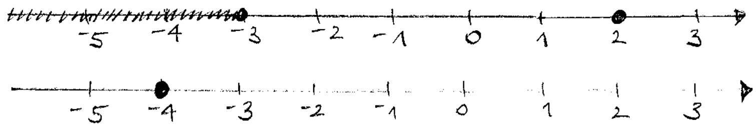
Seconda verifica settimanale di $\underline{\text{ANALISI MATEMATICA}}$ – CdL in STPCA – a.a. 2007/08 – Rovereto, $\underline{24/09 - 28/09/07}$

1) i) $A =]-\infty, -3] \cup \{2\}$; $B = \left\{ x \in \mathbb{Z} : \frac{(x^2-2)(x+4)}{x-1} = 0 \right\} = \{-4\}$.



$A \cup B = A$ (V) poichè $B \subset A$;

$A \cap B = \{-4\}$ (V) poichè $B \subset A$

$B \subseteq A$ (V) ; $1 \in B$ (F) $\{-\sqrt{2}, \sqrt{2}\} \subseteq B$ (F) poichè $-\sqrt{2}, \sqrt{2} \notin \mathbb{Z}$;

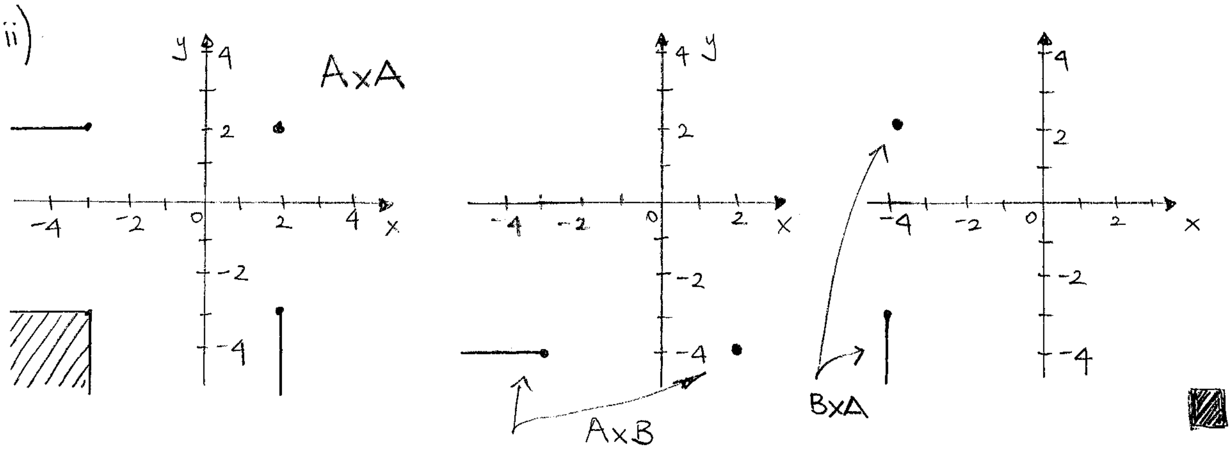
$\{-4\} \in B$ (F) poichè $-4 \in B$, oppure $\{-4\} \subseteq B$.

$-4 \in B$ (V) ; $\{-4\} \in \mathcal{P}(A)$ (V) (poichè $\{-4\}$ è un sottoinsieme di A)

$\{\{-4\}, \emptyset\} = \mathcal{P}(B)$ (V) (per def. di $\mathcal{P}(B)$) ; $[-3, 2[\subseteq \mathbb{R} \setminus A$ (F) poichè $-3 \notin \mathbb{R}\setminus A$

$B \setminus A = \emptyset$ (V) poichè $B = \{-4\}$ e $-4 \in A$. □

ii)



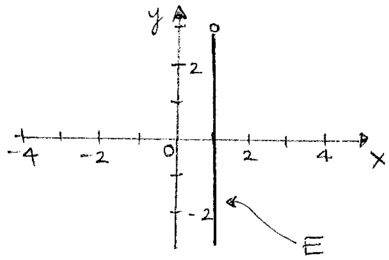
2) i) $A = \left\{ x \in \mathbb{R} : \frac{x^2 - x}{x^2+1} < 1 \right\} = \left\{ x \in \mathbb{R} : \frac{\cancel{x^2} - x - \cancel{x^2} - 1}{x^2+1} < 0 \right\} = \left\{ x \in \mathbb{R} : \frac{x+1}{x^2+1} > 0 \right\}$
$= \underline{]-1, +\infty[}$

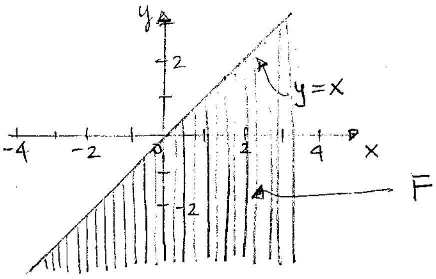
$B = \{ x \in \mathbb{R} : -2x \geq 3x - 1 \} = \{ x \in \mathbb{R} : 5x \leq 1 \} = \{ x \in \mathbb{R} : x \leq \frac{1}{5} \} = \underline{\underline{]-\infty, \frac{1}{5}]}}$.

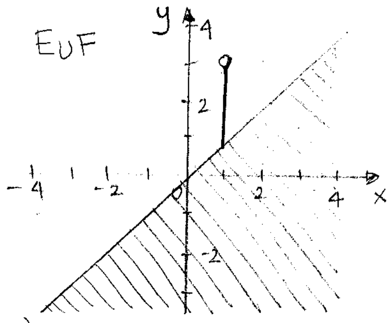
ii) $A \cup B = \underline{\underline{\mathbb{R}}}$; $A \cap B = \underline{\underline{]-1, \frac{1}{5}]}}$; $A \setminus B = \underline{\underline{]\frac{1}{5}, +\infty[}}$;

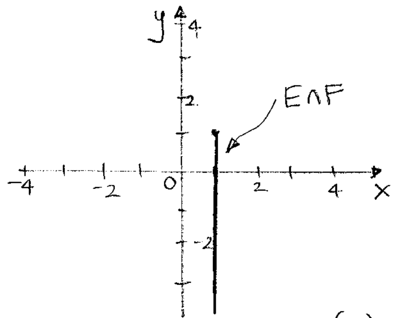
Tutti e tre questi insiemi sono intervalli di $\mathbb{R}$.

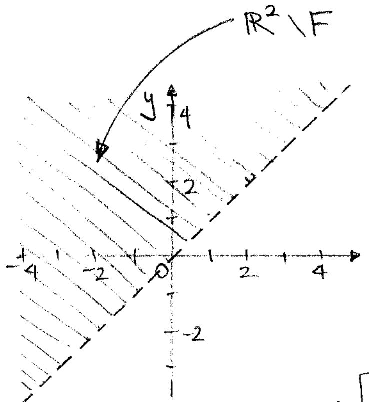
3) i)







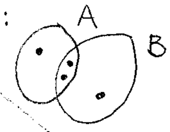




ii) $(1,3) \in \mathbb{R}^2 \setminus F$ (V) ; $\{(1,1)\} \in F$ (F) (vero è che $(1,1) \in F$) ;
$\{(x,0) \in \mathbb{R}^2 : x < 0\} \subseteq \mathbb{R}^2 \setminus F$ (V) ; $(1,-2) \in E \cap F$ (V).

4) i) $\exists!\, x : (x \in A)$ e $(x \in B)$ (F) (non è detto che esiste unico!)

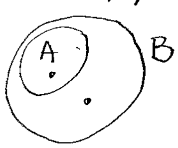
Es:



per esempio!

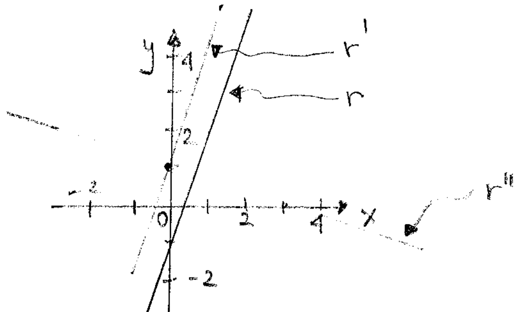
ii) $\exists\, x : (x \in A)$ e $(x \in B)$ (V)
(definizione che $A \cap B \neq \emptyset$).

iii) $\exists\, x \in A : x \notin B$ (F) Es:



iv) $(A \subseteq B) \Rightarrow (B \setminus A \neq \emptyset)$ (V) (altrimenti si avrebbe $A = B$).

5) i) $y = 3x - 1$



ii) r' ha pendenza $m = 3$. Affinché $P = (1,4) \in r'$ deve valere $4 = 3 + q$, ossia $q = 1$. L'eq. di r' risulta dunque $\underline{y = 3x + 1}$.

iii) $Q = (0,1)$. La pendenza della retta r'' è $m = -\frac{1}{3}$. Affinché $Q \in r''$ deve valere $1 = q$. L'eq. di r'' risulta dunque $y = -\frac{1}{3}x + 1$.