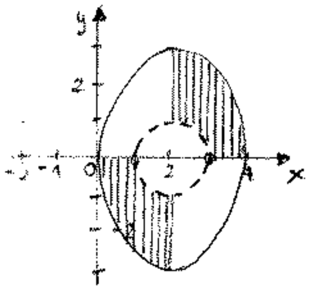
Quinta verifica settimanale di <u>ANALISI MATEMATICA</u> - CdL in STPA -
a.a. 2007/08 - Rovereto, <u>15 - 19 ottobre 2007</u>

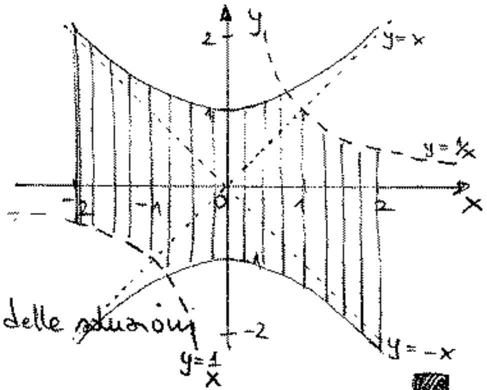
1) $$\begin{cases} xy \geq 2y \\ x^2 - 4x + y^2 + 3 > 0 \\ 9(x-2)^2 + 4y^2 \leq 36 \end{cases} \iff \begin{cases} y(x-2) \geq 0 \\ (x-2)^2 + y^2 > 1 \\ \dfrac{(x-2)^2}{4} + \dfrac{y^2}{9} \leq 1 \end{cases}$$

$$\iff \begin{cases} y \geq 0, \quad x \geq 2 \\ (x-2)^2 + y^2 > 1 \\ \dfrac{(x-2)^2}{4} + \dfrac{y^2}{9} \leq 1 \end{cases} \quad \text{oppure} \quad \begin{cases} y \leq 0, \quad x \leq 2 \\ (x-2)^2 + y^2 > 1 \\ \dfrac{(x-2)^2}{4} + \dfrac{y^2}{9} \leq 1 \end{cases}$$



= l'insieme delle soluzioni

$$\begin{cases} xy < 1 \\ -x^2 + y^2 - 1 \leq 0 \\ -2 \leq x \leq 2 \end{cases} \iff \begin{cases} xy < 1 \\ -x^2 + y^2 \leq 1 \\ -2 \leq x \leq 2 \end{cases}$$



= l'insieme delle soluzioni

2) i) $A = \{x \in \mathbb{R} : 0 \leq x^2 - 1 \leq 3\} = \{x \in \mathbb{R} : 1 \leq x^2 \leq 4\} = \underline{[-2,-1] \cup [1,2]}$

A è limitato inferiormente (infatti, $\forall x \in A$, $x \geq -2$)

limitato superiormente (infatti, $\forall x \in A$, $x \leq 2$);

$\underline{\min A = -2}$, $\underline{\max A = 2}$.

ii) $B = \{x \in \mathbb{R} : \frac{1}{x^2} > 4\} \iff \{x \in \mathbb{R} \setminus \{0\} : x^2 < \frac{1}{4}\} = \underline{\left]-\frac{1}{2}, \frac{1}{2}\right[\setminus \{0\}}$

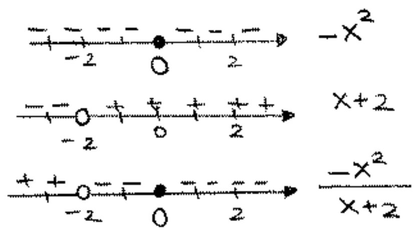
B è limitato inferiormente (infatti, $\forall x \in B$, $x \geq -\frac{1}{2}$)

limitato superiormente (infatti, $\forall x \in B$, $x \leq \frac{1}{2}$);

Osserviamo che $\nexists \min B$ e $\nexists \max B$. □

iii) $C = \left\{ x \in \mathbb{R} : \frac{x^2(x+1)}{x+2} \geq x^2 \right\} = \left\{ x \in \mathbb{R} : \frac{x^2(x+1) - x^2(x+2)}{x+2} \geq 0 \right\}$

$= \left\{ x \in \mathbb{R} : \frac{-x^2}{x+2} \geq 0 \right\} = \underline{\underline{]-\infty, -2[\cup \{0\}}}$;



C non è limitato inferiormente

(infatti, $\forall m \in \mathbb{R}\ \exists x \in C : x < m$)

C è limitato superiormente (infatti $\forall x \in C$, $x \leq 0$)

$\underline{\underline{\nexists \min C}}$, $\underline{\underline{\max C = 0.}}$ □

iv) $D = \{ x \in \mathbb{R} : x \geq 2,\ x^2 - 2x - 3 < 0 \} = \{ x \in \mathbb{R} : x \geq 2,\ x \in]-1, 3[\}$

$= \underline{\underline{[2, 3[}}$.

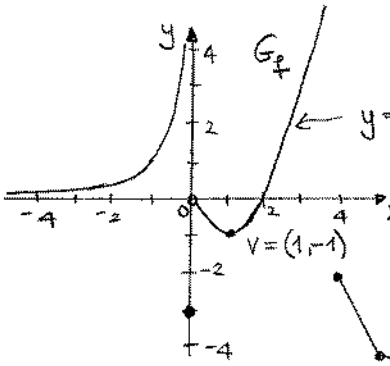
D è limitato inferiormente (infatti, $\forall x \in D$, $x \geq 2$)

D è limitato superiormente (infatti, $\forall x \in D$, $x < 3$).

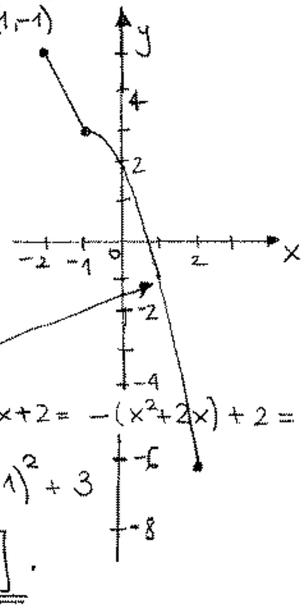
$\underline{\underline{\min D = 2}}$, $\underline{\underline{\nexists \max D}}$. ■

3) i)

$$f(x) = \begin{cases} -\frac{1}{x} & \text{se } x < 0 \\ -3 & \text{se } x = 0 \\ x^2 - 2x & \text{se } x > 0 \end{cases}$$



$$g(x) = \begin{cases} -2x + 1 & \text{se } -2 \leq x < -1 \\ -x^2 - 2x + 2 & \text{se } -1 \leq x \leq 2 \end{cases}$$



$y = -x^2 - 2x + 2 = -(x^2 + 2x) + 2 =$

$= -(x+1)^2 + 3$

ii) $\underline{\underline{f(-1) = 1}}$ $\underline{\underline{f\left(\frac{1}{2}\right)}} = \frac{1}{4} - \cancel{2} \cdot \frac{1}{\cancel{2}} = \underline{\underline{-\frac{3}{4}}}$. □

$g(-2) = \underline{\underline{5}} > g(2) = \underline{\underline{-6}}$. □

iii) $\underline{\underline{f(\mathbb{R}) = \{-3\} \cup [-1, +\infty[}}$ $\underline{\underline{g([-1, 1]) = [-1, 3]}}$. □

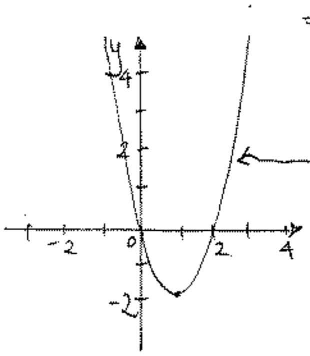
iv) f non è iniettiva (per esempio: $x_1 = -1/3 \neq x_2 = 3$, ma $f\left(\frac{1}{3}\right) = 3 = f(3)$)
f non è suriettiva ($f(\mathbb{R}) \subsetneq \mathbb{R}$) ;

g è iniettiva $\quad$ (se $x_1 \neq x_2$ allora $g(x_1) \neq g(x_2)$).

g è suriettiva $\quad$ $\left(g([-2,2]) = [-6,5]\right)$.

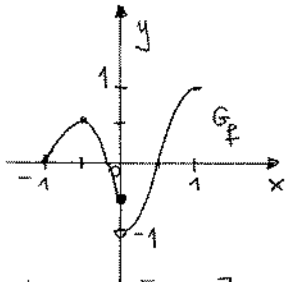
4) $f:\mathbb{R}\to\mathbb{R}$ $\quad f(x) = 2x^2-4x = 2x(x-2)$

$$= 2(x^2-2x) = 2(x-1)^2-2$$

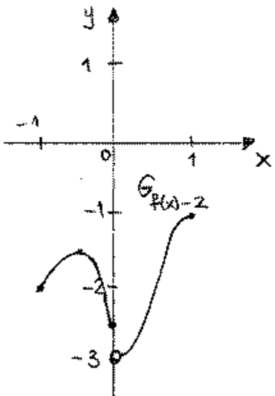


Possiamo scegliere $A = [1,+\infty[$ (oppure $A =]-\infty,1]$ o dei loro sottoinsiemi); come B dovremo prendere $B = f(\mathbb{R}) = [-2,+\infty[$.

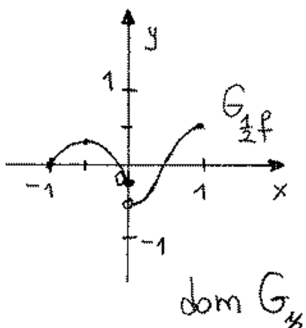
5)

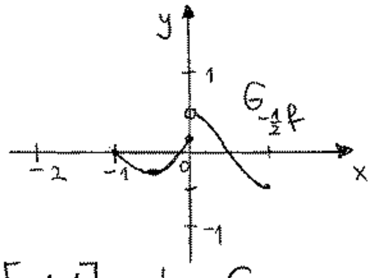


dom f = $[-1,1]$

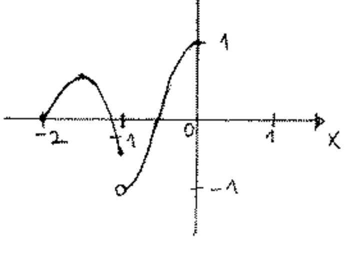


$\mathrm{dom}(f(x)-2) = [-1,1]$





$\mathrm{dom}\, G_{\frac12 f} = [-1,1] = \mathrm{dom}\, G_{-\frac12 f}$



$\mathrm{dom}\, G_{f(x+1)} = [-2,0]$.