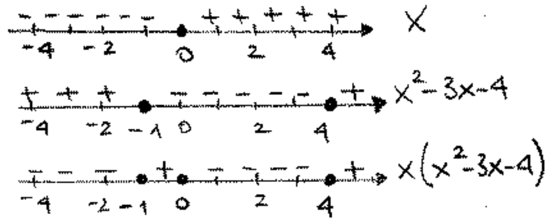
Secondo Appello di ANALISI MATEMATICA – CdL in STPCA –
a.a. 2007/08 – Rovereto, 21 gennaio 2008

FILA (A)

1) i) a) $\text{non}\left(\forall x,y \in \mathbb{N},\ \exists M \in \mathbb{R} :\ x+y < M\right) \iff \exists x,y \in \mathbb{N} : \forall M \in \mathbb{R},\ x+y \geq M.$ ☐

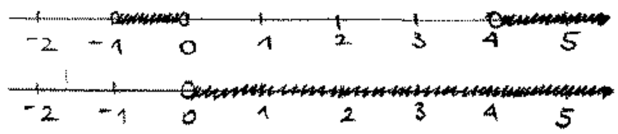
b) "Tutti gli studenti iscritti al primo anno del CdL in STPCA dell'Università di Trento hanno superato con le prove in itinere l'esame di Analisi Matematica e l'esame di Basi Neurali della Cognizione". ☐

ii) a) $A = \{x \in \mathbb{R} :\ x^3 - 3x^2 - 2x > 2x\} = \{x \in \mathbb{R} :\ x^3 - 3x^2 - 4x > 0\} =$
$= \{x \in \mathbb{R} :\ x(x^2 - 3x - 4) > 0\} = \underline{\underline{]-1, 0[\ \cup\ ]4, +\infty[}}$



$B = \left\{x \in \mathbb{R} :\ \sqrt{\dfrac{x+1}{x}} \geq 1\right\} = \underline{\underline{]0, +\infty[}}$

$\sqrt{\dfrac{x+1}{x}} \geq 1 \iff \dfrac{x+1}{x} \geq 1 \iff \dfrac{1}{x} \geq 0$

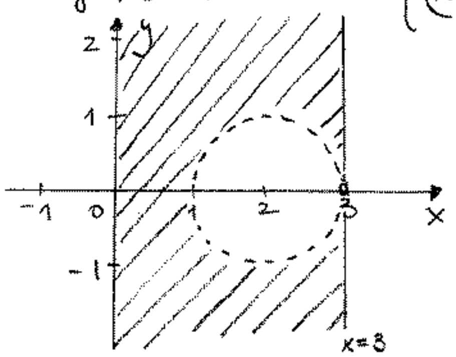


/// = A

/// = B ☐

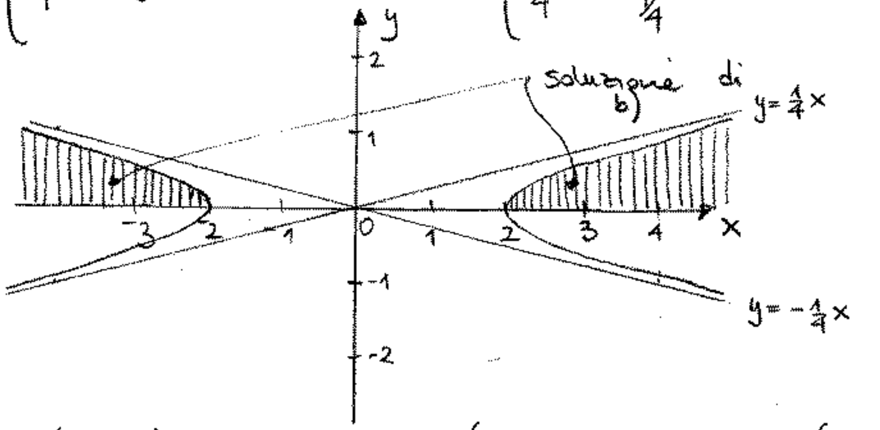
b) $\underline{\underline{A \cup B = \ ]-1, +\infty[\ \setminus \{0\}}}$.
$\underline{\underline{A \cap B = \ ]4, +\infty[}}$.

2) a) $\begin{cases} x^2 - 3x \leq 0 \\ x^2 - 4x + y^2 + 3 > 0 \end{cases} \iff \begin{cases} x(x-3) \leq 0 \\ (x-2)^2 - 4 + y^2 + 3 > 0 \end{cases} \iff \begin{cases} 0 \leq x \leq 3 \\ (x-2)^2 + y^2 > 1 \end{cases}$



/// Soluzione di a) ☐

b) $\begin{cases} y \geq 0 \\ \frac{x^2}{4} - 4y^2 \geq 1 \end{cases} \iff \begin{cases} y \geq 0 \\ \frac{x^2}{4} - \frac{y^2}{1/4} \geq 1 \end{cases}$



3) i) $\log_2\left(\frac{x-1}{x^2+1}\right) < 0 \iff \begin{cases} \frac{x-1}{x^2+1} > 0 \\ \frac{x-1}{x^2+1} < 1 \end{cases} \iff \begin{cases} x-1 > 0 \\ x-1 < x^2+1 \end{cases}$

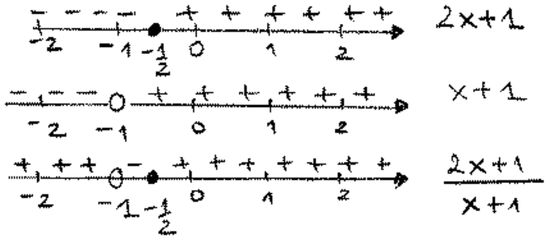
(nota: $x^2+1>0$ e quindi si può moltiplicare!)

$\iff \begin{cases} x > 1 \\ x^2 - x + 2 > 0 \end{cases} \iff \begin{cases} x > 1 \\ x \in \mathbb{R} \end{cases} \Rightarrow \underline{\underline{S =]1, +\infty[}}.$ $\square$

$\left|\frac{x}{x+1}\right| > 1 \iff \left\{ \frac{x}{x+1} < -1 \quad \text{o} \quad \frac{x}{x+1} > 1 \right.$

$\iff \left\{ \frac{x}{x+1} + 1 < 0 \quad \text{o} \quad \frac{x}{x+1} - 1 > 0 \right.$

$\iff \left\{ \frac{2x+1}{x+1} < 0 \quad \text{o} \quad \frac{-1}{x+1} > 0 \right.$



$\Rightarrow \underline{\underline{S =]-1, -\tfrac{1}{2}[\cup]-\infty, -1[}} = \underline{\underline{]-\infty, -\tfrac{1}{2}[\setminus \{-1\}}}.$ $\square$

$e^{|2x+2|}e^{-x^2} \leq \frac{1}{e}$ $\quad \underline{\underline{S =]-\infty, -1] \cup [3, +\infty[}}$ (vedi TPI, Fila B, Es. 1) ii)).

ii) $\displaystyle\int_{-1}^{1}\left(e^{-3x} + \frac{2x}{x^2+1}\right)dx = -\frac{1}{3}\int_{-1}^{1}(-3)e^{-3x}dx + \int_{-1}^{1}\frac{2x}{x^2+1}dx = -\frac{1}{3}\left[e^{-3x}\right]_{-1}^{1} + \log|x^2+1|\Big|_{-1}^{1}$

$$= -\frac{1}{3}\left(e^{-3}-e^{3}\right)+\left(\log 2-\log 2\right) = \underline{\underline{\frac{e^3}{3}-\frac{1}{3e^3}}}.$$ $\square$

$$\sum_{n=2}^{5}\int_0^1 x^{\frac{1}{n}}\,dx = \int_0^1 x^{\frac{1}{2}}\,dx+\int_0^1 x^{\frac{1}{3}}\,dx+\int_0^1 x^{1/4}\,dx+\int_0^1 x^{1/5}\,dx =$$

$$=\left[\frac{x^{3/2}}{3/2}\right]_0^1+\left[\frac{x^{4/3}}{4/3}\right]_0^1+\left[\frac{x^{5/4}}{5/4}\right]_0^1+\left[\frac{x^{6/5}}{6/5}\right]_0^1 = \frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6} = \underline{\underline{\frac{183}{60}}}.$$ $\blacksquare$

4) i) $g(x) = \log(f(x))$ • $\operatorname{dom} g = \{x\in\mathbb{R}:\ f(x)>0\} =]1,+\infty[$.

$h(x) = e^{f(x)}$ • $\operatorname{dom} h = \mathbb{R}$

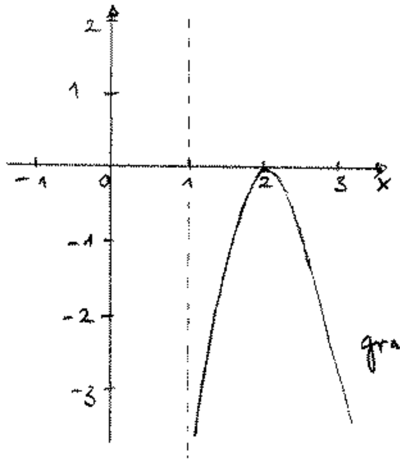


grafico qualitativo di g

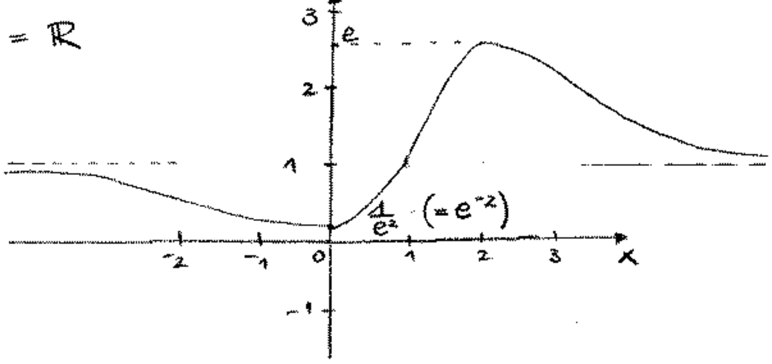


grafico qualitativo di h

$\square$

ii) f è una funzione limitata: infatti $-2 \le f(x) \le 1 \quad \forall x\in\mathbb{R}$;

g è limitata sup., ma non è limitata inferiormente: infatti $g(x)\le 0\ \forall x>1$;

e $\lim_{x\to 1^+} g(x) = -\infty$; $\lim_{x\to+\infty} g(x) = -\infty$.

h è limitata: infatti $e^{-2}\le h(x)\le e \quad \forall x\in\mathbb{R}$. $\square$

iii) $\min_{\mathbb{R}} f = -2 \qquad \max_{\mathbb{R}} f = 1$;

$\nexists \min_{]1,+\infty[} g \qquad \max_{]1,+\infty[} g = 0$;

$\min_{\mathbb{R}} h = e^{-2} \qquad \max_{\mathbb{R}} h = e$. $\blacksquare$

5) Vedi (TPI, Fila B, Esercizio 5). $\blacksquare$

6) $$D_{16,5}\cdot D_{22,5} = \frac{16!}{(16-5)!}\cdot\frac{22!}{(22-5)!} = \frac{16!}{11!}\cdot\frac{22!}{17!} = \underline{\underline{\frac{22!}{11!\ 17}}}.$$ $\blacksquare$

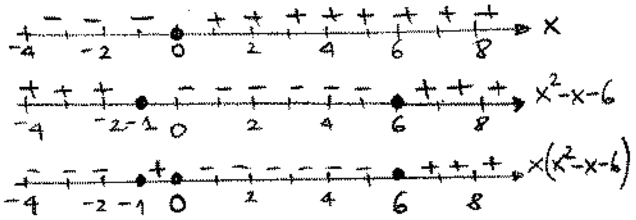
FILA (B)

1) i) a) non $\left(\exists x,y \in \mathbb{N} : \forall M \in \mathbb{R},\ x+y > M\right) \iff \forall x,y \in \mathbb{N}, \exists M \in \mathbb{R} : x+y \leq M.$

b) " C'è almeno uno studente iscritto al primo anno del CdL in STPCA dell'Università di Trento che non ha superato con le prove in itinere l'esame di Analisi Matematica e l'esame di Basi Neurali della Cognizione".

ii) a) $A = \{x \in \mathbb{R} : x^3 - x^2 - 2x \leq 4x\} = \{x \in \mathbb{R} : x^3 - x^2 - 6x \leq 0\} = \{x \in \mathbb{R} : x(x^2 - x - 6) \leq 0\}$

$=]-\infty, -1] \cup [0, 6]$

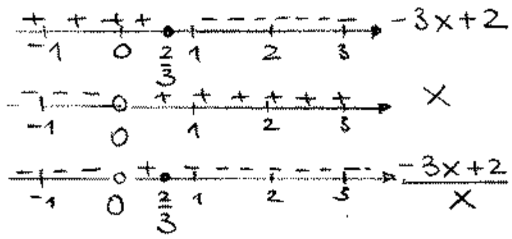


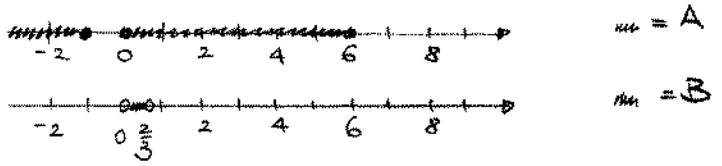
$B = \left\{x \in \mathbb{R} : \sqrt{\frac{x+2}{x}} > 2\right\}$

$\sqrt{\frac{x+2}{x}} > 2 \iff \frac{x+2}{x} > 4$

$=]0, \frac{2}{3}[$

$\iff \frac{-3x+2}{x} > 0$





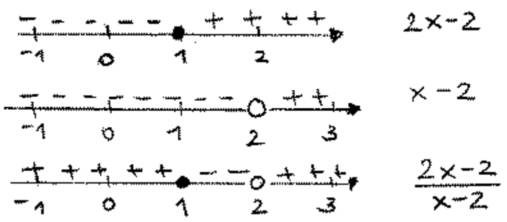
b) $A \cup B = A$ $\qquad A \cap B = B$ $\qquad A \setminus B =]-\infty, -1] \cup \{0\} \cup [\frac{2}{3}, 6]$

2) i) $\left|\frac{x}{x-2}\right| \geq 1 \iff \left\{ \frac{x}{x-2} \leq -1 \quad o \quad \frac{x}{x-2} \geq 1 \right.$

$\iff \left\{ \frac{x}{x-2} + 1 \leq 0 \quad o \quad \frac{x}{x-2} - 1 \geq 0 \right.$

$\iff \left\{ \frac{2x-2}{x-2} \leq 0 \quad o \quad \frac{2}{x-2} \geq 0 \right.$

$S = [1,2[\;\cup\;]2,+\infty[$.



$$\log_2\left(\frac{x-2}{x^2+4}\right) < 0 \iff \begin{cases} \frac{x-2}{x^2+4} > 0 \\ \frac{x-2}{x^2+4} < 1 \end{cases} \iff \begin{cases} x > 2 \\ x-2 < x^2+4 \end{cases}$$

(nota: $x^2+4>0$ e quindi si può moltiplicare!)

$$\iff \begin{cases} x > 2 \\ x^2 - x + 6 > 0 \end{cases} \Rightarrow \begin{cases} x > 2 \\ \mathbb{R} \end{cases} \qquad S =]2,+\infty[.$$

$$e^{|x+1|}e^{-x^2} < 1 \quad \Rightarrow S =]-\infty, \tfrac{1-\sqrt{5}}{2}[\;\cup\;]\tfrac{1+\sqrt{5}}{2}, +\infty[$$

(vedi TPI, Fila A, Es. 1) ii)).

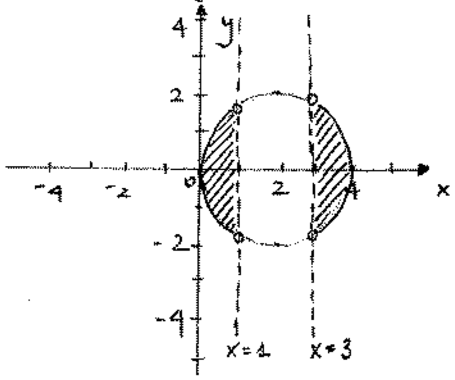
ii) $$\int_{-1}^{1}\left(2xe^{x^2} + \frac{x}{x^2+2}\right)dx = \left[e^{x^2} + \frac{1}{2}\log|x^2+2|\right]_{-1}^{1} = (e^1 - e^1) + \frac{1}{2}(\log 3 - \log 3) = 0$$

$$\sum_{n=2}^{6}\int_1^2 x^{-\frac{1}{n}}dx = \int_1^2 x^{-\frac{1}{2}}dx + \int_1^2 x^{-\frac{1}{3}}dx + \int_1^2 x^{-\frac{1}{4}}dx + \int_1^2 x^{-\frac{1}{5}}dx + \int_1^2 x^{-\frac{1}{6}}dx$$

$$= \left[\frac{x^{1/2}}{1/2}\right]_1^2 + \left[\frac{x^{2/3}}{2/3}\right]_1^2 + \left[\frac{x^{3/4}}{3/4}\right]_1^2 + \left[\frac{x^{4/5}}{4/5}\right]_1^2 + \left[\frac{x^{5/6}}{5/6}\right]_1^2$$

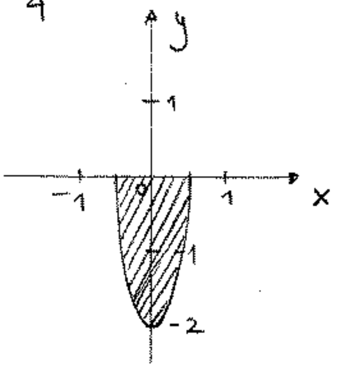
$$= 2(\sqrt{2}-1) + \frac{3}{2}(2^{2/3}-1) + \frac{4}{3}(2^{3/4}-1) + \frac{5}{4}(2^{4/5}-1) + \frac{6}{5}(2^{5/6}-1).$$

3) a) $$\begin{cases} x^2 - 4x + 3 > 0 \\ x^2 - 4x + y^2 \le 0 \end{cases} \iff \begin{cases} (x-1)(x-3) > 0 \\ (x-2)^2 + y^2 \le 4 \end{cases}$$



//// soluzione di a).

b) $\begin{cases} y \le 0 \\ 4x^2 + \frac{y^2}{4} \le 1 \end{cases} \iff \begin{cases} y \le 0 \\ \frac{x^2}{1/4} + \frac{y^2}{4} \le 1 \end{cases}$



/// soluzione di b).

4) vedi TPI, FILA (A), Esercizio 4).

5) i) $g(x) = \log(f(x))$ • $\text{dom}\, g = \{x \in \mathbb{R} : f(x) > 0\} =]-1, +\infty[$.

$h(x) = e^{f(x)}$ • $\text{dom}\, h = \mathbb{R}$

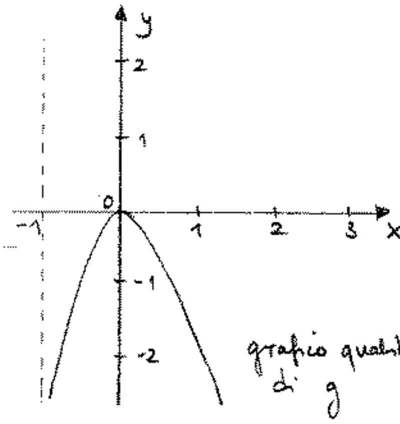


grafico qualitativo di g

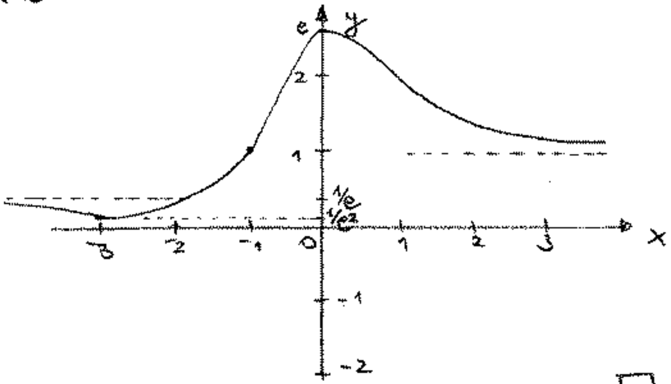


grafico qualitativo di f

ii) f è una funzione limitata: infatti $-2 \le f(x) \le 1 \quad \forall x \in \mathbb{R}$;

g è limitata superiormente, ma non limitata inferiormente: infatti

$g(x) \le 0 \quad \forall x > -1$; e $\lim_{x \to -1^+} g(x) = -\infty$, $\lim_{x \to +\infty} g(x) = -\infty$.

h è una funzione limitata: infatti $e^{-2} \le h(x) \le e \quad \forall x \in \mathbb{R}$.

iii) $\min_{\mathbb{R}} f = -2 \qquad \max_{\mathbb{R}} f = 1$;

$\nexists \min_{]-1,+\infty[} g \qquad \max_{]-1,+\infty[} g = 0$;

$\min_{\mathbb{R}} h = e^{-2} \qquad \max_{\mathbb{R}} h = e$.

6) $D_{15,6} \cdot D_{27,6} = \frac{15!}{(15-6)!} \cdot \frac{27!}{(27-6)!} = \frac{15!}{9!} \cdot \frac{27!}{21!}$.