lim_{x\to -1^-}\dfrac{1}{\underbrace{x^3+1}_{\text{infinit. neg}}} = \underline{\underline{-\infty}}$. $\qquad \lim_{x\to -1^+}\dfrac{1}{\underbrace{x^3+1}_{\text{infinit. positivo}}} = \underline{\underline{+\infty}}$ $\square$

iii) $\lim_{x\to+\infty} \frac{x^2+2^x}{3x^4+2^{x-1}} = \underline{\underline{2}}$ $\qquad \frac{\cancel{2^x}\left(\frac{x^2}{2^x}+1\right)}{\cancel{2^x}\left(3x^4/2^x+\frac{1}{2}\right)} \longrightarrow 2.$

$\lim_{x\to+\infty} \frac{x^2+2^7}{2^x+1} = \underline{\underline{0}}$ $\qquad : \frac{x^2\left(1+2^7/x^2\right)}{2^x\left(1+1/2^x\right)} \longrightarrow 0$

$\lim_{x\to-\infty} \frac{x^2+2^7}{2^x+1} = \underline{\underline{+\infty}}$ $\qquad : 2^x \to 0$ per $x\to-\infty$.

$\lim_{x\to+\infty} \frac{\log(1+x+x^3)}{4x} = \underline{\underline{0}}$ $\qquad \square$

iv) $\lim_{x\to+\infty} \frac{\log(1+x^2)}{e^{-x}} = \lim_{x\to+\infty} e^x \log(1+x^2) = \underline{\underline{+\infty}}$;

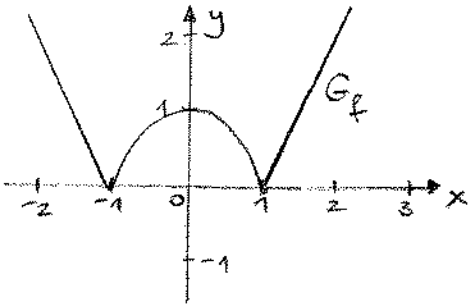
$\lim_{x\to-\infty} e^x \log(1+x^2) = \lim_{x\to-\infty} \frac{\log(1+x^2)}{e^{-x}} = \underline{\underline{0}}$;

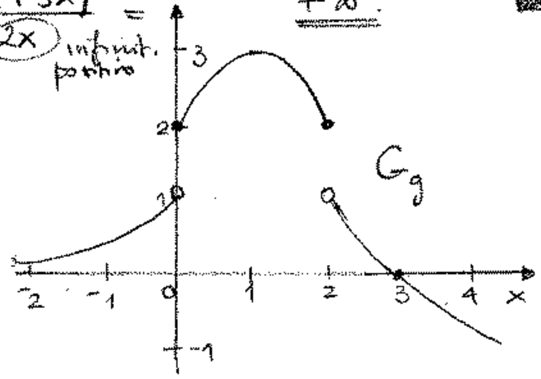
$\lim_{x\to0} \frac{\log(1+3x)}{4x} = \underline{\underline{\frac{3}{4}}}$ $\qquad : \frac{\log(1+3x)}{3x}\cdot\frac{3}{4} \xrightarrow[x\to0]{} \frac{3}{4}.$

$\lim_{x\to0^+} \frac{\log(2+3x)}{e^{\log(2x)}} = \lim_{x\to0^+} \frac{\log(2+3x)}{2x} = \underline{\underline{+\infty}}.$

$2x$ infinit. positivo

3)





$-x^2+2x+2 = -(x^2-2x)+2 = -(x-1)^2+3$

i) oss. che f è continua in tutti i pti $x \neq \pm 1$ (somma di funz. continue)

inoltre $\lim_{x\to-1^-}(2|x|-2) = \lim_{x\to-1^+}(-x^2+1) = f(-1) = 0$

$\lim_{x\to1^-}(-x^2+1) = \lim_{x\to1^+}(2|x|-2) = f(1) = 0$,

quindi f è continua su $\mathbb{R}$.

Oss. che g è continua in tutti i pt. $x\neq0$, $x\neq2$ (composizione, somma di funz. continue). Inoltre

$$\lim_{x\to0^-} e^x = 1 \neq \lim_{x\to0^+}(-x^2+2x+2) = f(0) = 2.$$

$$\lim_{x\to2^-}(-x^2+2x+2) = \underset{\substack{\| \\ f(2)}}{2} \neq \lim_{x\to2^+}\left(1-\log_2(x-1)\right) = 1 .$$

Quindi $x=0$ e $x=2$ sono pt. di discontinuità per g.

ii) $(f+g)(-3) = 2|-3|-2 + e^{-3} = \underline{4+\frac{1}{e^3}}$;

$(fg)\left(\frac{1}{2}\right) = \left(-\left(\frac{1}{2}\right)^2+1\right)\left(-\left(\frac{1}{2}\right)^2+\cancel{2}\cdot\frac{1}{\cancel{2}}+2\right) = \left(-\frac{1}{4}+1\right)\left(-\frac{1}{4}+3\right) = \frac{3}{4}\cdot\frac{11}{4} = \underline{\frac{33}{16}}$;

$\left(\frac{f}{g}\right)(2) = \frac{2}{2} = \underline{1}.$

$\lim\limits_{x\to-\infty} \frac{f(x)}{g(x)} = \lim\limits_{x\to-\infty} \frac{-2x-2}{e^x} = \underline{+\infty}$;

$\lim\limits_{x\to+\infty} \frac{g(x)}{f(x)} \quad \lim\limits_{x\to+\infty} \frac{1-\log(x-1)}{2x-2} = \underline{0}.$

4) $f: \mathbb{R}\setminus\{-2\} \to \mathbb{R}$ $\qquad f(x) = \frac{x^2}{x+2} = x-2+\frac{4}{x+2}$

$\underline{x=-2 \text{ asintoto verticale}}$;

$\underline{y=x-2 \text{ asintoto obliquo per } x\to-\infty \ (\text{per } x\to+\infty)}$.

$g: \mathbb{R}\setminus\{0\} \to \mathbb{R}$ $\qquad g(x) = \begin{cases} -\log|x| & \text{se } 0<|x|\le 1 \\ -\frac{1}{x^2}+1 & \text{se } |x|>1 \end{cases}$

$\underline{x=0 \text{ asintoto verticale}}$;

$\underline{y=1 \text{ asintoto orizzontale per } x\to-\infty \ (\text{per } x\to+\infty)}$.

5) i) $\lim\limits_{x\to-\infty} f(x)=0$; $\quad \lim\limits_{x\to-3^-} f(x) = 2$; $\quad \lim\limits_{x\to-3^+} f(x) = -3$;

$\lim\limits_{x\to-1^-} f(x) = +\infty$; $\quad \lim\limits_{x\to-1^+} f(x) = -2$; $\quad \lim\limits_{x\to+\infty} f(x) = -2.$

ii) pt. di discontinuità!

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−5 −4 −3 −2 −1 0 1 2 3 4 5

iii) su $]-\infty,-3[$ f è crescente;

su $]-3,-1[$ " ; su $[-1,0]$ f è decrescente;

su $[0,2]$ f è crescente, su $[2,+\infty[$ f è decrescente.

iv) pt. di discontinuità: $x=-3$, $x=-1$.

v) $\min\limits_{[-1,3]} f = -3 \qquad x=0$ pt. di minimo;

$\max\limits_{[-1,3]} f = 1 \qquad x=2$ pt. di massimo.