Ottava verifica settimanale di <u>ANALISI MATEMATICA</u> - CdL in STPCA - a.a. 2007/08 - Rovereto, <u>12 - 16</u> novembre 2007

1) i) $\left(\sqrt[4]{x}\right)^4 = x \quad \underline{\forall x \geq 0}$ ; $\quad \sqrt[5]{x^5} = x \quad \underline{\forall x \in \mathbb{R}}$;

$\sqrt{x^2} = |x| \quad \underline{\forall x \in \mathbb{R}}$;

$2^{\log_2(x-1)} = x-1 \quad \underline{\forall x > 1} \qquad \log_3\left(3^{x+1}\right) = x+1 \quad \underline{\forall x \in \mathbb{R}}$. □

ii) a) $2^{x^2}\cdot 2^{-3x} \leq 2^{2x} \iff 2^{x^2-3x} \leq 2^{2x}$

$\iff x^2-3x \leq 2x \qquad (2^x \uparrow)$

$\iff x^2-5x \leq 0 \implies \underline{S = [0,5]}$ ;

$\left(\frac{1}{3}\right)^{|x-1|}\cdot 3^x \geq 1 \iff 3^{-|x-1|+x} \geq 1$

$\iff -|x-1|+x \geq 0$

$\iff \begin{cases} x-1 \geq 0 \\ -(x-1)+x \geq 0 \end{cases}$ o $\begin{cases} x-1 < 0 \\ -(-x+1)+x \geq 0 \end{cases}$

$\iff \begin{cases} x \geq 1 \\ 1 \geq 0 \end{cases}$ o $\begin{cases} x < 1 \\ 2x-1 \geq 0 \end{cases}$

$\implies \underline{S = \left[\frac{1}{2}, +\infty\right[}$. □

ⓑ $\log(1+2x^2) - \log x^2 \geq \log 3 \iff \begin{cases} 1+2x^2 > 0 \quad \text{(sempre vero!)} \\ x \neq 0 \\ \log \dfrac{1+2x^2}{x^2} \geq \log 3 \end{cases}$

$\iff \begin{cases} x \neq 0 \\ \dfrac{1+2x^2}{x^2} \geq 3 \end{cases} \iff \begin{cases} x \neq 0 \\ 1+2x^2 \geq 3x^2 \end{cases} \iff \begin{cases} x \neq 0 \\ 1 \geq x^2 \end{cases}$

$\implies \underline{S = [-1,1]\setminus\{0\}}$. □

ⓒ $\log_{\frac{1}{2}}(|x|+1) + \log_{\frac{1}{2}} 4 \geq \log_{\frac{1}{2}} 2 - \log_{\frac{1}{2}}(2x)$

$\iff \begin{cases} 1+|x| > 0 \\ x > 0 \\ \log_{\frac{1}{2}} (1+|x|)(2x) \geq \log_{\frac{1}{2}} \frac{1}{2} \end{cases}$

$$\Longleftrightarrow \begin{cases} x>0 \\ 1+x>0 \\ (1+x)(2x) \leq \frac{1}{2} \end{cases} \Longleftrightarrow \quad \begin{matrix} x>0 \\ x>-1 \\ 2x^2+2x-\frac{1}{2} \leq 0 \end{matrix}$$

$$\Longleftrightarrow \begin{cases} x>0 \\ x>-1 \\ 4x^2+4x-1 \leq 0 \end{cases} \Longleftrightarrow \begin{cases} x>0 \\ x>-1 \\ \frac{-1-\sqrt{2}}{2} \leq x \leq \frac{-1+\sqrt{2}}{2} \end{cases}$$

$$\Longrightarrow \quad \underline{\underline{S = \left]0, \frac{-1+\sqrt{2}}{2}\right]}}. \qquad \square$$

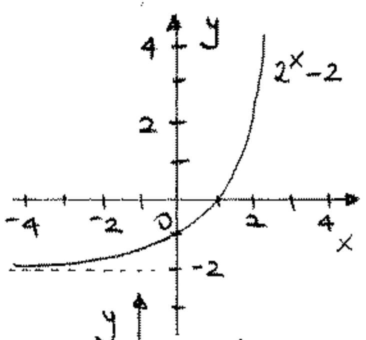
(d) $e^{2x} = e\, e^{x^2-4} \Longleftrightarrow e^{2x} = e^{x^2-3} \Longleftrightarrow x^2-3 = 2x$

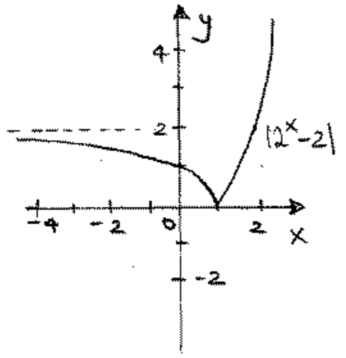
$\Longleftrightarrow x^2-2x-3=0 \Longrightarrow \underline{\underline{S=\{-1,3\}}}.$

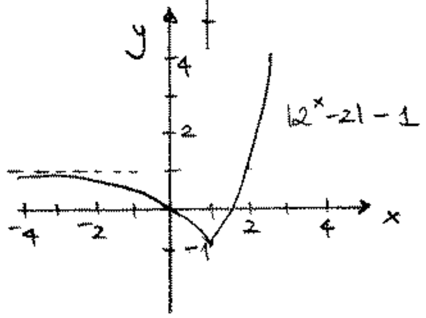
$$\log(2x) = \log(x^2-3) \Longleftrightarrow \begin{cases} x>0 \\ x^2-3>0 \\ 2x = x^2-3 \end{cases} \Longleftrightarrow \begin{cases} x>0 \\ x<-\sqrt{3} \ \text{ o } \ x>\sqrt{3} \\ x^2-2x-3=0 \end{cases}$$

$$\Longrightarrow S = \{3\}. \qquad \blacksquare$$

2) i) $|2^x-2|-1$ $\qquad \underline{\underline{\text{dom } f = \mathbb{R}}}$ ;

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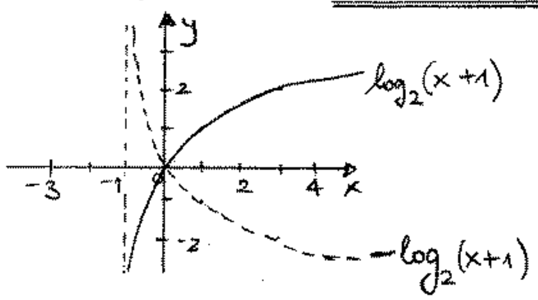
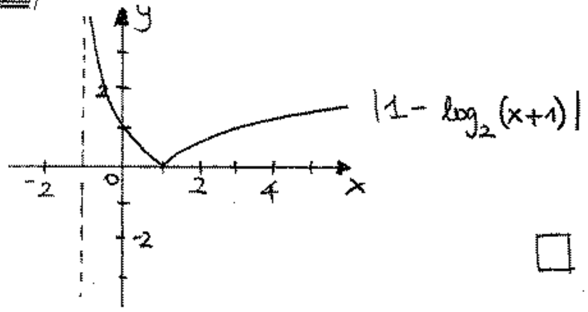
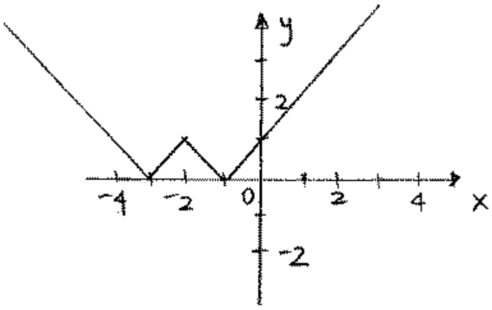
$|1-\log_2(x+1)|$ $\qquad \underline{\underline{\text{dom } f = ]-1,+\infty[}}$ ;




$\square$

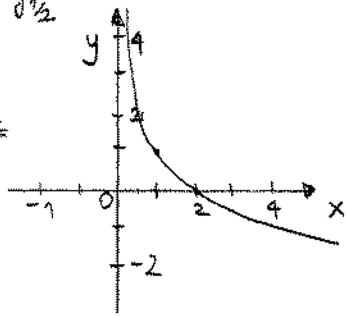
$||x+2|-1|$



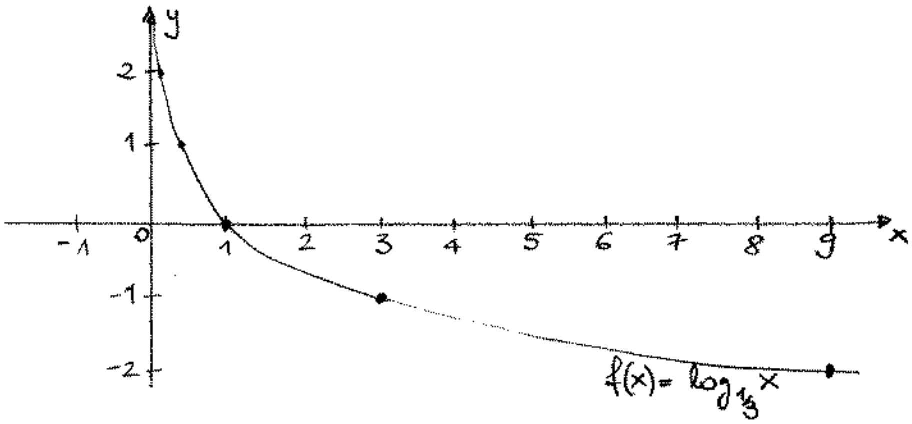

$\underline{\underline{\mathrm{dom}\, f = \mathbb{R}}}$ ;

$\log_{\frac{1}{2}} x - \log_{\frac{1}{3}} 3 = \log_{\frac{1}{2}} x + 1$  $\left( \log_{\frac{1}{3}} 3 = -1 \text{ !!} \right)$

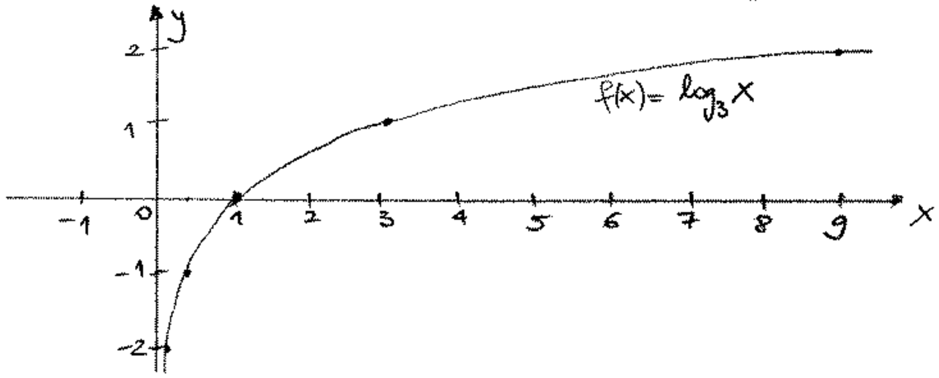
$\underline{\underline{\mathrm{dom}\, f = ]0, +\infty[}}$




ii) $\log_{\frac{1}{3}} \frac{1}{9} = 2 \quad \log_{\frac{1}{3}} \frac{1}{3} = 1 \quad \log_{\frac{1}{3}} 1 = 0 \quad \log_{\frac{1}{3}} 3 = -1 \quad \log_{\frac{1}{3}} 9 = -2$




iii) $\log_{3} \frac{1}{9} = -2 \quad \log_{3} \frac{1}{3} = -1 \quad \log_{3} 1 = 0 \quad \log_{3} 3 = 1 \quad \log_{3} 9 = 2$



3) Poniamo $f(x) = e^x + 2x - \frac{3}{2}$ continua in $\mathbb{R}$; osserviamo che $f(0) = -\frac{1}{2} < 0$ $\quad f(1) = e + \frac{1}{2} > 0$ ; quindi $\exists\, c \in \,]0,1[$ tale $f(c) = 0$ (tale $c$ è unico poiché $f(x)$ è strett. crescente essendo somma di due funzioni strett. crescenti : $e^x$ e $2x - \frac{3}{2}$ ).

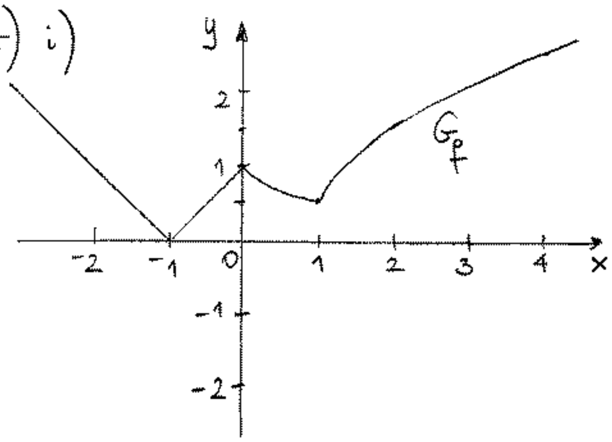
Notiamo che $a_1 = \frac{0+1}{2} = \frac{1}{2}$ e $f\left(\frac{1}{2}\right) = \sqrt{e} - \frac{1}{2} > 0$

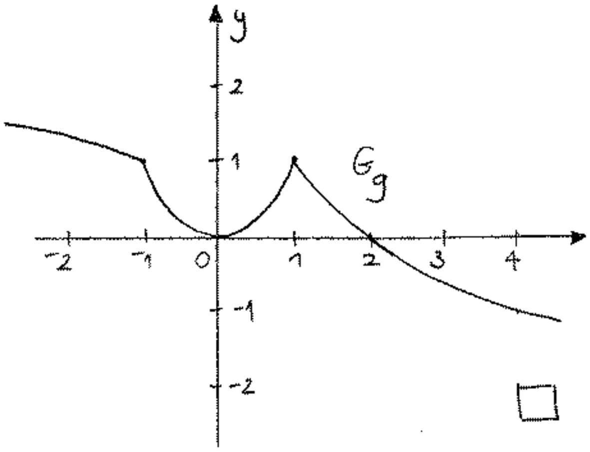
Quindi $c \in \,]0, \frac{1}{2}[$. Oss. inoltre che $a_2 = \frac{0 + \frac{1}{2}}{2} = \frac{1}{4}$ e $f\left(\frac{1}{4}\right) = \sqrt[4]{e} - 1 > 0$

Possiamo allora concludere che $c \in \,]0, \frac{1}{4}[$ .

Quindi $]a, b[ \;=\; ]0, \frac{1}{4}[$ !

4) i)




ii) $f$ e $g$ sono funzioni continue in $[-2,2]$; esse soddisfano il teorema di Weierstrass.

iii) $\min\limits_{[-2,2]} f = 0$ $\qquad x = -1$ pt. di min.

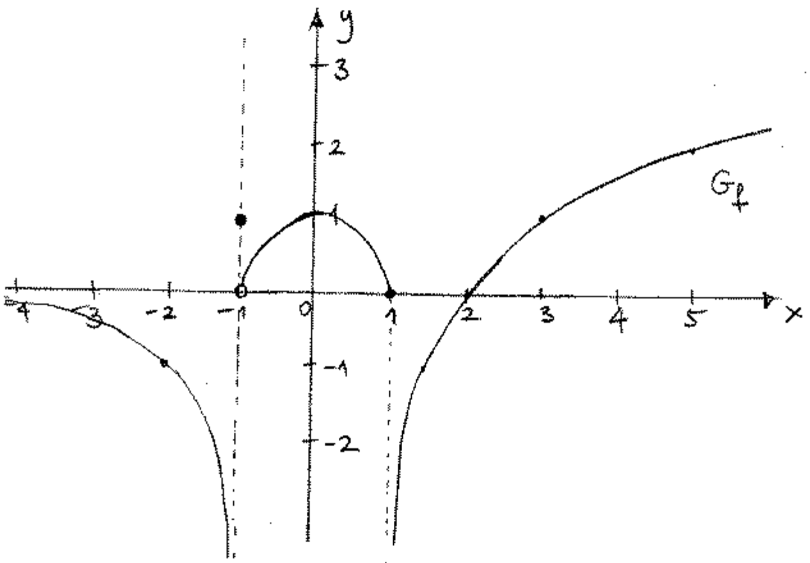
$\max\limits_{[-2,2]} f = f(2) = \frac{3}{2}$ $\qquad x = 2$ pt. di max.

$\min\limits_{[-2,2]} g = 0$ $\qquad x = 0$ , $x = 2$ pt. di min.

$\max\limits_{[-2,2]} g = -\sqrt[3]{-2} = \sqrt[3]{2}$ $\qquad x = -2$ pt. di max.

iv) $f\left([-1,2[\right) = [0, \frac{3}{2}[$ $\qquad g(\mathbb{R}) = \mathbb{R}$ .

5) i)




ii) $\lim\limits_{x\to-\infty} f(x)=0$ $\lim\limits_{x\to-1^-} f(x)=-\infty$ $\lim\limits_{x\to-1^+} f(x)=0$

$\lim\limits_{x\to1^-} f(x)=0$ $\lim\limits_{x\to1^+} f(x)=-\infty$ $\lim\limits_{x\to+\infty} f(x)=+\infty$ . □

iii) $\lim\limits_{x\to-1^+} f(x)=0 \neq f(-1)=1$

$\lim\limits_{x\to1^-} f(x)=0 = f(1)$ . ■