Undicesima verifica settimanale di **ANALISI MATEMATICA** — CdL in STPCA —
a.a. 2007/08 — Rovereto, 3 – 7 dicembre 2007

$$1)\,i)\ |x^2-4x| \le 3 \iff -3 \le x^2-4x \le 3 \iff \begin{cases} x^2-4x+3 \ge 0 \\ x^2-4x-3 \le 0 \end{cases}$$

$$\iff \begin{cases} x \le 1 \text{ o } x \ge 3 \\ 2-\sqrt{7} \le x \le 2+\sqrt{7} \end{cases} \Rightarrow \underline{\underline{S=\left[2-\sqrt{7},1\right]\cup\left[3,2+\sqrt{7}\right]}}$$

$$2x^2-2|x|>0 \iff \begin{cases} x \ge 0 \\ 2x^2-2x>0 \end{cases} \text{ o } \begin{cases} x<0 \\ 2x^2+2x>0 \end{cases}$$

$$\iff \begin{cases} x \ge 0 \\ x<0 \text{ o } x>1 \end{cases} \text{ o } \begin{cases} x<0 \\ x<-1 \text{ o } x>0 \end{cases}$$

$$\Rightarrow \underline{\underline{S= \left]-\infty,-1\right[\cup \left]1,+\infty\right[.}}$$

$$|x+1|+3x<-x^2+1 \iff \begin{cases} x+1 \ge 0 \\ x+1+3x<-x^2+1 \end{cases} \text{ o } \begin{cases} x+1<0 \\ -(x+1)+3x<-x^2+1 \end{cases}$$

$$\iff \begin{cases} x \ge -1 \\ x^2+4x<0 \end{cases} \text{ o } \quad \begin{matrix} x<-1 \\ x^2+2x-2<0 \end{matrix}$$

$$\iff \begin{cases} x \ge -1 \\ -4<x<0 \end{cases} \text{ o } \begin{cases} x<-1 \\ -1-\sqrt{3}<x<-1+\sqrt{3} \end{cases} \Rightarrow \underline{\underline{S=\left]-1-\sqrt{3},0\right[.}}$$

$$\log_2(1-|x|)<1 \iff \log_2(1-|x|)<\log_2 2 \iff \begin{cases} 1-|x|>0 \\ 1-|x|<2 \end{cases}$$

$$\iff \begin{cases} |x|<1 \\ |x|>-1 \leftarrow \text{questo è sempre vero!} \end{cases} \Rightarrow \underline{\underline{S=\left]-1,1\right[.}}$$

$\square$

$$ii)\ \log(x+2)+\log(2x-1)>\log 2 \iff \begin{cases} x+2>0 \\ 2x-1>0 \\ \log(x+2)(2x-1)>\log 2 \end{cases}$$

$$\iff \begin{cases} x>-2 \\ x>\frac{1}{2} \\ (x+2)(2x-1)>2 \end{cases} \iff \begin{cases} x>\frac{1}{2} \\ 2x^2+3x-4>0 \end{cases}$$

$$\iff \begin{cases} x>\frac{1}{2} \\ x<\frac{-3-\sqrt{41}}{4} \text{ o } x>\frac{-3+\sqrt{41}}{4} \end{cases} \Rightarrow \underline{\underline{S=\left]\frac{-3+\sqrt{41}}{4},+\infty\right[.}}$$

$$\left(\frac{1}{2}\right)^{-|x+1|+1}<2^x \iff 2^{|x+1|-1}<2^x \iff |x+1|-1<x$$

$$\Longleftrightarrow \quad \begin{cases} x+1 \geqslant 0 \\ x+1-1<x \end{cases} \quad \text{o} \quad \begin{cases} x+1<0 \\ -(x+1)-1<x \end{cases}$$

$$\Longleftrightarrow \quad \begin{cases} x \geqslant -1 \\ 0<0 \end{cases} \quad \text{o} \quad \begin{cases} x<-1 \\ -2<2x \end{cases} \Longleftrightarrow \begin{cases} x<-1 \\ -1<x \end{cases}$$

$$\Rightarrow \underline{\underline{S=\emptyset.}}$$

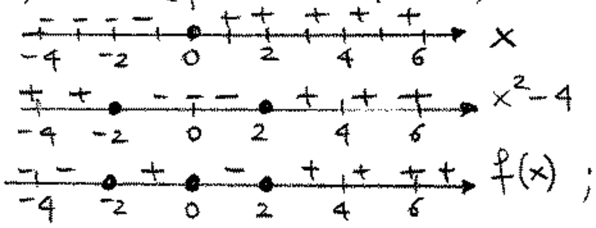
$$3^{\log_3(|x-1|-2)}=4 \quad \Longleftrightarrow \quad \begin{cases} |x-1|-2>0 \\ |x-1|-2=4 \end{cases} \Longleftrightarrow \begin{cases} |x-1|>2 \\ |x-1|=6 \end{cases}$$

$$\Rightarrow \underline{\underline{S=\{-5,7\}}}.$$

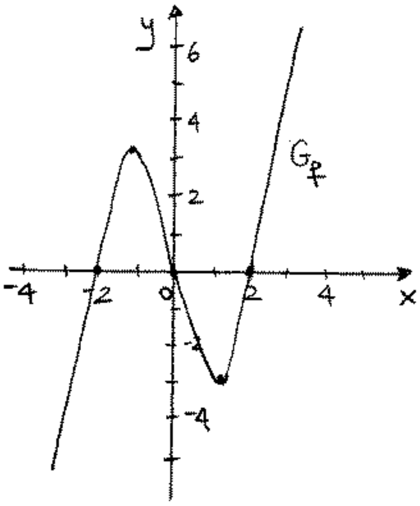
■

2) ⓐ $\underline{\underline{f(x)=x^3-4x}}$

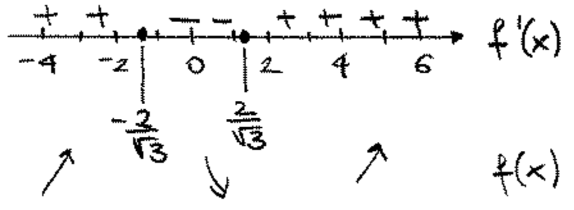
- $\operatorname{dom} f = \mathbb{R} =]-\infty,+\infty[$; (funzione dispari!)
- $x^3-4x = x(x^2-4)$



- $\lim_{x\to-\infty} f(x) = -\infty \qquad \lim_{x\to+\infty} f(x) = +\infty$;
- f è continua (prodotto e somma di funz. continue) ;
- $\operatorname{dom} f' = \mathbb{R} \qquad f'(x) = 3x^2-4$



$f'(x)=0 \Longleftrightarrow x=-\frac{2}{\sqrt3} \quad x=\frac{2}{\sqrt3}$ pt. critici

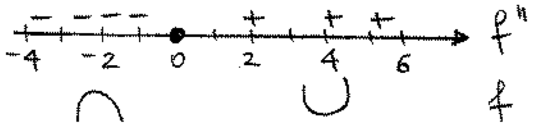


$x=-\frac{2}{\sqrt3}$ pt. di max loc. stretto

$$f\left(-\frac{2}{\sqrt3}\right) = -\frac{8}{3\sqrt3}+\frac{8}{\sqrt3} = \frac{16}{3\sqrt3}$$

$$f\left(\frac{2}{\sqrt3}\right) = \frac{8}{3\sqrt3}-\frac{8}{\sqrt3} = -\frac{16}{3\sqrt3} ;$$

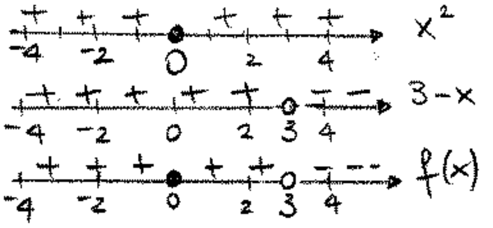
- $\operatorname{dom} f'' = \mathbb{R} \quad f''(x) = 6x$



□

$f(x) = \frac{x^2}{3-x}$

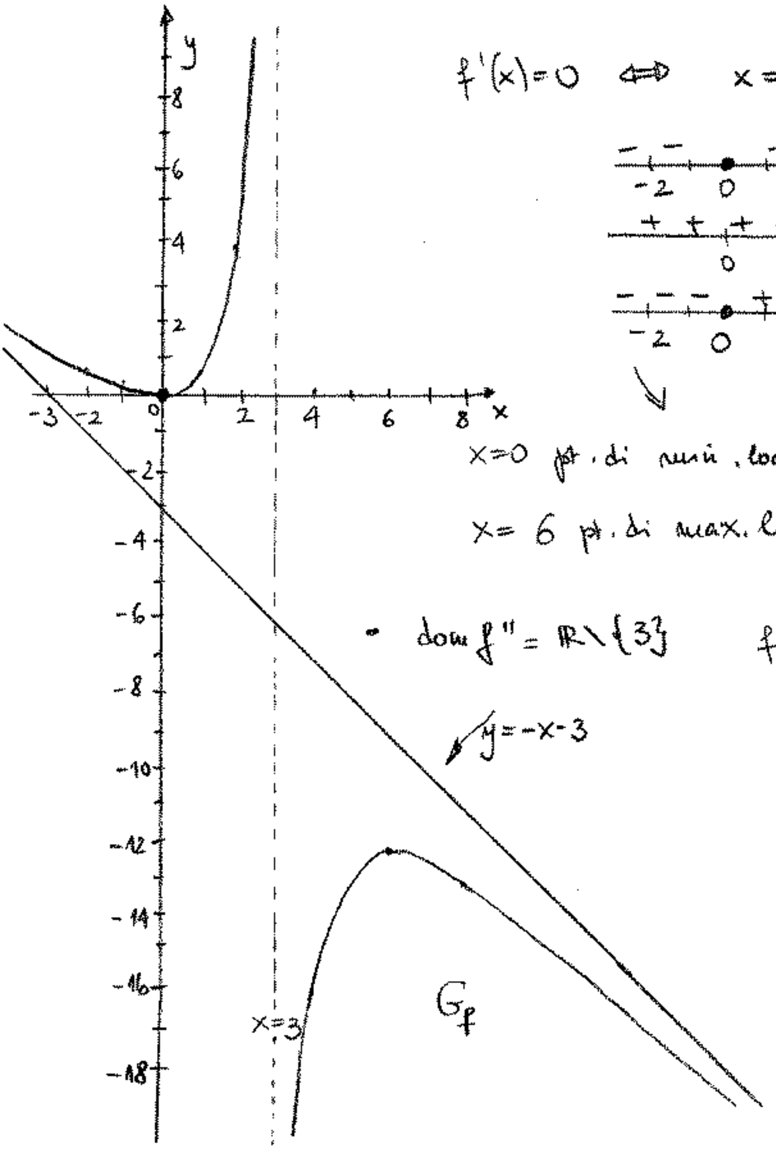
- $\operatorname{dom} f = \mathbb{R}\setminus\{3\} =]-\infty,3[\cup]3,+\infty[$;
- segno di f



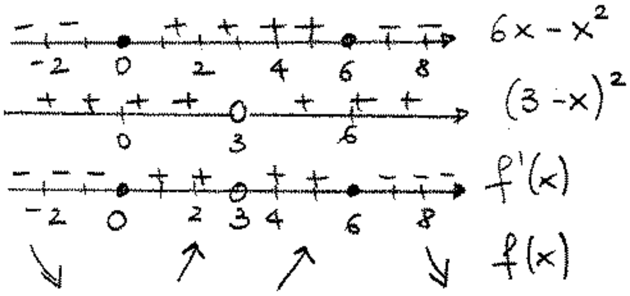
- $\lim\limits_{x\to-\infty} f(x) = +\infty$ $\lim\limits_{x\to+\infty} f(x) = -\infty$; oss. che $f(x) = -x-3+\frac{9}{3-x}$ e quindi $y=-x-3$ è asint. obliquo per f per $x\to-\infty$ (per $x\to+\infty$);

  $\lim\limits_{x\to3^-} f(x) = +\infty$ $\lim\limits_{x\to3^+} f(x) = -\infty$;

- f è continua nel suo dominio (rapporto di funz. continue);
- $\operatorname{dom} f' = \mathbb{R}\setminus\{3\}$ $f'(x) = \frac{2x(3-x)+x^2}{(3-x)^2} = \frac{6x-x^2}{(3-x)^2}$



$f'(x)=0 \iff x=0\ ,\ x=6$ pt. critici per f



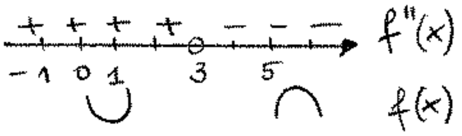
$x=0$ pt. di min. loc. (stretto) e $f(0)=0$

$x=6$ pt. di max. loc. (stretto) e $f(6) = \frac{36}{3-6} = -12$

- $\operatorname{dom} f'' = \mathbb{R}\setminus\{3\}$ $f''(x) = \frac{(6-2x)(3-x)^{\cancel{2}} + (6x-x^2)2\cancel{(3-x)}}{(3-x)^{\cancel{4}3}}$

$$= \frac{18-\cancel{6x}-\cancel{6x}+\cancel{2x^2}+\cancel{12x}-\cancel{2x^2}}{(3-x)^3}$$

$$= \frac{18}{(3-x)^3}$$



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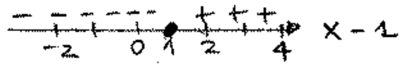
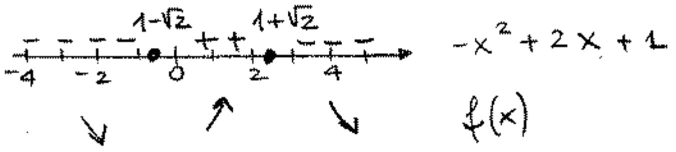
$f(x) = \dfrac{x-1}{x^2+1}$

- $\operatorname{dom} f = \mathbb{R} =]-\infty, +\infty[$
- segno di f



$x^2+1>0$

- $\lim\limits_{x\to-\infty} f(x) = 0 \qquad \lim\limits_{x\to+\infty} f(x) = 0$; $\Rightarrow y=0$ asintoto orizzontale per $x\to-\infty$ (per $x\to+\infty$);
- f è continua (rapporto di funz. continue e $x^2+1>0\ \forall x\in\mathbb{R}$)
- $\operatorname{dom} f' = \mathbb{R} \qquad f'(x) = \dfrac{(x^2+1)-(x-1)\cdot 2x}{(x^2+1)^2} = \dfrac{-x^2+2x+1}{(x^2+1)^2}$



$x = 1-\sqrt{2}$ pt. di min. loc. stretto $\qquad f(1-\sqrt{2}) = \dfrac{-\sqrt{2}}{4-2\sqrt{2}}$

$x = 1+\sqrt{2}$ pt. di max. loc. stretto $\qquad f(1+\sqrt{2}) = \dfrac{\sqrt{2}}{4+2\sqrt{2}}$

lo studio di $f''(x)$ è possibile, ma non facilissimo.

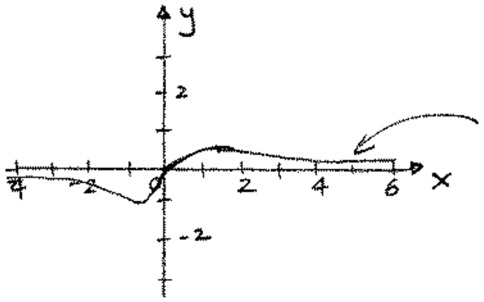
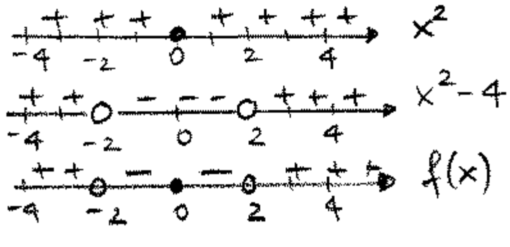


grafico qualitativo di f

☐

ⓑ $f(x) = \dfrac{x^2}{x^2-4}$ $\qquad$ f è pari!

- $\operatorname{dom} f = \mathbb{R}\setminus\{-2,2\} =]-\infty,-2[\,\cup\,]-2,2[\,\cup\,]2,+\infty[$
- segno di f



- $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to+\infty} f(x) = 1 \quad \Rightarrow y=1$ è asintoto orizzontale per f per $x\to-\infty$ (per $x\to+\infty$);

$\lim\limits_{x\to-2^-} f(x) = +\infty \qquad \lim\limits_{x\to-2^+} f(x) = -\infty$

$\lim\limits_{x\to2^-} f(x) = -\infty \qquad \lim\limits_{x\to2^+} f(x) = +\infty$

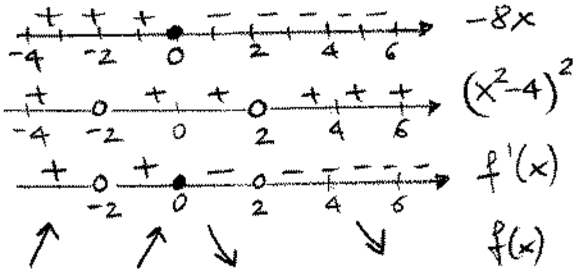
$x=-2$ asintoto verticale

$x=2$ asintoto verticale;

- f è continua in $\mathbb{R}^2\setminus\{-2,2\}$ (rapporto di funz. continue);

- dom $f' = \mathbb{R}\setminus\{-2,2\}$

$$f'(x) = \frac{2x(x^2-4) - x^2 \cdot 2x}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2}$$



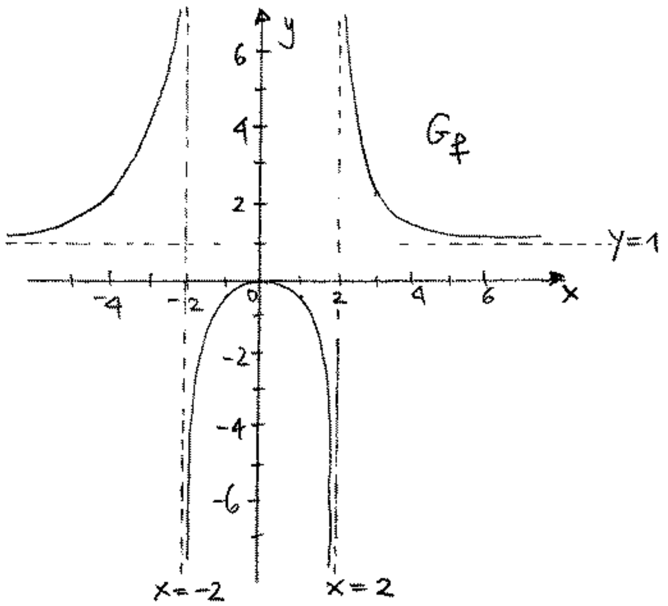
$x = 0$ pt. critico : è pt. di max loc. stretto

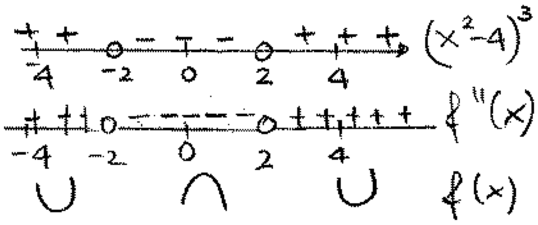
$f(0) = 0$

- dom $f'' = \mathbb{R}\setminus\{-2,2\}$

$$f''(x) = \frac{-8(x^2-4)^{\cancel{2}} + 8x \cdot 2\cancel{(x^2-4)}\, 2x}{(x^2-4)^{\cancel{4}3}} = \frac{32 + 24x^2}{(x^2-4)^3}$$

$$= \frac{8(4+3x^2)}{(x^2-4)^3}$$

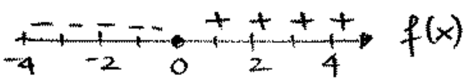




□

$f(x) = x\, e^{-x^2+x}$

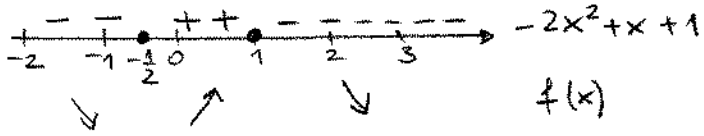
- dom $f = \mathbb{R} =]-\infty,+\infty[$
- segno di f



- $\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} f(x) = 0$ $\Rightarrow$ $y = 0$ asintoto orizzontale per $x\to-\infty$ (per $x\to+\infty$);

- dom $f' = \mathbb{R}$ $\quad f'(x) = e^{-x^2+x} + x e^{-x^2+x} \cdot (-2x+1)$

$$= e^{-x^2+x}\left(1-2x^2+x\right)$$

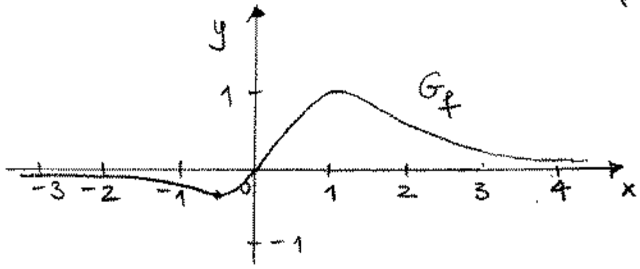


$x = -\frac{1}{2}$ pt. critico : pt. di min. loc. stretto

$$f\left(-\tfrac{1}{2}\right) = -\tfrac{1}{2} e^{-\frac{1}{4}-\frac{1}{2}} = -\tfrac{1}{2} e^{-3/4} = \frac{-1}{2\sqrt[4]{e^3}}$$

$x = 1$ pt. critico : pt. di max loc. stretto

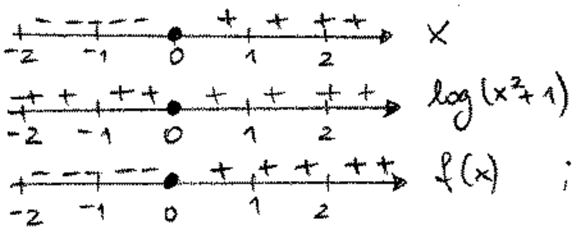
$$f(1) = e^{-1+1} = 1 .$$



lo studio di f'' è abbastanza complesso; in ogni caso si riesce a fare un grafico qualitativo di f. $\square$

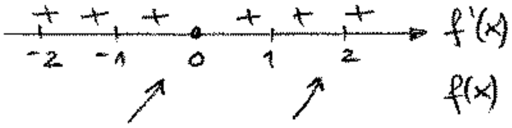
<u>$f(x) = x \log(x^2+1)$</u>

- dom $f = \mathbb{R} =]-\infty, +\infty[$ $\qquad$ f è dispari!
- segno di f



- $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to+\infty} f(x) = +\infty$;
- f è continua su tutto $\mathbb{R}$ (composizione e prodotto di funz. continue);
- dom $f' = \mathbb{R}$ $\quad f'(x) = \log(x^2+1) + \frac{2x^2}{x^2+1}$

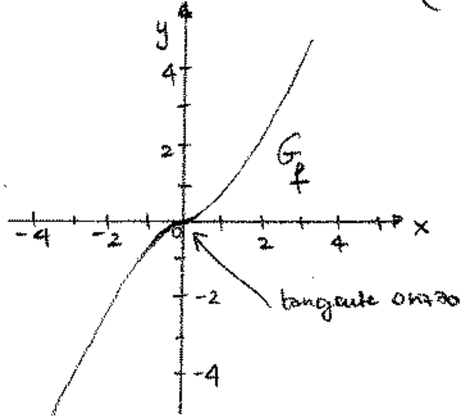
$$f'(x) = 0 \iff x = 0 \quad e \quad f'(x) > 0 \ \forall x \neq 0 .$$



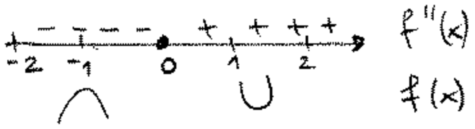
- dom $f'' = \mathbb{R}$ $\quad f''(x) = \frac{2x}{x^2+1} + \frac{4x(x^2+1) - 2x^2 \cdot 2x}{(x^2+1)^2} =$

$$= \frac{2x(x^2+1) + \cancel{4x^3} + 4x - \cancel{4x^3}}{(x^2+1)^2} =$$

$$f''(x) = \frac{2x\left[x^2+1+2\right]}{(x^2+1)^2} = \frac{2x(x^2+3)}{(x^2+1)^2}$$



tangente orizzontale! (nota: $f'(0)=0$!)



4) i) $-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\dots-\frac{1}{30} = \sum_{n=1}^{15}\frac{(-1)^n}{2n}$;

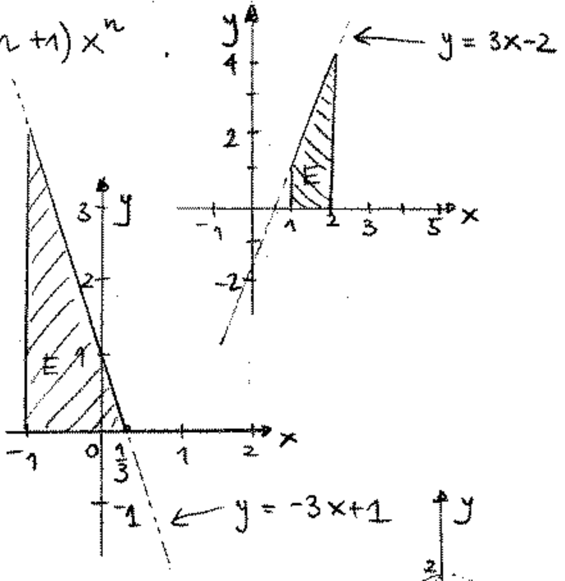
$1+4+9+\dots+256 = \sum_{n=1}^{16} n^2$;

ii) $\frac{1}{3}+\frac{1}{2}+\frac{1}{5}+\frac{1}{4}+\frac{1}{7}+\dots+\frac{1}{21}+\frac{1}{20} = \sum_{n=1}^{10}\left(\frac{1}{2n+1}+\frac{1}{2n}\right)$;

$2x+3x^2+4x^3+\dots+13x^{12} = \sum_{n=1}^{12}(n+1)x^n$.

5) $\int_1^2 (3x-2)\,dx = \text{area}(E) =$

$= \frac{4+1}{2}\cdot 1 = \underline{\underline{\frac{5}{2}}}$;

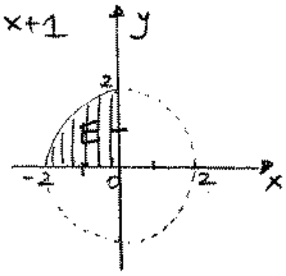


$\int_{-1}^{1/3}(-3x+1)\,dx = \text{area}(E) =$

$= \frac{4\cdot\left(1+\frac{1}{3}\right)}{2}$

$= \frac{16}{6} = \underline{\underline{\frac{8}{3}}}$;

$\int_{-2}^{0}\sqrt{4-x^2}\,dx = \text{area}(E) = \frac{\pi\cdot(2)^2}{4} = \underline{\underline{\pi}}$;



$\int_{-3}^{3}\sqrt{9-x^2}\,dx = \text{area}(E) = \frac{\pi\cdot(3)^2}{2}$

$= \underline{\underline{\frac{9\pi}{2}}}$.