FILA Ⓓ

1) i) $|3^{x+1}-1|<8 \iff -8<3^{x+1}-1<8 \iff -7<3^{x+1}<9$.

Poichè $3^{x+1}>-7$ per ogni $x\in\mathbb{R}$ (poichè $3^x>0\ \forall x\in\mathbb{R}$), basta soddisfare $3^{x+1}<3^2$; segue $S=\{x\in\mathbb{R}: x+1<2\}=\underline{\underline{]-\infty,1[}}$.

$\square$

$|x^2+x+1|\geq 3 \iff x^2+x+1\leq -3$ o $x^2+x+1\geq 3 \iff$

$\iff x^2+x+4\leq 0$ o $x^2+x-2\geq 0$. Notiamo che

$x^2+x+4>0\ \forall x\in\mathbb{R}$, e quindi $|x^2+x+1|\geq 3 \iff x^2+x-2\geq 0$,

ossia $\underline{\underline{S=]-\infty,-2]\cup[1,+\infty[}}$.

$\square$

ii) $$\log_2(x+2)+\log_2(1-x)\leq 0 \iff \begin{cases} x+2>0 \\ 1-x>0 \\ \log_2(x+2)(1-x)\leq \log_2 1 \end{cases} \iff \begin{cases} x+2>0 \\ 1-x>0 \\ (x+2)(1-x)\leq 1 \end{cases}$$

$$\iff \begin{cases} x>-2 \\ x<1 \\ x^2+x-1\geq 0 \end{cases} \iff \begin{cases} x>-2 \\ x<1 \\ x\leq \frac{-1-\sqrt{5}}{2} \text{ o } x\geq \frac{-1+\sqrt{5}}{2} \end{cases} \Rightarrow \underline{\underline{S=]-2,-\frac{1-\sqrt{5}}{2}[\cup]-\frac{1+\sqrt{5}}{2},1[}}.$$

$\square$

$$\log_{1/3}(x^2-4)\geq -2 \iff \log_{1/3}(x^2-4)\geq \log_{1/3}\left(\tfrac{1}{3}\right)^{-2} \iff \log_{1/3}(x^2-4)\geq \log_{1/3} 9$$

$$\iff \begin{cases} x^2-4>0 \\ x^2-4\leq 9 \end{cases} \iff \begin{cases} x^2-4>0 \\ x^2\leq 13 \end{cases} \Rightarrow \underline{\underline{S=[-\sqrt{13},-2[\cup]2,\sqrt{13}]}}.$$

$\square$

iii) $$e^{|2x-3|-x^2}\geq 1 \iff |2x-3|-x^2\geq 0 \iff$$

$$\iff \begin{cases} 2x-3\geq 0 \\ 2x-3-x^2\geq 0 \end{cases} \text{ o } \begin{cases} 2x-3<0 \\ -2x+3-x^2\geq 0 \end{cases} \iff$$

$$\iff \begin{cases} x\geq 3/2 \\ x^2-2x+3\leq 0 \end{cases} \text{ o } \begin{cases} x<3/2 \\ x^2+2x-3\leq 0 \end{cases} \iff$$

$$\iff \begin{cases} x < 3/2 \\ -3 \leq x \leq 1 \end{cases} \Rightarrow \underline{\underline{S=[-3,1]}}.$$

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$4^{-|x|}>0 \quad \underline{\underline{\forall x\in\mathbb{R}}}.$ ■

2) i) $\underline{\underline{\sum_{n=1}^{10}\frac{(-1)^n}{n(n+1)}}}$. ☐

ii) $\sum_{n=1}^{6}\text{area}(C_n)=\sum_{n=1}^{6}\pi\cdot\frac{4}{n^2}=4\pi\sum_{n=1}^{6}\frac{1}{n^2}=\underline{\underline{4\pi\left[1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36}\right]}}.$ ☐

iii) $\sum_{k=0}^{4}\frac{1}{k!}=\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}=2+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}=\underline{\underline{\frac{65}{24}}}$ ☐

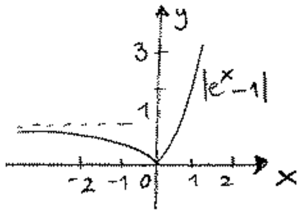
iv) $\int_3^4\frac{x-1}{\sqrt{x}+1}\,dx=\int_3^4\frac{(\sqrt{x}-1)\cancel{(\sqrt{x}+1)}}{\cancel{\sqrt{x}+1}}\,dx=\left[\frac{x^{3/2}}{3/2}-x\right]_3^4=\left[\left(\frac{2}{3}\cdot4^{3/2}-4\right)-\left(\frac{3^{3/2}}{3/2}-3\right)\right]=$

$=\left[\left(\frac{16}{3}-4\right)-\left(\frac{2}{\cancel{3}}\cdot\cancel{3}\sqrt{3}-3\right)\right]=\underline{\underline{\frac{13}{3}-2\sqrt{3}}}$ ☐

$\int_1^2\frac{x^4+3x}{x^3}\,dx=\int_1^2\left(x+\frac{3}{x^2}\right)dx=\left[\frac{x^2}{2}-\frac{3}{x}\right]_1^2=\left[\left(2-\frac{3}{2}\right)-\left(\frac{1}{2}-3\right)\right]=\underline{\underline{3}}$ ☐

$\int_0^1\left(e^{4x}+\frac{1}{x+e}\right)dx=\left[\frac{e^{4x}}{4}+\log|x+e|\right]_0^1=\left[\left(\frac{e^4}{4}+\log(1+e)\right)-\left(\frac{1}{4}+1\right)\right]=$

$=\underline{\underline{\frac{e^4}{4}-\frac{5}{4}+\log(1+e)}}.$ ☐



$\int_{-1}^{2}|e^x-1|\,dx=\int_{-1}^{0}(-e^x+1)\,dx+\int_0^2(e^x-1)\,dx=$

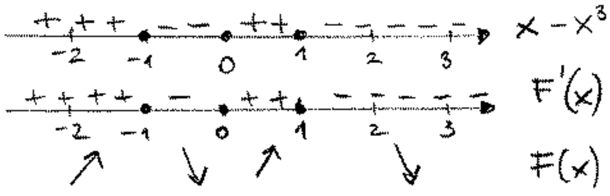
$=\left[-e^x+x\right]_{-1}^{0}+\left[e^x-x\right]_0^2=\left[(-1)-\left(-\frac{1}{e}-1\right)\right]+\left[(e^2-2)-(1)\right]$

$=\underline{\underline{\frac{1}{e}+e^2-3}}.$ ■

3) Per il teorema fondamentale del calcolo integrale si ha $F'(x)=f(x)\ \forall x\in[-2,2]$ e quindi $F'(x)=(x-x^3)e^{-x}$. Ne segue che $F'(x)=0 \iff \underline{x=0,\ x=-1,}$ $\underline{x=1}$, che sono dunque i punti critici di F.

Dalla monotonia di F segue che

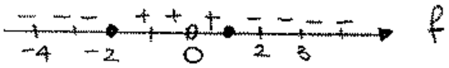
$\underline{x=-1 \text{ pt. di max. loc.}}$ (stretto) di F,



$x=0$ pt. di min. loc. (stretto) di F, $x=1$ pt. di max. loc. (stretto) di F.

4) i) $f(x) = \dfrac{-x^2-x+2}{x^2}$. Basta notare che in FILA Ⓑ si è studiato $-f(x)$ e quindi abbiamo immediatamente

- $\mathrm{dom}\, f = \mathbb{R}\setminus\{0\}$
- segno di f



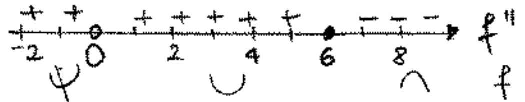
- $y=-1$ asintoto orizzontale per f per $x\to-\infty$ (per $x\to+\infty$)

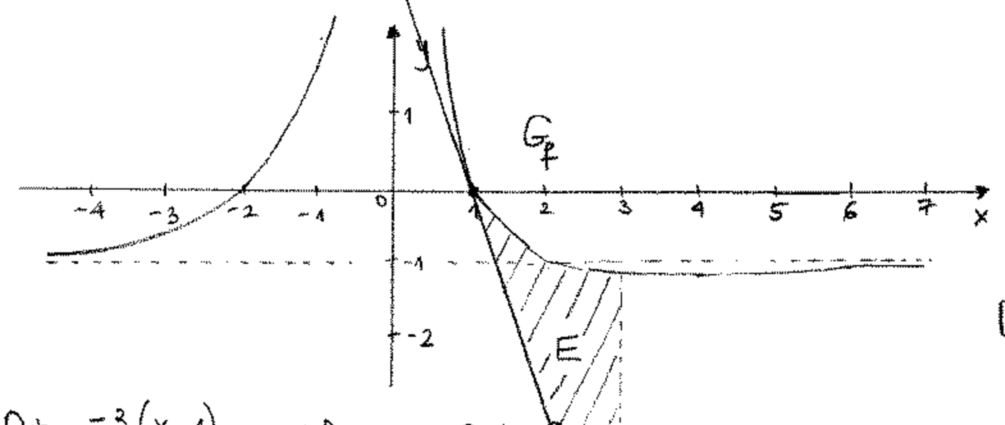
  $x=0$ asintoto verticale
- $f'(x) = \dfrac{x-4}{x^3}$

$x=4$ pt. critico di f, $x=4$ pt. di min. loc. (stretto)

$$f(4) = -\frac{9}{8}.$$

- $f''(x) = \dfrac{-2(x-6)}{x^4}$





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ii) $y = 0 + -3(x-1) \Rightarrow \underline{y = -3x+3}$ ← r

iii) $$\mathrm{area}(E) = \int_1^3 \left(\frac{-x^2-x+2}{x^2} - (-3x+3)\right)dx =$$

$$= -\int_1^3 \left(-3x+3+1+\frac{1}{x}-\frac{2}{x^2}\right)dx = -\int_1^3\left(-3x+4+\frac{1}{x}-\frac{2}{x^2}\right)dx =$$

$$= -\left[-3\frac{x^2}{2}+4x+\log|x|+\frac{2}{x}\right]_1^3 = -\left[\left(-\frac{27}{2}+12+\log 3+\frac{2}{3}\right)-\left(-\frac{3}{2}+4+2\right)\right]$$

$$= -\left[\left(-\frac{5}{6}+\log 3\right)-\frac{9}{2}\right] = \underline{\frac{16}{3}-\log 3}.$$

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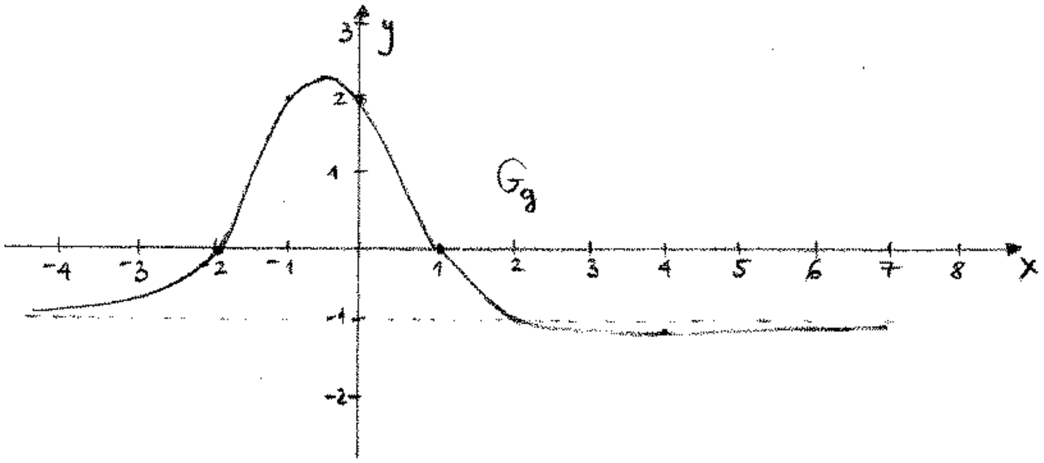
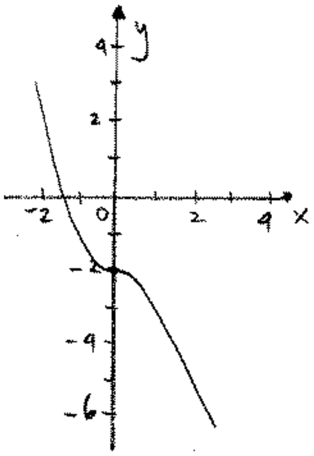
iii)



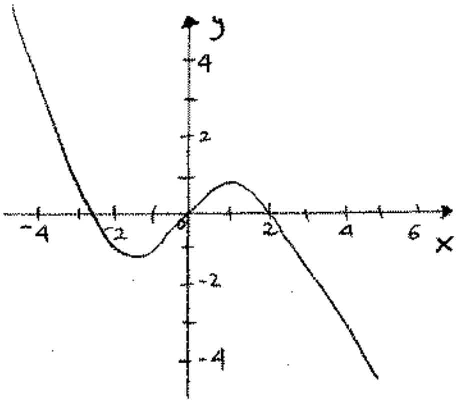
$\min_{[-3,-1]} g = g(-3) = \frac{-9+3+2}{9} = \underline{\underline{-\frac{4}{9}}}$ $\underline{x=-3}$ pt. di minimo di g su $[-3,-1]$;

$\max_{[-3,-1]} g = g(-1) = \frac{-1+1+2}{1} = \underline{\underline{2}}$ $\underline{\underline{x=-1}}$ pt. di massimo di g su $[-3,-1]$. ■

5) i)



ii)



Esempi possibili. ■

6) $6 \cdot D_{6,3} + 6 \cdot D_{6,3} = 2 \cdot 6 \cdot \frac{6!}{3!} =$

$= 12 \cdot \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}} = 12 \cdot 120 = \underline{\underline{1440}}$ ■