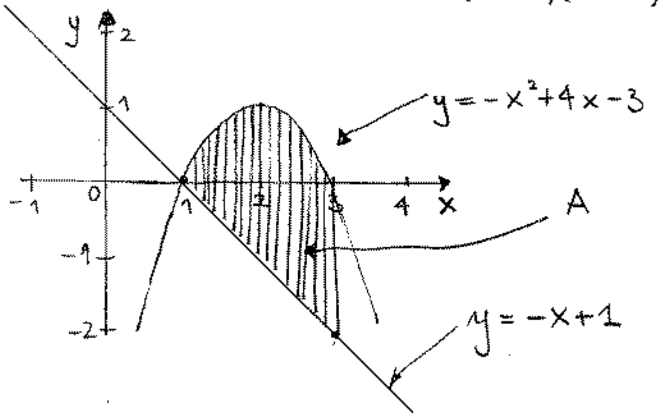
Quarta verifica settimanale di ANALISI MATEMATICA — CdL in STPCA — a.a. 2007/08 — Rovereto, 8 – 12/10/07

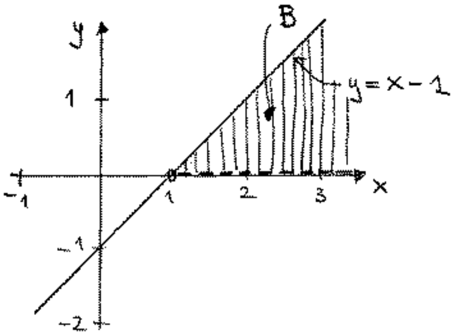
1) i) $A = \{(x,y) \in \mathbb{R}^2 : \ 1 \le x \le 3, \ -x+1 \le y \le -x^2+4x-3\}$

$B = \{(x,y) \in \mathbb{R}^2 : \ 0 < y \le x-1\}$

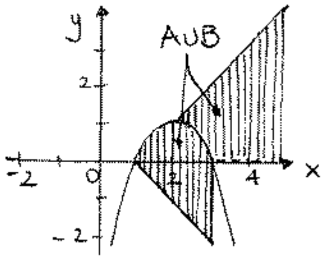
Oss. $-x^2+4x-3 = -(x^2-4x)-3 = -(x-2)^2+4-3 = -(x-2)^2+1$

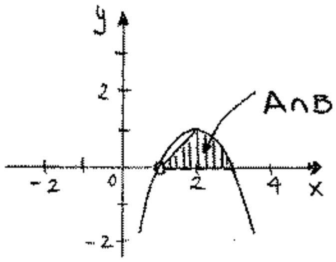
$-x^2+4x-3=0 \iff -(x-1)(x-3)=0 \Rightarrow x=1, \ x=3.$

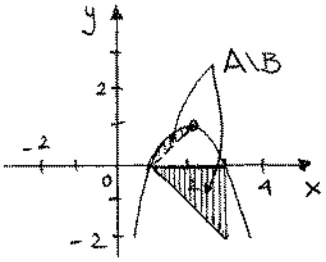




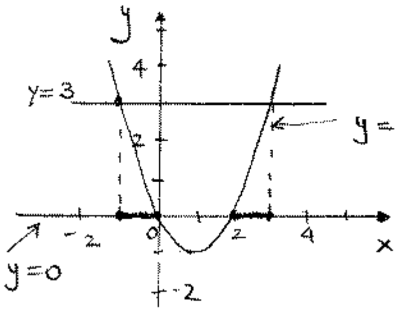
ii)







2) i) • $0 \le x^2-2x \le 3 \iff \begin{cases} x^2-2x \ge 0 \\ x^2-2x-3 \le 0 \end{cases} \iff \begin{matrix} x \le 0 \ \text{ o } \ x \ge 2 \\ -1 \le x \le 3 \end{matrix}$



$\Rightarrow \quad \underline{\underline{S = [-1,0] \cup [2,3]}}$

• $-x^2+1 = x+k \iff x^2+x+(k-1)=0$

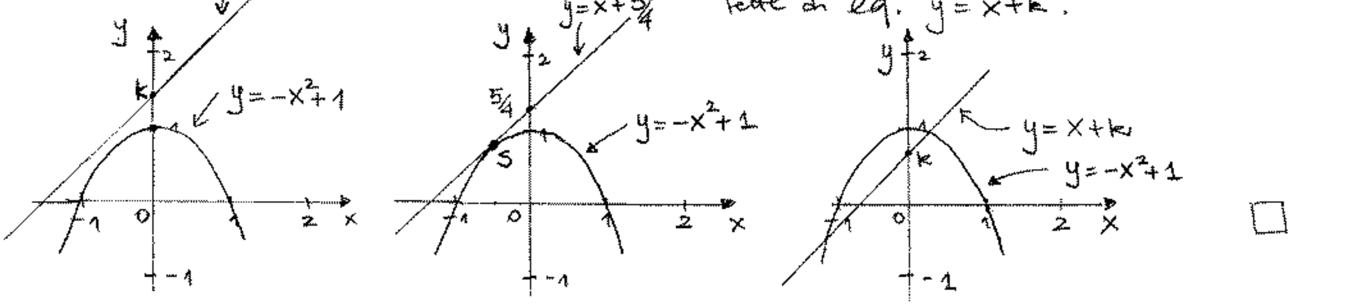
$$x_{1/2} = \frac{-1 \pm \sqrt{1-4(k-1)}}{2} =$$

$$= \frac{-1 \pm \sqrt{5-4k}}{2}$$

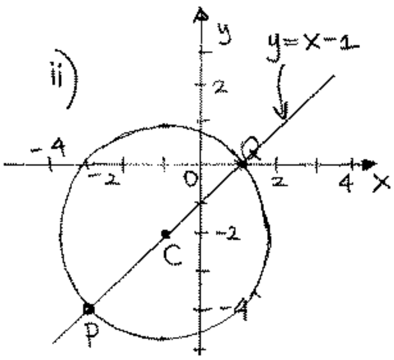
Segue allora che per $k > 5/4$ $\nexists$ sono pt. di intersezioni tra la parabola di eq. $y = -x^2+1$ e le rette di eq. $y = x+k$;

per $k = 5/4$ $\exists$ un pt. di intersezione tra la parabola di eq. $y = -x^2+1$ e la retta di eq. $y = x + 5/4$; il pt. di intersezione è $S = (-\frac{1}{2}, \frac{3}{4})$;

per $k < 5/4$ $\exists$ sono due pt. di intersezioni tra la parabola di eq. $y = -x^2+1$ e le rette di eq. $y = x+k$.



ii)



$C = (-1,-2)$ $\quad r = 2\sqrt{2}$

$(x+1)^2 + (y+2)^2 = 8$ eq. della circonferenza

L'intersezione con la retta di eq. $x - y - 1 = 0$ si ottiene risolvendo il sistema

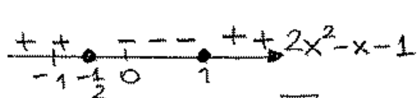
$$\begin{cases} (x+1)^2 + (y+2)^2 = 8 \\ y = x-1 . \end{cases}$$

Abbiamo
$$\begin{cases} (x+1)^2 + (x+1)^2 = 8 \\ y = x-1 \end{cases} \iff \begin{cases} 2(x+1)^2 = 8 \\ y = x-1 \end{cases} \iff \begin{cases} (x+1)^2 = 4 \\ y = x-1 \end{cases}$$

$$\iff \begin{cases} (x+1) = \pm 2 \\ y = x-1 \end{cases} \iff \begin{array}{l} P = (-3,-4) \\ Q = (1,0); \quad \underline{\underline{d(P,Q) = 4\sqrt{2}.}} \end{array}$$

3) • $\dfrac{(x-2)(x+1)}{x^2+1} < -1 \iff \dfrac{x^2-x-2}{x^2+1} + 1 < 0 \iff \dfrac{x^2-x-2+x^2+1}{x^2+1} < 0$

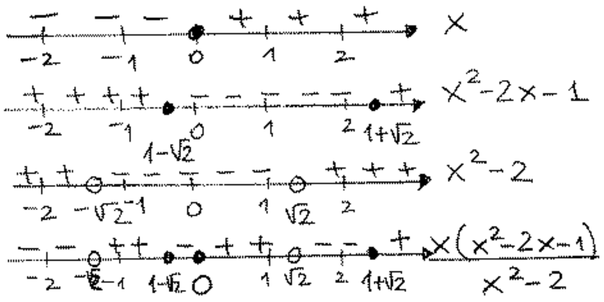
$$\iff \frac{2x^2-x-1}{x^2+1} < 0$$



$\underline{\underline{S =]-\frac{1}{2}, 1[.}}$

• $\dfrac{(x^2-2x)x}{x^2-2} \leq \dfrac{x}{x^2-2} \iff \dfrac{(x^2-2x)x - x}{x^2-2} \leq 0$

$\iff \dfrac{x(x^2-2x-1)}{x^2-2} \leq 0$

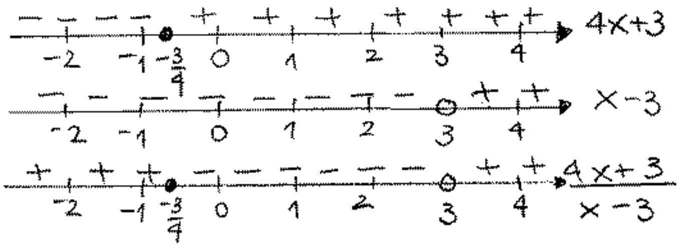


$S = \left]-\infty, -\sqrt{2}\right[\cup \left[1-\sqrt{2}, 0\right] \cup$
$\cup \left]\sqrt{2}, 1+\sqrt{2}\right]$

• $\dfrac{x(x+2)}{x-3} < x+1 \iff \dfrac{x^2+2x-(x+1)(x-3)}{x-3} < 0$

$\iff \dfrac{\cancel{x^2}+2x-\cancel{x^2}+3x-x+3}{x-3} < 0$

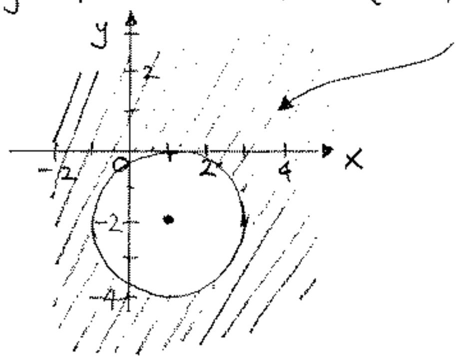
$\iff \dfrac{4x+3}{x-3} < 0$



$S = \left]-\frac{3}{4}, 3\right[$

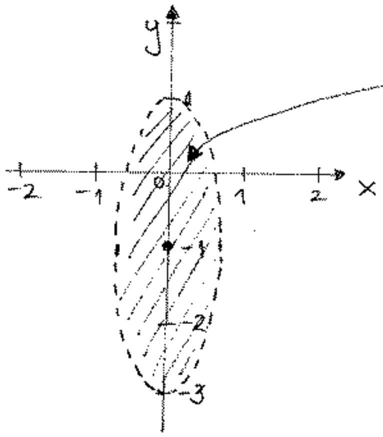
• $x^3+3x^2+6x \leq 0 \iff x(x^2+3x+6) \leq 0$ $S = \left]-\infty, 0\right]$

4) i) $3(x-1)^2 + 3(y+2)^2 \geq 12 \iff (x-1)^2 + (y+2)^2 \geq 4$



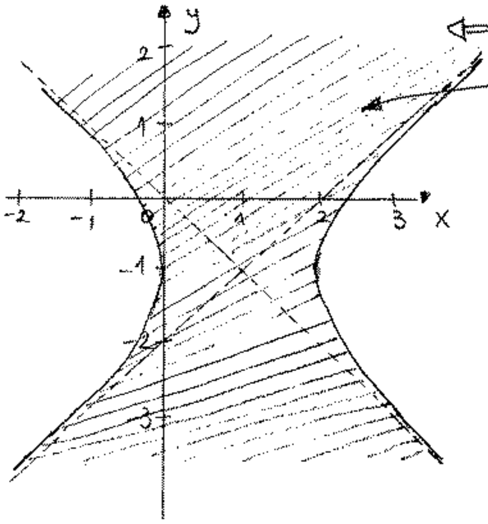
ii) $9x^2+y^2+2y<3 \iff 9x^2+(y+1)^2<4$

$\iff \frac{x^2}{4/9}+\frac{(y+1)^2}{4}<1$



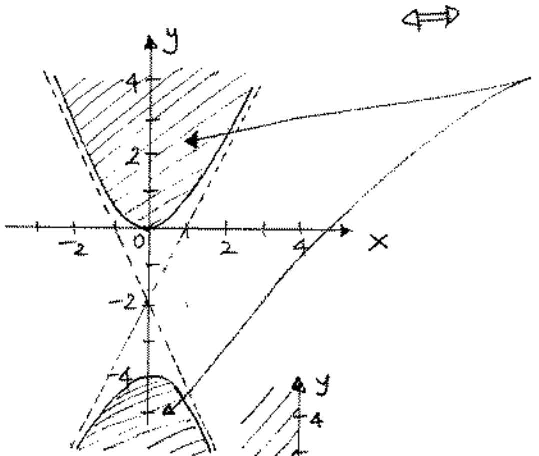
iii) $x^2-2x-y^2-2y-1\leq 0 \iff (x-1)^2-1-(y+1)^2\leq 0$

$\iff (x-1)^2-(y+1)^2\leq 1$

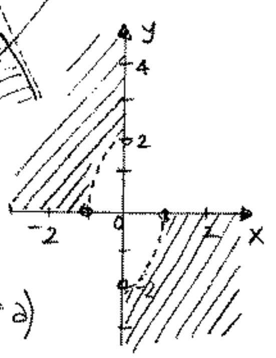


iv) $-4x^2+y^2+4y\geq 0 \iff -4x^2+(y+2)^2-4\geq 0$

$\iff -x^2+\frac{(y+2)^2}{4}\geq 1$



5) a)



//// = a)

b)