FILA (B)

1) i) $|4^{x-2}-1|<3 \iff -3<4^{x-2}-1<3 \iff -2<4^{x-2}<4$

Poiché $4^{x-2}>-2$ per ogni $x\in\mathbb{R}$ (essendo $4^{x-2}>0\ \forall x\in\mathbb{R}$), basta soddisfare $4^{x-2}<4$; ne segue $S=\{x\in\mathbb{R} : x-2<1\}$, ossia $\underline{S=\,]-\infty,3[}$.

$\square$

$|x(x-2)|>3 \iff x^2-2x<-3$ o $x^2-2x>3 \iff$

$\iff x^2-2x+3<0$ o $x^2-2x-3>0$. Notiamo che $x^2-2x+3>0\ \forall x\in\mathbb{R}$, quindi $|x(x-2)|>3 \iff x^2-2x-3>0$, ossia $\underline{S=\,]-\infty,-1[\,\cup\,]3,+\infty[}$.

$\square$

ii) $\log_{\frac12}(x+1) + \log_{\frac12}(2-x) > 0 \iff \begin{cases} x+1>0 \\ 2-x>0 \\ \log_{\frac12}(x+1)(2-x) > \log_{\frac12} 1 \end{cases} \iff \begin{cases} x+1>0 \\ 2-x>0 \\ (x+1)(2-x)<1 \end{cases}$

$\iff \begin{cases} x>-1 \\ x<2 \\ x^2-x-1>0 \end{cases} \iff \begin{matrix} x>-1 \\ x<2 \\ x<\frac{1-\sqrt5}{2} \text{ o } x>\frac{1+\sqrt5}{2} \end{matrix} \Rightarrow \underline{S = \left]-1, \frac{1-\sqrt5}{2}\right[\cup \left]\frac{1+\sqrt5}{2}, 2\right[}.$ $\square$

$\log_4(x^2-2) \le \frac12 \iff \log_4(x^2-2) \le \log_4 4^{\frac12} \iff \begin{cases} x^2-2>0 \\ x^2-2 \le 2 \end{cases}$

$\iff \begin{cases} x^2>2 \\ x^2 \le 4 \end{cases} \Rightarrow \underline{S = \left[-2, -\sqrt2\right[\cup \left]\sqrt2, 2\right]}.$ $\square$

iii) $e^{|x+2|-x^2} > 1 \iff |x+2|-x^2>0 \iff \begin{cases} x+2 \ge 0 \\ x+2-x^2>0 \end{cases}$ o $\begin{cases} x+2<0 \\ -(x+2)-x^2>0 \end{cases}$

$\iff \begin{cases} x \ge -2 \\ -1<x<2 \end{cases}$ o $\begin{cases} x<-2 \\ x^2+x+2<0 \end{cases} \Rightarrow \underline{S =]-1, 2[}.$

$3^{-|x+2|} > 0 \qquad \underline{S = \mathbb{R}} \quad$ (poiché $3^x>0 \ \forall x \in \mathbb{R}$). $\blacksquare$

2) i) $\underline{\sum_{n=1}^{10} \frac{1}{n(n+1)}}$. $\square$

ii) $\sum_{n=1}^{5} \text{area}(T_n) = \sum_{n=1}^{5} \frac12 \cdot \frac{n}{2} \cdot \frac{1}{n^3} = \frac14 \sum_{n=1}^{5} \frac{1}{n^2} = \underline{\frac14\left(1+\frac14+\frac19+\frac1{16}+\frac1{25}\right)}$ $\square$

iii) $\sum_{k=1}^{4} \frac{1}{(k-1)!} = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!} = 2+\frac12+\frac16 = \underline{\frac83}.$ $\square$

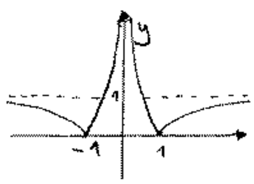
iv) $\int_2^4 \frac{x-1}{\sqrt x - 1}\,dx = \int_2^4 \frac{\cancel{(\sqrt x-1)}(\sqrt x+1)}{\cancel{\sqrt x-1}}\,dx = \left[\frac{x^{3/2}}{3/2}+x\right]_2^4 = \left[\left(\frac23 4^{3/2}+4\right)-\left(\frac23 \cdot 2^{3/2}+2\right)\right] =$

$= \left(\frac{16}{3}+4\right) - \left(\frac23 \cdot 2\sqrt2+2\right) = \underline{\frac{22}{3} - \frac43\sqrt2}.$ $\square$

$\int_1^3 \frac{x^5-3x}{x^2}\,dx = \int_1^3 \left(x^3-\frac3x\right)dx = \left[\frac{x^4}{4} - 3\log|x|\right]_1^3 = \left(\frac{81}{4}-3\log3\right)-\frac14 = \underline{20-3\log3}.$ $\square$

$\int_0^1 \left(2^x + \frac{x^2}{x^3+1}\right)dx = \left[\frac{2^x}{\log 2} + \frac13 \log|x^3+1|\right]_0^1 = \left[\left(\frac{2}{\log 2}+\frac13\log 2\right)-\left(\frac{1}{\log 2}\right)\right] =$

$= \underline{\frac{1}{\log 2} + \frac13 \log 2}.$ $\square$



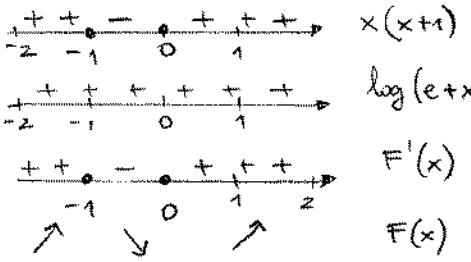
$\int_{1/2}^{3} \left|\frac{1}{x^2}-1\right| dx = \int_{1/2}^{1} \left(\frac{1}{x^2}-1\right)dx + \int_1^3 \left(-\frac{1}{x^2}+1\right)dx =$

$= \left[-\frac1x - x\right]_{1/2}^{1} + \left[+\frac1x + x\right]_1^3 = \left[\left(\cancel{-2}+\cancel{2}+\frac12\right) + \left(+\frac13+3-1-1\right)\right] = \underline{\frac{11}{6}}.$ $\blacksquare$

3) Per il teorema fondamentale del calcolo integrale si ha

$F'(x) = f(x)$; quindi $F'(x) = x(x+1)\log(e+x^2)$. Ora $F'(x)=0 \iff \underline{x=-1}$ o $\underline{x=0}$,

sono i punti critici di F.



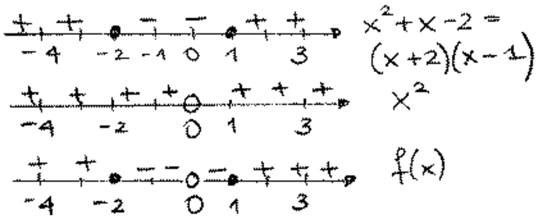
Dalla monotonia di F segue che

$\underline{x=-1 \quad \text{pt. di max. loc.}}$ (stretto) di F,

$\underline{x=0 \quad \text{pt. di min. loc.}}$ (stretto) di F.

4) i) $f(x) = \dfrac{x^2+x-2}{x^2}$ • $\operatorname{dom} f = \mathbb{R}\setminus\{0\} =]-\infty,0[\,\cup\,]0,+\infty[$.

• segno di f



• $\displaystyle\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\cancel{x^2}\left(1+\frac{1}{x}-\frac{2}{x^2}\right)}{\cancel{x^2}} = 1 = \lim_{x\to+\infty} f(x)$ $\Rightarrow$ $y=1$ asintoto orizzontale per f per $x\to-\infty$ (per $x\to+\infty$).

$\displaystyle\lim_{x\to0^-} f(x) = \lim_{x\to0^-} \frac{\overbrace{(x^2+x-2)}^{\to -2}}{\underbrace{(x^2)}_{\text{infinitesimo positivo}}} = -\infty$

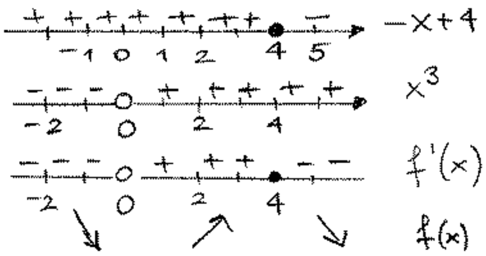
$\Rightarrow$ $x=0$ asintoto verticale per f.

$\displaystyle\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{\overbrace{(x^2+x-2)}^{\to -2}}{\underbrace{(x^2)}_{\text{infinitesimo positivo}}} = -\infty$

• $\operatorname{dom} f' = \mathbb{R}\setminus\{0\}$ $\quad f'(x) = \dfrac{(2x+1)x^2 - (x^2+x-2)2x}{x^4} = \dfrac{\cancel{2x^3}+x^2-\cancel{2x^3}-2x^2+4x}{x^4}$

$$= \frac{-x^2+4x}{x^4} = \frac{-x+4}{x^3}$$

$x=4$ pt. critico di f;

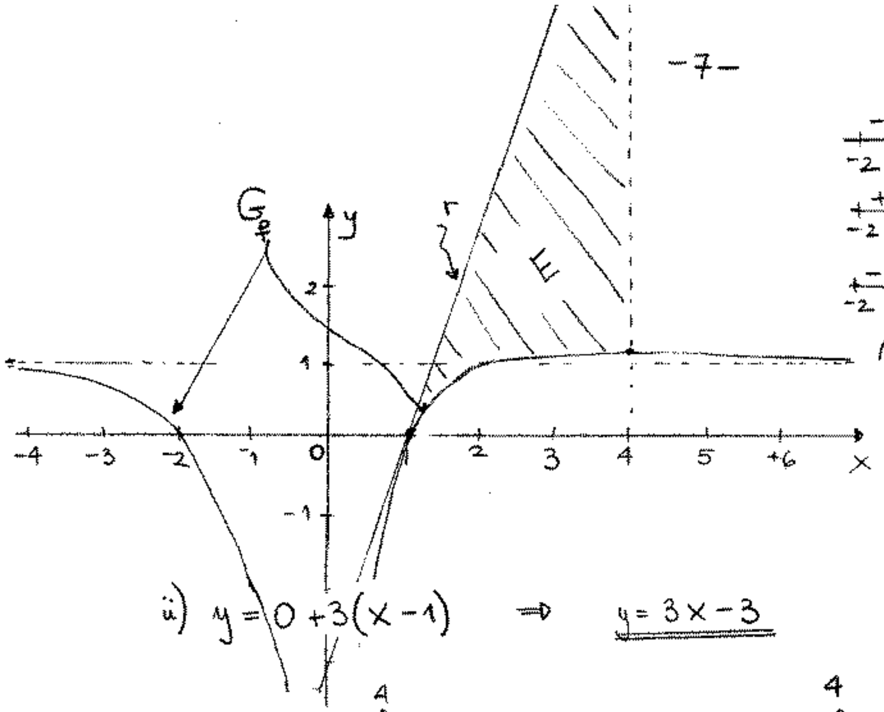


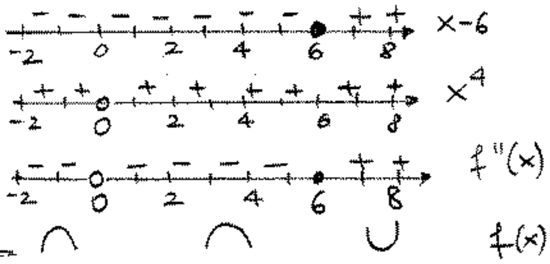
$f(4) = \dfrac{16+4-2}{x^2} = \dfrac{18}{16} = \underline{\underline{\dfrac{9}{8}}}$.

$x=4$ pt. di max. loc. (stretto)

• $\operatorname{dom} f'' = \mathbb{R}\setminus\{0\}$ $\quad f''(x) = \dfrac{(-1)(x^3)-(-x+4)3x^2}{x^6} = \dfrac{-x^3+3x^3-12x^2}{x^6}$

$$= \frac{2\cancel{x^2}(x-6)}{\cancel{x^6}\,x^4} = \frac{2(x-6)}{x^4}$$



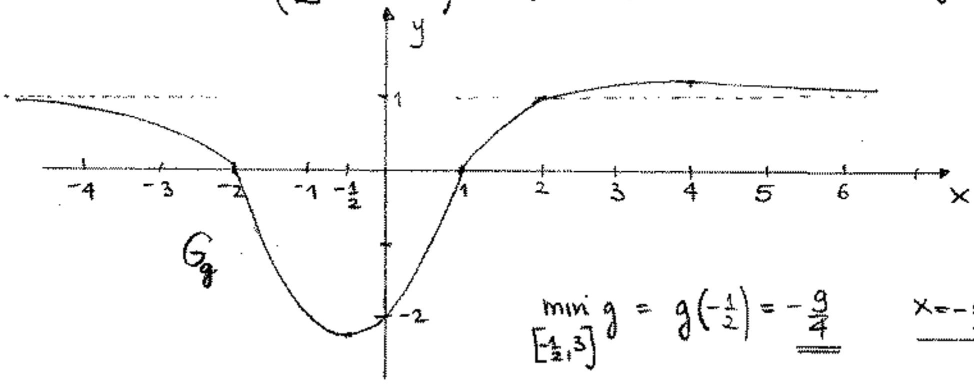


ii) $y = 0 + 3(x-1) \Rightarrow \underline{y = 3x-3}$

iii) $\text{area}(E) = \int_1^4 \left(3x-3 - \frac{x^2+x-2}{x^2}\right)dx = \int_1^4 \left(3x-3-1-\frac{1}{x}+\frac{2}{x^2}\right)dx =$

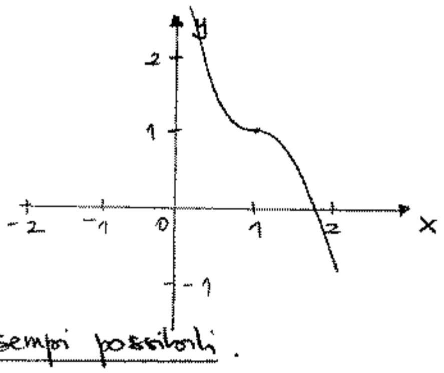
$$= \int_1^4 \left(3x-4-\frac{1}{x}+\frac{2}{x^2}\right)dx = \left[3\frac{x^2}{2} - 4x - \log|x| - \frac{2}{x}\right]_1^4 = \left(24-16-\log 4 - \frac{1}{2}\right)$$

$$- \left(\frac{3}{2} - 4 - 2\right) = \left(\frac{15}{2} - \log 4\right) + \frac{9}{2} = \underline{12 - \log 4}.$$

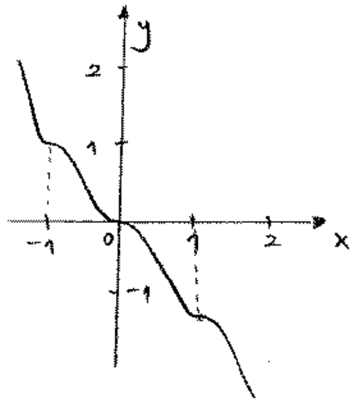
iv) 

$\min\limits_{[-\frac{1}{2},3]} g = g\left(-\frac{1}{2}\right) = \underline{-\frac{9}{4}}$ $\quad$ $\underline{x=-\frac{1}{2}}$ pt. di min. di g su $\left[-\frac{1}{2}, 3\right]$;

$\max\limits_{[-\frac{1}{2},3]} g = g(3) = f(3) = \frac{9+3-2}{9} = \underline{\frac{10}{9}}$ $\quad$ $\underline{x=3}$ pt. di max di g su $\left[-\frac{1}{2}, 3\right]$.

5) i) 

Esempi possibili.

ii) 

6) $P_{10}^{3,3,4} + P_{10}^{5,1,4} + P_{10}^{5,3,2} = \underline{\frac{10!}{3!\,3!\,4!} + \frac{10!}{5!\,4!} + \frac{10!}{5!\,3!\,2!}}$